A new hybrid particle swarm optimization approach for structural design optimization in the automotive industry

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Abstract
This paper presents an innovative optimization approach to solve structural design optimization problems in the automotive industry. The new approach is based on Taguchi’s robust design approach and particle swarm optimization algorithm. The proposed approach is applied to the structural design optimization of a vehicle part to illustrate how the present approach can be applied for solving design optimization problems. The results show the ability of the proposed approach to find better optimal solutions for structural design optimization problems.

Keywords
Structural design optimization, particle swarm optimization, Taguchi’s method, robust design

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Introduction
The optimal design of structures includes sizing, shape and topology optimization. There has been extensive research focused on shape optimization due to its great contribution to cost, material and time savings in the procedures of the engineering design. The purpose of shape optimization is to determine the optimal shape of a continuum medium to maximize or minimize a given criterion (often called an objective function), such as minimize the weight of the body, maximize the stiffness of the structure or remove the stress concentrations, subjected to the stress or displacement constraint conditions.

Computer-aided optimization has been commonly used to obtain more economical designs. Numerous algorithms have been developed to solve structural design optimization problems in the last four decades. The early works on this topic mostly use various mathematical techniques. These methods are not only time consuming in solving complex nature problems, but also they may not be used efficiently for finding global or near global optimum solutions. In the past few decades, a number of innovative approaches, such as tabu search, genetic algorithm, simulated annealing, particle swarm optimization algorithm, ant colony algorithm and immune algorithm, have been developed and widely applied in various fields of science.1–13

Fast convergence speed and robustness in finding the global minimum are not easily achieved at the same time. Fast convergence requires a minimum number of calculations, increasing the probability of missing important points; on the other hand, the evaluation of more points for finding the global minimum decreases the convergence speed. This leads to the question: ‘how to obtain both fast convergence speed and global search capability at the same time?’ There have been a number of attempts to answer this question, while hybrid algorithms have shown outstanding reliability and efficiency in application to the engineering optimization problems.14–20

Therefore, researchers are paying great attention to hybrid approaches in order to answer this question, particularly to avoid premature convergence towards a local minimum and to reach the global optimum results.

There is an increasing interest in applying the new approaches and in further improving the performance of optimization techniques for the solution of structural design optimization problems. Although

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some improvements regarding structural design optimization issues are achieved, the complexity of design problems presents shortcomings. The main goal of the present research is to develop an effective approach to solve real-world design optimization problems. A new hybrid approach based on robustness issues is used to help better initialize particle swarm optimization algorithm search. The aim has been to reach optimum designs by using Taguchi’s robust parameter design approach coupled with particle swarm optimization algorithm. In this new hybrid approach, signal-to-noise (S/N) values are calculated and ANOVA (analysis of variance) tables for each of the objectives are formed using S/N ratios, respectively. According to results of ANOVA tables, appropriate interval levels of design parameters are found, and then initial search population of the particle swarm optimization algorithm process is defined according to these interval levels. Optimum results of the design problem are then obtained using particle swarm optimization algorithm.

The developed new hybrid optimization approach is applied to a vehicle part design optimization problem taken from the automotive industry to demonstrate the application of the present approach to real-world design problems. The results of the proposed approach show that the proposed optimization method converges rapidly to the global optimum solution and provides reliable and accurate solutions.

The organization of the paper is as follows. The third section presents a detailed review on optimization approaches. The particle swarm optimization (PSO), Taguchi’s method and the proposed hybrid approach are presented in the third section. A case study from automobile industry is solved in the fourth section. The paper is concluded in the fifth section.

Literature review

Recently, new approaches in the area of optimization research are presented to further improve the solution of optimization problems with complex nature.

Over the past few years, studies on evolutionary algorithms have shown that these methods can be efficiently used to eliminate most of the difficulties of classical methods. Evolutionary algorithms are widely used to solve engineering optimization problems with complex nature. Various research works have been carried out to enhance the performance of evolutionary algorithm.1–23

For instance, Yildiz and Saitou2 developed a novel approach for multi-component topology optimization of continuum structures using a multi-objective genetic algorithm to obtain Pareto optimal solutions that exhibits trade-offs among stiffness, weight, manufacturability and assemble ability. Yildiz and Saitou present a method for synthesizing structural assemblies directly from the design specifications without going through the two-step process. Given an extended design domain with boundary and loading conditions, the method simultaneously optimizes the topology and geometry of an entire structure and the location and configuration of joints considering structural performance, manufacturability and assemble ability. The developed approach is applied to multi-component topology optimization of a vehicle floor frame.

The PSO algorithm originally developed by Eberhart and Kennedy24 is inspired with social behavior, such as bird flocking or fish schooling, which is used successfully in the solution of optimization problems. The PSO algorithm has been used in many areas of optimization studies. The use of PSO algorithms in the optimum solution of problems resulted in better solutions compared to classical methods.25–32

The PSO algorithm was applied to the shape optimization of a torque arm and to the size optimization of truss structures taken from literature by Fourie and Groenwold.6 In their PSO algorithm, the concept of craziness is redefined and elitism operator borrowed by GA was used. Their results showed that PSO algorithms were better than GA and the gradient-based recursive quadratic programming algorithm.

Perez and Behdinan26 proposed a particle swarm approach for structural design optimization. The effectiveness of the improved PSO algorithm on structural optimization is shown through the use of four classical truss optimization examples. Results from the three tested cases using an improved PSO illustrate the ability of the algorithm to find optimal results which are better than, or at the same level of, other structural optimization methods.

Venter and Sobieszczanski-Sobieski32 used PSO algorithm for multidisciplinary optimization of a transport aircraft wing.

Some researchers have used the robustness issues to solve optimization problems.33–37 Robinson et al.38 present a review paper which focuses largely on the work done since 1992 and a historical perspective of parameter design is also given.

Kunjur and Krishnamurty39 presented a robust optimization approach that integrates optimization concepts with statistical robust design techniques. Although Taguchi’s methods have been successfully applied to processes in the design and manufacturing, they are also criticized for their efficiency.36,40

Hybrid methods are also used to enhance the performance of evolutionary algorithm. Yildiz et al.15 developed a hybrid robust genetic algorithm (HRGA) based on Taguchi’s method and genetic algorithm. After the approach was validated by multi-objective welded beam design problems, it was applied to structural design optimization problem of an automobile component from industry.

The immune algorithm is hybridized with a hill-climbing local search algorithm by Yildiz.16 and the hybrid approach is applied to multi-objective disc brake and manufacturing optimization problems from the literature.
Yildiz\textsuperscript{18} developed a new hybrid particle swarm optimization approach to solve optimization problems in design and manufacturing areas by hybridizing the particle swarm algorithm with receptor editing property of immune system.

Tsai et al.\textsuperscript{41} proposed a hybrid algorithm in which the Taguchi’s method is inserted between crossover and mutation operations of a genetic algorithm. The Taguchi method is incorporated in the crossover operations to select the better genes to achieve crossover and, consequently, enhance the performance of the genetic algorithm.

It is known that the PSO algorithm is more efficient than genetic algorithm (GA) at exploring the solution space, but it does not guarantee the global optimum as in other evolutionary methods. The introduction of hybrid methods comes from the increasing need to tackle complex real-world problems. Some of the hybrid approaches in literature have been made on the hybrid particle swarm.\textsuperscript{20,42,43}

Yildiz and Solanki\textsuperscript{20} developed a new hybrid optimization approach based on PSO and immune algorithm. The proposed approach is used for multi-objective optimization of vehicle crashworthiness.

Fan et al.\textsuperscript{42} proposed a hybrid approach algorithm based on the Nelder–Mead simplex search method and PSO algorithm. The approach was applied to the optimization of multi-modal functions. Their results indicated that the proposed algorithm was better than hybrid GA, continuous GA, simulated annealing (SA) and tabu search in finding optimal solutions for multi-modal functions.

Xia and Wu\textsuperscript{43} proposed a hybrid approach based on the hybridization of the PSO and SA and they applied this to the multi-objective flexible job-shop scheduling problem as a case study.

The proposed hybrid approach is applied to the design optimization of a vehicle component to illustrate how the present approach can be applied for solving structural design optimization problems.

**Hybrid particle swarm optimization algorithm for structural optimization**

In this paper, a new hybrid optimization approach, named HRPSO (hybrid robust particle swarm optimization algorithm), is developed to solve structural design optimization problems. In the proposed optimizations approach, the refinement of the population space is introduced by Taguchi’s method. The bounds selected on the design variables are first used for the initial swarm, then they are applied throughout particle swarm algorithm for finding optimal design parameters. The aim is to overcome the limitations caused by larger population regarding computational cost and quality of solutions for global optimization. First, some brief explanations about the particle swarm optimization algorithm and Taguchi’s method are given and, finally, the proposed hybrid approach is explained.

**Particle swarm optimization algorithm**

The PSO algorithm was developed by Eberhart and Kennedy.\textsuperscript{24} It is a biologically inspired algorithm which models the social dynamics of bird flocking. A large number of birds flock synchronously, change direction suddenly, scatter and regroup iteratively, and finally perch on a target. This form of social intelligence not only increases the success rate for food foraging but also expedites the process. The PSO algorithm facilitates simple rules simulating bird flocking and serves as an optimizer for continuous non-linear functions.

The general principles of the PSO algorithm are outlined as follows.

- **Particle representation.** The particle in the PSO is a candidate solution to the underlying problem and moves iteratively about the solution space. The particle is represented as a real-valued vector containing an instance of all parameters that characterize the optimization problem. We denote the \textit{i}th particle by $P_i = (p_{i1}, p_{i2}, \ldots, p_{id})^T \in \mathbb{R}^d$ where $d$ is the number of parameters.

- **Swarm.** The PSO explores the solution space by flying a number of particles, called swarm. The initial swarm is generated at random and the size of swarm is usually kept constant through iterations. At each iteration, the swarm of particles search for target optimal solution by referring to previous experiences.

- **Personal best experience and swarm’s best experience.** The PSO enriches the swarm intelligence by storing the best positions visited so far by every particle. In particular, particle \textit{i} remembers the best position among those it has visited, referred to as \textit{pbest}, and the best position by its neighbors. There are two versions for keeping the neighbors’ best position, namely \textit{lbest} and \textit{gbest}. In the local version, each particle keeps track of the best position \textit{lbest} attained by its local neighboring particles. For the global version, the best position \textit{gbest} is determined by any particles in the entire swarm. Hence, the \textit{gbest} model is a special case of the \textit{lbest} model.

- **Particle movement.** The PSO is an iterative algorithm according to which a swarm of particles fly about the solution space until the stopping criterion is satisfied. At each iteration, particle \textit{i} adjusts its velocity $v_{ij}$ and position $p_{ij}$ through each dimension \textit{j} by referring to, with random multipliers, the personal best position ($p_{besti}$) and the swarm’s best position ($l_{besti}$, if the local version is adopted) using equations (1) and (2) as follows

$$v_{ij} = v_{ij} + c_1 r_1 (p_{bestj} - p_{ij}) + c_2 r_2 (l_{bestj} - p_{ij}) \tag{1}$$
and

\[ p_{ij} = p_{ij} + v_{ij} \]  

(2)

where \( c_1 \) and \( c_2 \) are the cognitive coefficients and \( r_1 \) and \( r_2 \) are random real numbers drawn from \( U(0, 1) \). Thus, the particle flies toward \( p_{best} \) and \( l_{best} \) in a navigated way while still exploring new areas by the stochastic mechanism to escape from local optima. Clerc and Kennedy\(^{25}\) have pointed out that the use of a constriction factor is needed to ensure convergence of the algorithm by replacing equation (2) with the following

\[ v_{ij} = K \left( v_{ij} + c_1 r_1 (p_{best_{ij}} - p_{ij}) + c_2 r_2 (l_{best_{ij}} - p_{ij}) \right) \]  

(3)

and

\[ K = \frac{2}{2 - \phi - \sqrt{\phi^2 - 4\phi}} \]  

(4)

where \( \phi = c_1 + c_2, \phi > 4 \). Typically, \( \phi \) is set to 4.1 and \( K \) is thus 0.729.

- **Stopping criterion.** The PSO algorithm is terminated with a maximal number of iterations or the best particle position of the entire swarm cannot be improved further after a sufficiently large number of iterations.

**Taguchi method**

The Taguchi method is a universal approach, which is widely used in robust design (Robinson et al.,\(^{38}\) Tsai et al.\(^{41}\) and Phadke\(^{45}\)). There are three stages to achieve Taguchi’s objective: (1) concept design, (2) robust parameter design (RPD), and (3) tolerance design. The robust parameter design is used to determine the levels of factors and to minimize the sensitivity of noise. That is, a parameter setting should be determined with the intention that the product response has minimum variation while its mean is close to the desired target. Taguchi’s method is based on statistical and sensitivity analysis for determining the optimal setting of parameters to achieve robust performance. The responses at each parameter setting were treated as a measure that would be indicative of not only the mean of some quality characteristic, but also the variance of that characteristic. The mean and the variance would be combined into a single performance measure known as the S/N ratio. Taguchi classifies robust parameter design problems into different categories depending on the goal of the problem and for each category as follows:

**Smaller the better.** For these kind of problems, the target value of \( y \), that is, quality variable, is zero. In this situation, S/N ratio is defined as follows

\[ S/N = -10 \log \left( \frac{1}{n} \sum y_i^2 \right) \]  

(5)

**Larger the better.** In this situation, the target value of \( y \), that is, quality variable, is infinite and S/N ratio is defined as

\[ S/N = -10 \log \left( \frac{1}{n} \sum \frac{1}{y_i^2} \right) \]  

(6)

**Nominal the best.** For these kind of problems, the certain target value is given for \( y \) value.

In this situation S/N ratio is defined as

\[ S/N = -10 \log \left( \sum \frac{1}{y_i^2} \right) \]  

(7)

Taguchi’s method uses an orthogonal array and analysis of mean to analyze the effects of parameters based on statistical analysis of experiments. To compare performances of parameters, the statistical test known as the analysis of variance (ANOVA) is used. Further details and technical merits about robust parameter design can be found in work by Phadke\(^{45}\).

Most of the structural design optimization problems in the automotive industry have usually uncontrollable variations in their design parameters with complex nature. There is a need to overcome the shortcomings due to the traditional optimization methods and also to further improve the strength of recent approaches to achieve better results for the real-world design problems. Therefore, in this research, an integrated approach to solve structural design optimization problem is proposed based on Taguchi’s parameter design and particle swarm optimization algorithm. The architecture of proposed hybrid approach is given in Figure 1.

Taguchi’s robust parameter design is introduced to help to define robust initial population levels of design parameters and to reduce the effects of noise factors to achieve better initialize particle swarm optimization algorithm search. The problem with larger populations is that the particle swarm optimization algorithm may
tend to converge and stick around certain solutions which may not be the best ones. This is handled with the help of robust parameter levels which are embedded into particle swarm optimization algorithm process as being initial population intervals. In other words, the design space is restricted and refined based on the effect of the various design variables on objective functions. The purpose of the ANOVA tables is to help differentiate the robust designs from the non-robust ones.

Finally, optimum results of multi-objective problem are obtained by applying particle swarm optimization algorithm through cloning, mutation and receptor editing operations. The present approach is considered in two stages as follows:

1. Stage 1: Determine efficient solution space for structural design optimization variables using Taguchi’s method.
2. Stage 2: Apply particle swarm optimization algorithm to find structural design variables.

In the first stage, Taguchi’s robust parameter design procedure is used to find the levels of variables for efficient search space as follows:

- identify the objectives, constraints and design parameters;
- determine the settings of the design parameter levels;
- conduct the experiments using orthogonal array;
- compute S/N ratios and ANOVA analysis;
- find the optimal settings of design parameters.

The main issue of experimental analysis is the ANOVA analysis which is formed using S/N ratios for each of the objectives. According to the results of ANOVA, appropriate levels of design parameters are found, and then the initial search population of particle swarm optimization algorithm process is defined according to levels.

Finally, optimum results of the optimization problem are obtained by applying particle swarm optimization algorithm process in two steps as follows:

- define initial population set;
- use particle swarm operators to create the next generation;
- evaluate objective function and constraints;
- repeat the loop until the optimum solutions are found.

In this paper, a new hybrid approach is proposed to improve the performance of the particle swarm optimization algorithm. Our argument behind the proposed approach is that the strength of one algorithm can be used to improve the performance of another approach in the optimization process. The combination of Taguchi’s method and particle swarm optimization algorithm results in a solution which leads to better parameter values for structural design optimization problems. The algorithm of proposed hybrid approach can be outlined as follows:

BEGIN
Step 1: Taguchi’s method
Begin
  1.a Choose convenient orthogonal array from Taguchi’s standard orthogonal arrays
  1.b Define levels and intervals
  1.c For i:= 1 to NOE (number of experiments) do begin
    Compute objective function values end;
  1.d Choose convenient S/N ratio type (smaller the best or larger the best or nominal the best) based on minimization or maximization of objective functions
  1.e For i:= 1 to NOE do begin
    Compute S/N ratios end;
  1.f Constitute Anova table for objective functions using S/N ratios
  1.g Determine optimum levels and intervals using percentage contribution to performance using Anova table
Use these levels and intervals for forming initial population end;
  ‘Generate optimal solution set using particle swarm optimization algorithm and computed robust initial population space’
Begin
Input
Use Initial population found in previous part of the program as input to particle swarm optimization algorithm
Step 2: Particle swarm algorithm
  2.a Generate initial swarm population with random positions and velocities
While maximum iterations or minimum error criteria is not attained
  for i:= 1 to NOP (number of particles) do begin
    2.b Calculate fitness value for each particle
    2.c For each particle,
    If the fitness value is better than the best fitness value ($l_{best}$) in history set current value as the new $l_{best}$
    2.d Determine the best fitness value of all the particles as the $g_{best}$
    2.e Calculate velocity of every particle according to equation (1)
    2.f For each particle
    Update position of every particle according to equation (3) end;
END.
The bar bending stress ($\sigma_s$) is calculated from the following equation

$$\sigma_s(x) = \frac{6PL}{h_4x_3^3}$$  \hspace{1cm} (17)

The bar buckling load is found from the following equation

$$P_c(x) = \frac{4.013E\sqrt{\frac{x_2^4}{16}}}{L^2}(1 - \frac{\chi^3}{2L}\sqrt{\frac{E}{4G}})$$  \hspace{1cm} (18)

The bar displacement is computed using the following equation

$$\delta(x) = \frac{4PL^3}{Eh_4x_4^5}$$  \hspace{1cm} (19)

**Step 1:** Four design variables are used to define the objective and seven constraint functions. The design variables are $h(x_1)$, $l(x_2)$, $t(x_2)$, and $b(x_3)$. The objective is to minimize the cost $f_1(x)$ under the given loading conditions subject to constraints. The bending stress, buckling load, and weld stress are defined with notations as $\sigma(x)$, $P_c(x)$, and $\tau(x)$. The values of loads and stresses are given as $P = 6000$ lb, $\tau_{\text{max}} = 13,600$ psi, and $\sigma_{\text{max}} = 30,000$ psi.

**Step 2:** In this step, experiments are designed to evaluate the effects of four design variables related to the objective function. The selection of an orthogonal array for a given problem depends on the number of factors and their levels. Taguchi has tabulated 18 basic orthogonal arrays, which are called standard orthogonal arrays. In most of the problems, one of the standard orthogonal arrays is considered to plan a matrix experiment. The suitable orthogonal array with regard to four design variables at four levels each is chosen as $L_{16}$.

The equations for calculating S/N ratios for quality characteristics are logarithmic functions based on the mean square quality characteristics. In this problem, smaller the better characteristic is considered to compute S/N ratios based on the objectives as smaller the better characteristic is considered to compute the S/N ratios for each experiment are computed using the following equation (see Table 1)

$$\text{S/N ratio} = -10 \log(\sum \frac{y_i^2}{n})$$  \hspace{1cm} (20)

$L_{16}$ orthogonal array is used to simulate the experiments. The levels and S/N ratios are tabulated for 16 experiments as shown in Table 1. The intervals of parameters for four levels are given as $0.125 < x_1 < 5$, $0.1 < x_2 < 10$, $0.1 < x_3 < 10$, $0.125 < x_4 < 5$. The ANOVA for objective is formed using S/N ratios as shown in Tables 2 and 3.

**Step 3:** In the previous step, the experiments are designed to evaluate the effects of four design parameters with respect to objective function. In this step, the intervals of the design parameters are found using ANOVA regarding the effects of factors on the
The parameters used by the proposed hybrid approach for optimization process are the following:

(a) number of individuals: 50;
(b) maximum number of generations: 500;
(c) number of objective function evaluations: 25,000.

The best solutions obtained by the above mentioned approaches are listed in Tables 2 and 3, and their statistical simulation results are given in Table 4 for welded beam design problem.

When considering the number of function evaluations, the best solution computed and the statistical analysis results are taken into account together, it is concluded that HRPSO provided better solutions for this problem among the other results given in Tables 2, 3 and 4.

The worst solution found by HRPSO is better than the best solutions found by Siddall and Ragsdell and Phillips. The use of the HRPSO improves the convergence rate by computing the best value 1.72485 with the smallest function evaluation 25,000 and standard deviation values. As can be seen from Table 4, HRPSO gives the best results reported in the literature for welded beam design problem.

### Structural design optimization using improved hybrid particle swarm algorithm

The hybrid approach proposed in the third section is applied to solve a structural design optimization problem taken from automotive industry for the optimal design of an automobile component in this section. The objective functions are due to the volume and the frequency of the part which is to be designed for minimum volume and avoiding critical frequency subject to strength constraints. In this research, then structural optimization is performed using the present approach. In the first stage, the experiments are designed to evaluate the effects of four design variables related to objective functions. The four design variables $X_1$, $X_2$, $X_3$ and $X_4$ are selected as shown in Table 1.

#### Table 1. S/N ratios of the welded beam design optimization problem.

<table>
<thead>
<tr>
<th>Exp. no.</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>S/N (Objective)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.125</td>
<td>0.1</td>
<td>0.125</td>
<td>3.375</td>
<td>39.82</td>
</tr>
<tr>
<td>2</td>
<td>0.125</td>
<td>3.4</td>
<td>3.4</td>
<td>1.75</td>
<td>-14.04</td>
</tr>
<tr>
<td>3</td>
<td>0.125</td>
<td>6.7</td>
<td>6.7</td>
<td>3.375</td>
<td>-27.09</td>
</tr>
<tr>
<td>4</td>
<td>0.125</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>-35.25</td>
</tr>
<tr>
<td>5</td>
<td>1.75</td>
<td>0.1</td>
<td>3.4</td>
<td>3.375</td>
<td>-18.19</td>
</tr>
<tr>
<td>6</td>
<td>1.75</td>
<td>3.4</td>
<td>0.1</td>
<td>5</td>
<td>-21.52</td>
</tr>
<tr>
<td>7</td>
<td>1.75</td>
<td>6.7</td>
<td>10</td>
<td>0.125</td>
<td>-27.57</td>
</tr>
<tr>
<td>8</td>
<td>1.75</td>
<td>6.7</td>
<td>1.75</td>
<td>1.75</td>
<td>-33.51</td>
</tr>
<tr>
<td>9</td>
<td>3.375</td>
<td>0.1</td>
<td>6.7</td>
<td>5</td>
<td>-27.59</td>
</tr>
<tr>
<td>10</td>
<td>3.375</td>
<td>3.4</td>
<td>10</td>
<td>3.375</td>
<td>-37.02</td>
</tr>
<tr>
<td>11</td>
<td>3.375</td>
<td>6.7</td>
<td>0.1</td>
<td>1.75</td>
<td>-36.53</td>
</tr>
<tr>
<td>12</td>
<td>3.375</td>
<td>10</td>
<td>3.4</td>
<td>0.125</td>
<td>-42.02</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
<td>0.1</td>
<td>10</td>
<td>1.75</td>
<td>-23.30</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>3.4</td>
<td>6.7</td>
<td>0.125</td>
<td>-39.51</td>
</tr>
<tr>
<td>15</td>
<td>5</td>
<td>6.7</td>
<td>3.4</td>
<td>5</td>
<td>-46.10</td>
</tr>
<tr>
<td>16</td>
<td>5</td>
<td>10</td>
<td>0.1</td>
<td>3.375</td>
<td>-48.83</td>
</tr>
</tbody>
</table>

The feasible range of design variables without shape distortions is considered as $90 < X_1 < 150$, $185 < X_2 < 230$, $10 < X_3 < 25$ and $30 < X_4 < 45$.

Matrix experiments are designed using $L_{16}$ orthogonal arrays and $S/N$ ratios are conducted for each objective as given in Table 5. Smaller the better and larger the better characteristics are applied to compute $S/N$ ratios based on each objective as smaller the better and larger the better for frequency.

#### Table 2. ANOVA of the objective function for the welded beam problem.

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
<th>S/N</th>
<th>% Contribute</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7.31</td>
<td>-29.67</td>
<td>-34.82</td>
<td>-39.90</td>
<td>2478.34</td>
<td>39.21</td>
<td></td>
</tr>
<tr>
<td>-17.25</td>
<td>-30.09</td>
<td>-31.93</td>
<td>-30.79</td>
<td>567.47</td>
<td>8.98</td>
<td></td>
</tr>
</tbody>
</table>

The initial population of PSO algorithm is randomly generated for individuals within the range of the solution space bounded by $0.125 < X_1 < 1.75$, $0.1 < X_2 < 3.4$, $0.1 < X_3 < 3.4$, $0.1 < X_4 < 3.4$, $0.1 < X_1 < 1.75$, $0.1 < X_2 < 3.4$, $0.1 < X_3 < 10$, $0.125 < X_4 < 5$ (level 1 < $X_1$ < level 2, level 1 < $X_2$ < level 2, level 1 < $X_3$ < level 4, level 1 < $X_4$ < level 4).
Table 3. Comparison of the best solution welded beam design problem by different methods.

<table>
<thead>
<tr>
<th>Design variables</th>
<th>HRPSO</th>
<th>Akay and Karaboga(^{51})</th>
<th>Huang et al.(^{50})</th>
<th>He and Wang(^{49})</th>
<th>Coello and Montes(^{48})</th>
<th>Ragsdell and Phillips(^{47})</th>
<th>Siddall(^{46})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_1)</td>
<td>0.205730</td>
<td>N/A</td>
<td>0.203137</td>
<td>0.202369</td>
<td>0.205986</td>
<td>0.245500</td>
<td>0.2444</td>
</tr>
<tr>
<td>(x_2)</td>
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<td>3.471328</td>
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<td>N/A</td>
<td>9.033498</td>
<td>9.048210</td>
<td>9.020224</td>
<td>6.19600</td>
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<td>0.000000</td>
<td>N/A</td>
<td>-44.57856</td>
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<td>(g_4(x))</td>
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<td>N/A</td>
<td>-3.423726</td>
<td>-3.429347</td>
<td>-3.430043</td>
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<tr>
<td>(g_5(x))</td>
<td>-0.080730</td>
<td>N/A</td>
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<td>-0.079381</td>
<td>-0.080986</td>
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<td>(g_6(x))</td>
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<td>(g_7(x))</td>
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<td>-11.68135</td>
<td>-58.6664</td>
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<td>-3.02256</td>
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<tr>
<td>(f(x))</td>
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<td>1.724852</td>
<td>1.733462</td>
<td>1.728024</td>
<td>1.72822</td>
<td>2.385937</td>
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Figure 3. Design variables.

Table 5. Experimental results and S/N ratios for volume and frequency.

<table>
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<th>Exp. no.</th>
<th>(X_1) (mm)</th>
<th>(X_2) (mm)</th>
<th>(X_3) (mm)</th>
<th>(X_4) (mm)</th>
<th>F1 (Volume)</th>
<th>F2 (Frequency)</th>
<th>S/N1 Volume</th>
<th>S/N2 Frequency</th>
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<td>1</td>
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<td>-102.52</td>
<td>13.62</td>
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<tr>
<td>11</td>
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<td>215</td>
<td>10</td>
<td>35</td>
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<td>-99.64</td>
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<td>-100.52</td>
<td>10.83</td>
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<tr>
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<td>-100.29</td>
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<tr>
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<td>150</td>
<td>230</td>
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<td>40</td>
<td>94,617.3</td>
<td>3.4</td>
<td>-99.51</td>
<td>10.83</td>
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</table>
ANOVA for each objective is formed using S/N ratios for the first and second objective functions as shown in Tables 6 and 7. The optimal settings for $X_1$, $X_2$, $X_3$ and $X_4$ variables are computed as $X_1=90$, $185 < X_2 < 230$, $10 < X_3 < 20$ and $X_4 = 30$.

Then, the problem is solved using particle swarm optimization algorithm. The parameters used by the proposed hybrid approach for optimization process are the following:

(a) number of individuals: 30;
(b) maximum number of generations: 80;
(c) number of objective function evaluations: 2400.

In the present approach, PSO algorithm begins its search with a set of solutions for which its population range is defined by Taguchi’s method as given above. The structural layout results of the present approach using PSO algorithm and robust design based on Taguchi’s method for the design of vehicle part is given in Figure 4.

The results of present hybrid approach for the design of vehicle part are given in Table 8. The numbers of function evaluations are also listed for each approach in Table 6. The HRPSO has the smallest function evaluation number. It is seen that shape design optimization performance is improved compared to other approaches.

### Conclusions

This research describes a new design optimization strategy based on particle swarm optimization algorithm and Taguchi’s parameter design to develop an innovative design optimization approach for solving structural design optimization problems. Taguchi’s robust parameter design is introduced to help to define robust initial population levels of design parameters to achieve better initialize particle swarm optimization algorithm

| Table 6. Results of the analysis of variance for volume. |
|----------------|----------------|----------------|----------------|----------------|---------------|---------------|---------------|---------------|
| Level 1 | Level 2 | Level 3 | Level 4 | S | DOF | M | F | Cont. (%) |
| $X_1$ | $102.89$ | $102.15$ | $101.14$ | $100.14$ | $17.19$ | $3$ | $5.73$ | $69.98$ | $30.6$ |
| $X_2$ | $100.4$ | $101.19$ | $101.75$ | $102.98$ | $11.82$ | $3$ | $3.94$ | $48.10$ | $21.1$ |
| $X_3$ | $99.71$ | $101.12$ | $102.09$ | $103.4$ | $26.66$ | $3$ | $8.88$ | $108.5$ | $47.5$ |
| $X_4$ | $101.29$ | $101.71$ | $101.66$ | $101.67$ | $0.11$ | $3$ | $0.03$ | $0.47$ | $0.2$ |
| Error | $0.24$ | $3$ | $0.08$ | $2.4$ | $0.24$ | $3$ | $0.08$ | $2.4$ |
| Total | $56.05$ | $15$ | $100$ | $100$ |

| Table 7. Results of the analysis of variance for frequency. |
|----------------|----------------|----------------|----------------|----------------|---------------|---------------|---------------|
| Level 1 | Level 2 | Level 3 | Level 4 | S | DOF | M | F | Cont. (%) |
| $X_1$ | $18.51$ | $16.09$ | $13.42$ | $10.9$ | $129.9$ | $3$ | $43.3$ | $650.8$ | $95.81$ |
| $X_2$ | $14.45$ | $14.66$ | $14.89$ | $14.92$ | $0.69$ | $3$ | $0.23$ | $3.5$ | $0.515$ |
| $X_3$ | $14.07$ | $14.78$ | $15.04$ | $15.03$ | $2.75$ | $3$ | $0.91$ | $13.8$ | $2.033$ |
| $X_4$ | $14.21$ | $14.67$ | $15.04$ | $15.01$ | $2.02$ | $3$ | $0.67$ | $10.1$ | $1.491$ |
| Error | $0.2$ | $3$ | $0.06$ | $0.151$ | $0.2$ | $3$ | $0.06$ | $0.151$ |
| Total | $135.5$ | $15$ | $100$ | $100$ |

| Table 8. Comparison of the design optimization results for vehicle component. |
|----------------|----------------|----------------|----------------|----------------|---------------|---------------|---------------|
| $X_1$ (mm) | $X_2$ (mm) | $X_3$ (mm) | $X_4$ (mm) | Volume (cm$^3$) | Frequency (Hertz) | Stress (MPa) | Function evaluations |
| Topology design | $110$ | $190$ | $17$ | $32$ | $117,001$ | $6.3$ | $280.8$ | — |
| CAD optimum design | $130.8$ | $200.5$ | $15$ | $40.3$ | $107,973$ | $4.6$ | $286.6$ | $50,000$ |
| Genetic algorithm | $98.9$ | $188.6$ | $11$ | $36$ | $101,223$ | $6.8$ | $271.2$ | $10,000$ |
| PSO algorithm | $98.9$ | $188.6$ | $11$ | $36$ | $95,823$ | $7.2$ | $283.2$ | $4500$ |
| Hybrid PSO approach | $90$ | $185$ | $10.1$ | $30$ | $87,685$ | $8.6$ | $297.3$ | $2400$ |
search. The design solution space of particle swarm optimization algorithm is refined based on the effect of the various design variables on objective functions. The proposed approach is applied to a vehicle component taken from the automotive industry. It is seen that better results can be achieved with the present hybrid approach.

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**References**

34. Chen SX, Low TS and Leow B. Design optimization of a nonlinear magnetic media system using finite element

**Appendix**

**Analysis of variance (ANOVA)**

ANOVA is a standard statistical technique to interpret the experimental results. The percentage contribution of various process parameters to the selected performance characteristic can be estimated by ANOVA. Thus information about how significant the effect of each controlled parameter is on the quality characteristic of interest can be obtained. ANOVAs for raw data have been performed to identify the significant parameters and to quantify their effect on the objective function. The ANOVA based on the raw data identifies the factors which affect the average response rather than reducing variation. In ANOVA, total sum of squares (SS_T) is calculated by

$$SS_T = \sum_{i=1}^{N} (Y_i - \bar{Y})^2$$ (21)

where $N$ is the number of experiments in the orthogonal array, $N = 9$, $Y_i$ is the experimental result for the $i$th experiment and $\bar{Y}$ is given by

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^{N} Y_i$$ (22)

The total sum of the squared deviations $SS_T$ is decomposed into two sources: the sum of the squared deviations $SS_P$ due to each process parameter and the sum of the squared error $SS_e$. $SS_P$ can be calculated as

$$SS_P = \sum_{j=1}^{J} \left(\frac{SY_j}{t}\right)^2 - \frac{1}{N} \left[\sum_{i=1}^{N} Y_i\right]^2$$ (23)

where $p$ represent one of the experiment parameters, $j$ the level number of this parameter $p$, $t$ the repetition of each level of the parameter $p$, $SY_j$ the sum of the experimental results involving this parameter $p$ and level $j$. The sum of squares from error parameters $SS_e$ is

$$SS_e = SS_T - SS_A - SS_B - SS_C$$ (24)

The total degrees of freedom is $D_T = N - 1$, and the degrees of freedom of each tested parameter is $D_p = t - 1$. The variance of the parameter tested is $V_p = SS_p/D_p$. Then, the $F$-value for each design parameter is simply the ratio of the mean of squares deviations to the mean of the squared error ($F_p = V_p/V_c$). The percentage contribution $p$ can be calculated as

$$p_p = \frac{SS_p}{SS_T}$$ (25)

Table 9 shows experimental results for $L_o$ orthogonal array. Table 10 shows the results of ANOVA for the objective function.

It can be seen from Table 10 that $X_2$ and $X_3$ are the most significant parameters for the objective function.
Table 9. Experimental results.

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Table 10. Results of the analysis of variance for the objective function.

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</tr>
<tr>
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