Reduced-Complexity Cluster Modelling for the 3GPP Channel Model

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Abstract—The realistic performance of a multi-input multi-output (MIMO) communication system depends strongly on the spatial correlation properties introduced by clustering in the propagation environment. Simulating realistic correlated channels is essential to predict the performance of real MIMO systems. Since the modeling method of the correlated channels suggested by the Third Generation Partnership Project (3GPP) channel model can result in considerable implementation complexity for large networks, this paper presents a computationally efficient method to approximately calculate the spatial correlation matrix for channel models such as the 3GPP channel model, which are based on clusters of scatterers. This proposed approximation method is on the basis of using the Taylor series expansion to the steering vectors for uniform linear arrays (ULAs) and a moderate angle spread of the cluster. The approximation method is evaluated in terms of the mean square error (MSE) of the approximated correlation matrices generated by the proposed approximation method and by the exact method suggested by the 3GPP channel model. The mean square error (MSE) metric is used to measure this difference. The cumulative distribution function (CDF) of mutual information is also calculated by the direct approximation to the spatial correlation matrix for the MIMO channels, with much lower computational complexity [16].

Our main contribution is to propose a computationally efficient method for simulating the spatial correlation matrix and channel matrix for the MIMO channels. The key insight is that an approximation of the steering vectors for uniform linear arrays (ULAs) by the Taylor series expansion approach and moderate angle spreads allow us to deduce a closed-form approximation to the spatial correlation matrix for the MIMO channels which depends only on the moments of the scatterer distributions. Instead of using the full number of the multi-path components, we simulate the spatial correlated MIMO channels on a cluster-by-cluster basis with each cluster modelled by a few terms, hence greatly reducing computation time. Because the spatial correlation matrix is a function of clusters, which are random, system level simulations will require averaging over many spatial correlation matrix realizations. The advantages of the proposed method make it particularly suitable for large networks, where many users and channels need to be simulated.

To evaluate the accuracy, validity and reliability of the proposed method, we compare the difference of the spatial correlation matrices generated by the proposed approximation method and by the exact method suggested by the 3GPP channel model. The mean square error (MSE) metric is used to measure this difference. The cumulative distribution function (CDF) of mutual information is also calculated by the direct method and using the approximation. It is shown that, if the angle spread of the cluster is within $10^\circ$, the difference is negligible. Therefore, the proposed approximation method can calculate the spatial correlation matrix for MIMO channels with acceptable accuracy and low complexity, provided the angle spreads of the clusters are within $10^\circ$.

Keywords—MIMO, Cluster Modelling, Reduced-Complexity Modelling, 3GPP Channel Model

I. INTRODUCTION

Multi-input multi-output (MIMO) has emerged as one of the most promising future technologies in mobile radio communications. The use of multiple antennas at both link ends in wireless communications promises high spectral efficiency and reliability. Although the theoretical properties of MIMO communication systems have been acknowledged for some time, the pragmatic application of MIMO communication in realistic propagation channels has not been developed until recently [1]-[4]. These results show that realistic MIMO channels have significant spatial correlation due to the distribution of scatterers in the propagation environment. In general, spatial correlation has an adverse effect on capacity and error rate performance, and it has been shown in [5] that channel models disregarding clustering effects overestimate channel capacity. Thus simulating realistic correlated channels is essential to predict the performance of real MIMO systems.

In this paper, we use the Third Generation Partnership Project (3GPP) channel model to simulate spatially correlated MIMO channels. The 3GPP channel model is one of the geometry-based stochastic channel models [6]-[9] that adopts the concept of scattering clusters (referred to in the 3GPP document [6] as ‘paths’) containing a number of stochastically varying multi-path components (‘sub-paths’ in the 3GPP terminology). It requires defined directions and amplitudes for all multi-path components to generate each channel realization, which can result in considerable implementation complexity for large networks. Therefore, we propose a practical alternative to the exact calculation suggested by the 3GPP channel model to simulate the spatial correlation matrix for the MIMO channels, with much lower computational complexity [16].
The paper is organised as follows: some background on the 3GPP channel model and the exact method it provides to simulate the channel matrix and spatial correlation matrix are introduced in Section II. Section III presents the Taylor series expansion approach to simplify the modelling of a cluster. The evaluation of this approximation using the MSE metric and its application to calculating the CDF of the mutual information of the MIMO channels are described in Section IV. Finally, the conclusions are drawn in Section V.

II. EXACT CALCULATIONS OF THE CHANNEL MATRIX AND SPATIAL CORRELATION MATRIX DEFINED BY THE 3GPP CHANNEL MODEL

In the 3GPP channel model, there is a fixed number of 6 “paths” in every scenario, each of the paths being made up of 20 spatially separated multi-path components [6]. Therefore the 3GPP channel model requires the generation of 120 multi-path components with defined directions and amplitudes for each channel realization. Here we use \( n_p \) to denote the total number of ‘paths’, which correspond to clusters in this paper, and \( n_s \) to stand for the number of multi-path components (‘sub-paths’ in 3GPP) in each path. The MIMO channel matrix for the channels with \( n_T \) transmit antennas and \( n_R \) receive antennas is defined as:

\[
H_{n_R,n_T} = \Psi_R \Xi \Psi_T^\dagger = \sum_{p=1}^{n_p} \xi_p \Phi_R \phi_{R,p} \Phi_T^\dagger \phi_{T,p} \tag{1}
\]

where \( H_{n_R,n_T} \) denotes the \((n_R - n_T)\) channel matrix with \( n_s \cdot n_p \) multi-path components involved in every channel realization. The matrices \( \Psi_R \) and \( \Psi_T \) denote the steering vector matrices at the receiver side and the transmitter side respectively, in this paper, a ULA is used for both transmitter and receiver side, therefore they have dimensions \((n_T - n_s) \cdot n_p\) and \((n_r - n_s) \cdot n_p\) individually, and their columns are the vectors \( \Phi_R \phi_{R,p} \) and \( \Phi_T \phi_{T,p} \) respectively, \( p = 1 \ldots n_p \cdot n_s \), \( \phi_{R,p} \) and \( \phi_{T,p} \) stand for the angle-of-arrival (AoA) and angle-of-departure (AoD) for the \( p \)th multi-path component respectively. The matrix \( \Xi \) is an \((n_s \cdot n_p - n_s \cdot n_p)\) diagonal matrix containing the multi-paths’ gains, with diagonal elements \( \xi_p \).

In this paper, the following notations are defined as: the symbol \((.)^\dagger\) means matrix transposition; \((.)^\ast\) stands for complex conjugation; \((.)^H\) stands for matrix Hermitian; \((.)^\odot\) denotes the expectation of the given term, \( \text{vec}(.) \) is to vectorize a given matrix: that is, to form a vector by stacking the columns of the matrix.

The definitions of \( \Phi_R(\phi_{R,p}) \), \( \Phi_T(\phi_{T,p}) \) and \( \xi_p \) are as follows [10]:

\[
\Phi_R(\phi_{R,p}) = \left\{ \exp\left(2\pi \frac{il_R}{\lambda} \sin(\phi_{R,p})\right) : i = 1 \ldots n_R \right\} \tag{2}
\]

\[
\Phi_T(\phi_{T,p}) = \left\{ \exp\left(2\pi \frac{kl_T}{\lambda} \sin(\phi_{T,p})\right) : k = 1 \ldots n_T \right\} \tag{3}
\]

where \( \lambda \) is the wavelength of the radio wave, \( l_R \) and \( l_T \) denote the antenna intervals in the ULA at the receiver side and transmitter side respectively. In this paper we assume half-wavelength spacing. The variables \( i \) and \( k \) are used to denote the positions of the antenna elements in the antenna arrays at the receiver side and transmitter side respectively.

\[
\xi_p = \sqrt{P_p \cdot G_R(\phi_{R,p})} e^{j\theta_p} \tag{4}
\]

where \( P_p \) stands for the power of the \( p \)th multi-path component, \( G_R(\phi_{R,p}) \) and \( G_R(\phi_{R,p}) \) are the antenna gains of the transmit antenna and receive antenna respectively for the \( p \)th multi-path component, \( \phi_{p} \) denotes the phase of the \( p \)th multi-path component arriving at the receiver. In this paper we assume omnidirectional antenna elements.

Since we will present the method to simulate the spatial correlated MIMO channels on a cluster-by-cluster basis with each cluster modelled by a few terms in Section III, in the following, we provide the exact calculations of the channel matrix and spatial correlation matrix for only one of the \( n_p \) clusters involved.

The exact calculation of the MIMO channel matrix for one cluster involved can be rewritten as:

\[
H_{n_R,n_T} = \Psi_R \Xi \Psi_T^\dagger = \sum_{p=1}^{n_p} \xi_p \Phi_R \phi_{R,p} \Phi_T^\dagger \phi_{T,p} \tag{5}
\]

The channel realization between the \( a \)th receive antenna and the \( b \)th transmit antenna is calculated as:
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where $\phi_{R.0}$ stands for the center AoA of the cluster, $\Psi_{R.a}(\phi_{R.0})$, $\Psi_{R.c}(\phi_{R.0})$ and $\Psi_{R.d}(\phi_{R.0})$ are the first, second and third order differentials of $\Psi_{R.a}(\phi_{R.p})$ at $\phi_{R.0}$.

Secondly we substitute (2) for the terms in (9), which will lead (9) to a power series form:

$$
\Psi_{R.a}(\phi_{R.p}) = \Psi_{R.a}(\phi_{R.0}) \left( 1 + \alpha_{1.a}(\phi_{R.0}) (\phi_{R.p} - \phi_{R.0}) + \cdots \right)
$$

where the detailed forms of $\alpha_{1.a}(\phi_{R.0})$, $\alpha_{2.a}(\phi_{R.0})$.. are calculated by MAPLE [15], and, assuming $\lambda/2$ element spacing:

$$
\alpha_{1.a}(\phi_{R.0}) = 2 \pi \cos(\phi_{R.0}) \lambda
$$

Similarly, the terms $\Psi_{T.b}(\phi_{T.p})$, $\Psi_{R.c}(\phi_{R.p})$ and $\Psi_{R.d}(\phi_{T.p})$ in (8) can also be expressed as power series forms. The highest order of the power series in (10) is directly determined by the order used in the Taylor series expansion.

B. The Approximation Method to Simulate the Spatial Correlation Matrix

We use the power series forms to substitute for the corresponding terms in (8), then the approximated calculation of $R_{ab,cd}$ can be deduced, as shown in (11).

$$
R_{ab,cd} = \left[ \begin{array}{c}
1 + A_{10}(\phi_{R.0}) \sum_{p=1}^{n_r} \xi_p^2 (\Delta \phi_{R,p})^2 + \cdots \\
A_{11}(\phi_{T.0}) \sum_{p=1}^{n_s} \xi_p^2 (\Delta \phi_{T,p})^2 + \cdots \\
A_{20}(\phi_{R.0}) \sum_{p=1}^{n_r} \xi_p^2 (\Delta \phi_{R,p})^2 + \cdots \\
A_{21}(\phi_{T.0}) \sum_{p=1}^{n_s} \xi_p^2 (\Delta \phi_{T,p})^2 + \cdots \\
\end{array} \right]
$$

III. THE APPROXIMATION METHOD TO SIMULATE THE SPATIAL CORRELATION MATRIX AND CHANNEL MATRIX

A. The Approximation of the Steering Vectors for ULAs by the Taylor Series Expansion Approach

If the angle spread of the cluster is moderate, then the theory of Taylor series [11] can be applied into the steering vector functions (2) and (3), and the steering vectors for all the multi-path components within that cluster can be calculated with respect to the center angle of the cluster. In the following, the approximation of the steering vectors for ULAs by the Taylor series expansion approach will be illustrated in detail.

First of all, according to the Taylor series expansion, we can express the calculation of the steering value for the $p^{th}$ multi-path component at the $a^{th}$ receive antenna in (2) in the form of:

$$
\Psi_{R,a}(\phi_{R,p}) = \Psi_{R,a}(\phi_{R,0}) + \Psi'_{R,a}(\phi_{R,0})(\phi_{R,p} - \phi_{R,0}) + \Psi''_{R,a}(\phi_{R,0})(\phi_{R,p} - \phi_{R,0})^2 + \cdots
$$

$$
= \Psi_{R,a}(\phi_{R,0}) \Psi_{R,b}(\phi_{R,0}) (\phi_{R,p} - \phi_{R,0}) + \Psi''_{R,a}(\phi_{R,0})(\phi_{R,p} - \phi_{R,0})^2 + \cdots
$$
In (11), \( \hat{R}_{ab,cd} \) stands for the approximated value of \( R_{ab,cd} \), \( \Delta \phi_{R,p} \), \( \Delta \phi_{T,p} \) denote the offsets of the \( p \)\( ^{th} \) multi-path component with respect to the center AoA and AoD individually, \( A_{00}(\phi_{R,0}) \) stands for the coefficient of the term that includes the power-weighted first order of \( \Delta \phi_{R,p} \) and the zero-th order of \( \Delta \phi_{T,p} \), that is, the subscript indices of \( A \) are made up of two digits, the first one denotes the order of \( \Delta \phi_{R,p} \), the second one denotes the order of \( \Delta \phi_{T,p} \). In the expression of \( \hat{R}_{ab,cd} \), the polynomial terms are sorted in the ascending order of the total order of variables \( \Delta \phi_{R,p} \) and \( \Delta \phi_{T,p} \). The values of \( A \)s are obtained by firstly expanding the multiplication of the four power series terms, secondly sorting the polynomial terms in the ascending order and thirdly combining the polynomial terms with the same variable part together to extract their comprehensive coefficient. Thus we can see that the calculation of \( A \)s are only in terms of the center AoA, center AoD of the cluster, and the positions of the antenna elements in the antenna arrays.

Obviously, for the cluster with a moderate angle spread, the higher order used in the Taylor series expansion, the greater the total number of the polynomial terms will be in the calculation of \( \hat{R}_{ab,cd} \), thus the value of \( \hat{R}_{ab,cd} \) will be closer to that of \( R_{ab,cd} \). However, the calculation of \( \hat{R}_{ab,cd} \) with a large number of polynomial terms included in (11) will result in significant computational complexity. For example, with the second order Taylor series expansion, the total number of the polynomial terms in (11) is 25; if the third order Taylor series expansion is used, the total number of the polynomial terms in (11) will be 49; the fourth order Taylor series expansion can lead to 81 polynomial terms in total included in (11). Interestingly, whatever the order of the Taylor series expansion used is, the first six polynomial terms in (11) are the same, which suggests that it may be sufficient to use these six terms only. They are the constant, \( A_{00}(\phi_{R,0}) \sum_{p=1}^{n_p} \xi_p^2 (\Delta \phi_{R,p}) \),

\[ A_{01}(\phi_{T,0}) \sum_{p=1}^{n_p} \xi_p^2 (\Delta \phi_{T,p}) \], \( A_{02}(\phi_{R,0}) \sum_{p=1}^{n_p} \xi_p^2 (\Delta \phi_{R,p})^2 \), \( A_{03}(\phi_{T,0}) \sum_{p=1}^{n_p} \xi_p^2 (\Delta \phi_{T,p})^2 \), and \( A_{11}(\phi_{R,0},\phi_{T,0}) \sum_{p=1}^{n_p} \xi_p^2 (\Delta \phi_{R,p},\Delta \phi_{T,p}) \). Note that the summations are, respectively, the first moments, the second moments and the product moment of the distributions of the AoA’s and AoD’s. Therefore, we are interested in whether these six terms are sufficient to calculate \( \hat{R}_{ab,cd} \), and if so, in what angle spread range of the cluster the approximation method works well. The answers to these questions will be given in Section IV.

C. The Generation of the MIMO Channel Matrix Reconstructed from the Spatial Correlation Matrix

The calculation of the MIMO channel matrix reconstructed from the spatial correlation matrix can be expressed as follows [12]:

\[
\mathbf{H}_{n_R,n_T} = \mathbf{R}_T^{1/2} \mathbf{H}_w \mathbf{R}_R^{1/2}
\]

(12)

where \( \mathbf{H}_w \) is an \( (n_R - \text{by} - n_T) \) matrix of complex Gaussian coefficients, \( \mathbf{R}_R \) and \( \mathbf{R}_T \) are the spatial correlation matrices at the receiver side and transmitter side respectively, expressing the correlation of the receive/transmit signals across the array elements. The channel correlation between a pair of receive antennas denoted as \( R_{R,T} \) is defined as:

\[
R_{R,T} = \sum_{k=1}^{n_R} \sum_{p=1}^{n_p} \xi_p^2 \Psi_{R,p}(\phi_{R,p}) \Psi_{T,k}(\phi_{T,k}) \Psi_{R,p}'(\phi_{R,p}) \Psi_{T,k}'(\phi_{T,k})
\]

(13)

Using the approximation method above, we can calculate the approximated values of \( R_R \) and \( R_T \) denoted as \( \hat{R}_R \) and \( \hat{R}_T \) respectively, hence the approximated value of the MIMO channel matrix denoted as \( \hat{H}_{n_R,n_T} \) can be generated using (12).

IV. THE EVALUATION OF THE APPROXIMATION METHOD

A. The Evaluation of the Approximation Method by the MSE Metric

The mean square error (MSE) metric [13] is used to measure the performance degradations caused by the approximate calculation of the spatial correlation matrix, as a function of the number of truncated polynomial terms used. The spatial correlation matrix generated from the exact method suggested by the 3GPP channel model is taken as reference for the performance analysis, since this method does not use any approximation. The calculation of the MSE in units of dB is defined as:

\[
mse = 10 \log_{10} \left( \sum_{a=1}^{n_R} \sum_{b=1}^{n_T} \sum_{c=1}^{n_R} \sum_{d=1}^{n_T} \left( \text{vec}(R_{ab,cd}) - \text{vec}(\hat{R}_{ab,cd}) \right)^2 \right)
\]

(14)

The simulation results of the MSE versus the number of the truncated polynomial terms in (11) are shown in Fig.2.
that the first six terms are used, as mentioned in Section III B; absissa 10 implies that the first ten terms are used, which includes $|\xi_p|^2 (\Delta \phi_{R,p} \Delta \phi_{T,p})^2$, $|\xi_p|^2 (\Delta \phi_{R,p'})^2 (\Delta \phi_{T,p'})^2$ besides the first six terms; absissa 15 means that the first fifteen terms are used, which consists of terms $|\xi_p|^2 (\Delta \phi_{R,p} \Delta \phi_{T,p})^3$, $|\xi_p|^2 (\Delta \phi_{R,p'})^3 (\Delta \phi_{T,p'})^3$ and the first ten terms; etc. In Fig.2, from (a) to (c), the angle offsets of the cluster components are drawn from a uniform random distribution on $(-0.5 \sim 0.5)$ degrees, $(-3 \sim 3)$ degrees and $(-5 \sim 5)$ degrees respectively.

Fig. 2 shows that the MSE increases with angle spread, and, in general decreases with increasing number of terms included in the approximation (11) and with increasing order of the Taylor series. However for large angular spread and low order Taylor series the error increases with number of terms beyond 6 or 10 terms. This is because of the mismatch between the order of the Taylor series and the number of terms in total: the higher order terms are not accurate. However we note that even for angle spread $10^\circ$ and fourth order series the reduction in error for more than 6 terms is negligible. This is because for a symmetrical distribution of multi-path components within the cluster the third order moments are generally small. Also the reduction in error between 1 and 3 terms is very small: this is because the first order moments are very small if the nominal AoA/AoD of the cluster is close to the actual mean AoA of the component multipaths. For a 6 term approximation, compared to a single term the error is reduced by 60dB for $1^\circ$ angle spread, by 40 dB for $6^\circ$, and by 30 dB for $10^\circ$.

This suggests that 6 terms give the optimum trade-off between accuracy and complexity, and that this approximation is adequate for angle spread up to $10^\circ$, for ULA’s with half-wavelength antenna element spacing. In the following section, we apply this approximation method to calculate the CDF of the mutual information of MIMO channels, to illustrate its utility.

B. The Performance of the Approximation Method in Calculating the CDF of the Mutual Information of MIMO Channels

The mutual information of a MIMO channel is defined by the function below [14]:

$$
C = W \log_2 \det \left( I + \frac{S}{N} \mathbf{H}_{\eta g,\eta s} \mathbf{H}^H_{\eta g,\eta s} \right)
$$

where $S/N$ is the signal-to-noise ratio (SNR), and $I$ is the identity matrix with dimensions $(n_R - b y - n_R)$, the bandwidth $W$ is set to unit so that $C$ denotes the bandwidth efficiency in units of $\text{bit/s/Hz}$; $\mathbf{H}_{\eta g,\eta s} \mathbf{H}^H_{\eta g,\eta s}$ is the channel correlation matrix at the receiver side, which we will calculate on a cluster-by-cluster basis referring to (13). The simulations are for a MIMO system having two transmit and two receive
antennas, with antenna elements in the arrays 2/3 spaced apart. We compare the simulated CDFs of the mutual information predicted by \( \hat{R}_{ab,cd} \) and \( R_{ab,cd} \). In Fig.3, from (a) to (b), the simulated CDFs are depicted for angle spreads of 6° and 10° respectively, and the SNR is 10 dB. For each case, the CDF deduced from \( \hat{R}_{ab,cd} \) is calculated in three ways to show the influence of the number of the truncated polynomial terms on the performance of the approximation calculation.

The simulation results in Fig.3 show that in all these cases using the first six truncated polynomial terms in (11) can provide an estimate of the CDF of the mutual information very close to the exact prediction. Although some error remains, the computational complexity has been greatly reduced. Six terms must be calculated per cluster, instead of 20 in the 3GPP model.

V. CONCLUSIONS

We have presented a computationally efficient method based on the 3GPP channel model to approximately calculate the spatial correlation matrix for the MIMO channels, for ULA antennas and angle spread of the cluster within 10° (assuming half-wavelength element spacing). By evaluating the approximation method with the MSE metric and analyzing its performance in calculating the CDF of the mutual information of the MIMO channels, we found that the proposed approximation method can generate the spatial correlation matrix close to that generated by the exact method suggested by the 3GPP channel model. The computational reduction is significant.

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Comparison of the CDFs of the mutual information (outage capacity) predicted by the exact method and the approximation method.