Compressive sensing of underground structures using GPR

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**Abstract**

Feature detection in sensing problems usually involves two processing stages. First, the raw data collected by a sensor, such as a Ground Penetrating Radar (GPR), is inverted to form an image of the subsurface area. Second, the image is searched for features like lines using an algorithm such as the Hough Transform (HT), which converts the problem of finding spatially spread patterns in the image space to detecting sparse peaks in the HT parameter space. This paper exploits the sparsity of features to combine the two stages into one direct processing step using Compressive Sensing (CS). The CS framework finds the HT parameters directly from the raw sensor measurements without having to construct an image of the sensed media. In addition to skipping the image formation step, CS processing can be done with a minimal number of raw sensor measurements, which decreases the data acquisition cost. The utility of this CS-based method is demonstrated for finding buried linear structures in both simulated and experimental GPR data.

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**1. Introduction**

The problem of sensing a medium by several sensors to extract interesting features is a very important and broad topic. The basic framework of these problems is generally very similar for applications from digital imaging [1], magnetic resonance imaging (MRI) [2], synthetic aperture radar (SAR) [3] and subsurface imaging [4,5] even though the data acquisition processes, sensing geometries and sensed properties are different.

Feature detection in sensing problems usually involves two-stage processing. First, the medium is sensed by one or more sensors and the data collected by the sensors is used to form an image of the medium. For example, while the data taken by a digital camera is directly the image of the scene, data collected by an MRI machine or a ground penetrating radar (GPR) [6–8] must be inverted to an image of the interior of the body or subsurface using various imaging algorithms [9–11]. The second stage involves applying feature detection algorithms to the constructed images. Detecting features like lines or parameterized shapes in images arises in many diverse areas of image processing, pattern recognition, computer vision and subsurface imaging since these types of shapes occur in many natural and man-made objects. The Hough Transform (HT) [12] and Generalized Hough Transform (GHT) [13–15] are well-known methods to detect lines and other parameterized shapes in an image. The GHT uses a parameterized model of each feature to transform the feature in the original image space into a single peak in the parameter space. The better a feature corresponds to the model, the more values (or votes) will be accumulated at a peak point. Since each parameter space cell represents a possible feature in the image domain, the GHT can be seen as a sparse representation of the parameterized shapes in the image.

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The imaging algorithms for the first stage generally require either fine spatial sampling or a high number of time/frequency measurements. This might be expensive and time consuming especially in applications like MRI or subsurface imaging where the cost of each measurement is relatively high. Recent results in Compressive Sensing (CS) [16–18] introduce solutions for these problems. CS shows that a sparse or compressible signal, i.e., an image, can be reconstructed from a relatively small number of “random” linear measurements, by solving a convex $\ell_1$ norm minimization problem, which in most cases can be cast as a linear program. The required number of compressive measurements for correct reconstruction is linearly related to the sparsity level of the signal. The effectiveness of CS has been shown in a variety of very interesting applications such as image reconstruction [19,20], medical imaging [21], radar and subsurface imaging [22–24]. However, to find features the images must first be reconstructed from the compressive measurements and only then can the feature detection algorithms be applied. In this paper we show how to combine the sensing and feature retrieval stages into a single unified framework using compressive sensing. If the sensed medium contains only a sparse combination of the features, it is possible to directly find these features from a relatively small number of raw compressed sensor measurements without first reconstructing an image of the medium.

We first discuss how to detect parameterized shapes in images by using only a small number of compressed measurements of the image. As already mentioned it is not always possible to obtain compressed measurements of the image itself directly. Rather sensors measure varying properties of the medium that must be inverted to obtain an image. Then we investigate the problem of finding linear underground structures, e.g., pipes or tunnels, using GPR, which is great interest in industrial, civilian and military applications [25–27]. Using simulated and experimentally measured data we show that linear underground structures could be detected from small number of raw GPR measurements without first creating an image of the medium.

### 2. Detecting parameterized shapes in images using CS

The Hough Transform (HT) [12] is the classical approach for line detection in images. The HT uses a parameterized model for the line and maps each image point to the parameter space points which could have possibly produced the image point. Votes from the colinear points form a peak in the parameter space at the correct line parameters. Thus, the HT converts the problem of finding spatially spread patterns into detecting peaks in the parameter space.

In [14], a line is parameterized by its distance from origin, $\rho$, and the angle $\theta$ of the normal vector to line as

$$\rho = x_p \cos \theta + y_p \sin \theta$$

Thus each image pixel $(x_p, y_p)$ is mapped to a $(\rho, \theta)$ parameter space point, or vice versa. In the extended version for any parameterized shape, the Generalized Hough Transform (GHT) [13–15] maps arbitrary parameterized curves into peaks in the parameter space. The GHT can be formulated as

$$R(\pi) f(x, y, z) = \int f(\varphi_x(\xi, \pi), \varphi_y(\xi, \pi), \varphi_z(\xi, \pi)) \, d\xi$$

where $\pi$ is a $p$-dimensional vector defining the curve parameters and $\varphi_x(\xi, \pi)$, $\varphi_y(\xi, \pi)$ and $\varphi_z(\xi, \pi)$ are functions that define the parametric curve. The GHT transforms from the target space to the $p$-D parameter space defined by $\pi$.

Unlike this classical view, we approach the shape detection problem in terms of the inverse GHT by asking the question: which combination of parameter domain cells represents the image data best? This inverse approach facilitates the compressive sensing framework and allows us to include any possible prior information about the parameter space such as sparsity. If we consider that each cell in the parameter space corresponds to one image shape, a dictionary of possible shapes can be generated. Then a variety of algorithms, such as basis pursuit algorithms [28–30], can be used to find the best subset of dictionary elements to represent the image.

In order to express the target space in terms of a dictionary of possible shapes, we discretize the parameter vector $\pi$ along each of its $p$ dimensions. Then we can enumerate a finite set of possible parameter vectors $\mathcal{P} = \{\pi_1, \pi_2, \ldots, \pi_N\}$, where $N$ determines the discretization. This allows us to write the image $f$ as

$$f = H p$$

where $H$ is the dictionary of possible shapes and $p$ is a vector that represents the discretized parameter space.\footnote{Here the image $f$ and the parameter space $p$ is concatenated to a single vector form.} Each column of $H$ is one possible parameterized shape in the target domain, i.e., the $k$th column of $H$ corresponds to the shape with parameters $\pi_k$. Sparseness in parameter space indicates that the number of nonzero elements of $p$ is small. This is generally a fair assumption since the total number of shapes in an image is much lower than the number of all possible shapes. We say that the image is $K$-sparse if its GHT has no more than $K$ nonzero peaks, or, equivalently, the vector $p$ has no more than $K$ nonzero elements.

Consider our $K$-sparse signal $p$ of length $N$ as our parameter space signal where the image $f$ is represented as in (3). In CS rather than sampling all the pixels in the image $f$ we measure linear projections of $f$ into a second set of basis vectors $\phi_m$, $m = 1, 2, \ldots, M$. Here many fewer samples than the size of $p$ are taken, $M \ll N$. In matrix notation we measure
The result of CS theory says that the sparse parameter domain signal \( p \) can be recovered exactly from \( M = \mathcal{O}(\mu^2(\Phi, H) \log N) \) CS measurements (with overwhelming probability \([17]\)) \( s \) by solving the \( \ell_1 \) minimization problem

\[
\hat{p} = \arg\min_{p} \| p \|_1 \quad \text{s.t.} \quad s = \Phi H p
\]

where \( \mu(\Phi, H) \) is the coherence between \( \Phi \) and \( H \) defined as in \([31]\).

The optimization problem in (5) is valid for the noiseless case. For noisy compressive measurements in the form

\[
s^N = Ap + z, \quad z_k \sim \mathcal{N}(0, \sigma^2)
\]

where \( A = \Phi H \), a relaxed version of (5) can be solved:

\[
\min \| p \|_1 \quad \text{s.t.} \quad \| A^T(s^N - Ap) \|_\infty < \epsilon
\]

The optimization problem in (7) is called the Dantzig Selector \([32]\), and it is shown in \([18,32–34]\) that (7) is a stable method for recovering the sparse vector \( p \).

An important part of this method is to properly select the parameters \( N \) and \( \epsilon \). The parameter space vector length \( N \) defines the resolution in the parameter space. Increasing \( N \) makes the grid finer, but it also increases the algorithm complexity. Making the initial resolution too rough might introduce substantial bias into the estimates. Similar case applies to HT as well. The method doesn’t require an exact match between a line in the image and the dictionary but for correct reconstruction the dictionary should be created with a resolution in parameter space such that the line data has the highest projection to the closest line in the dictionary within the constraint defined in (7). In detecting lines our results indicate that using 1 pixel spacing in \( \rho \) and 2 degrees in \( \theta \) usually suffices. The second important parameter \( \epsilon \) controls the tradeoff between the sparsity of the solution and the closeness of the solution to the data. If \( \epsilon \) is not set properly regularization result either will not fully reconstruct the parameter space vector and miss some shapes (underfitting), or try to explain a significant portion of the noise by introducing spurious peaks or false shapes. Selection of the relaxation parameter \( \epsilon \) can be done either by estimating the noise variance or using cross-validation \([35,36]\).

The HT calculates votes for each parameter space value. Having shapes other than lines in the image is a big problem for HT since each different shape will be voting for unrelated line parameters. Since the proposed method doesn’t apply a voting mechanism and tries to fit the image data as sparse as possible within a constraint, it has higher performance than HT. In addition proposed method has the advantage of jointly detecting several types of parameterized shapes, i.e., circles and lines, by possibly using a combined dictionary of shapes \([37]\).

2.1. Example: Detecting lines by CS

An important application of shape detection is detecting linear structures in images. As an example, an image containing three lines with parameters \( \rho = [-3, 21, -27] \) and \( \theta = [33, 132, 153] \) degrees is constructed, as shown in Fig. 1(a). A noisy version of the image is used for testing. The image is \( 50 \times 50 \), i.e., has 2500 pixel values. For detecting lines in the image only 400 projections (compressive samples) of the image with random Gaussian vectors are used. This could be done with a proposed new camera architecture that directly acquires random projections of the signal without collecting every pixel \([20]\). The compressive samples, \( s \), of the noisy image are shown in Fig. 1(c). We assume that these measurements are the only information we have acquired about the image, and our goal is to find the linear structures in the image.

If the problem stated in (7) is solved, the parameter space image shown in Fig. 2(a) is obtained. It can be seen that the resultant image is sparse with 3 peaks corresponding to the true line parameters. The detected lines are the same as those in Fig. 1(a). If we had the information about all the pixels of the image and had applied the standard Hough transform to the original image in Fig. 1(b) we would obtain the parameter space image shown in Fig. 2(b). Even though the HT image shows 3 significant peaks, it is more noisy, and it requires all the pixels in the image.
3. Compressive sensing of underground features using GPR

The previous section relates the parameter space \( p \) to the image \( f \) linearly as in (3). However, in most cases it might not be possible to directly measure random projections, \( s \), of the image. Instead the image could be sensed through sensors like MRI, GPR, seismic, etc. Here, we investigate the problem of finding linear underground structures like pipes or tunnels using GPR. The method could be extended to other sensing problems for different features of interest.

GPR surveys the subsurface by transmitting short electromagnetic pulses into the ground and processing the reflections. More information about GPR principles can be obtained from [6]. To handle the GPR problem in a compressive sensing framework, the feature parameter space \( p \) should be linearly related to the sensor measurements \( \xi \). From (3) we know how \( p \) is related to the target space, i.e., the image \( f \). Thus we need to construct the relationship between \( \xi \) and \( f \). Such a relation depends on the data acquisition process and the target models, and is created by discretizing the target space and synthesizing the GPR model data for each discrete spatial position; the result forms a dictionary of GPR responses. Thus, we assume that targets like pipes or tunnels are combinations of point-like reflectors at discrete spatial positions and these reflectors do not interact so superposition is valid. Although modeling the GPR response from a linear structure like this is only an approximation, it is a commonly used simplification because the response for a point reflector can be easily calculated [24]. The point-like target assumption is not crucial. If the received data can be calculated for other types of target models like cylinders, then the CS-based ideas presented in this paper can still be used.

A detailed analysis for developing the \( \xi - f \) relation and constructing the target space image \( f \) from compressed time/frequency domain GPR measurements is given in [38,23]. Here, we only summarize how to construct the \( \xi - f \) relation for time-domain GPRs in order to develop the proposed feature detection method. For imaging, the GPR sensor is moved in space to collect reflected time data at different scan positions. The collection of these scan points constitute a synthetic aperture that forms the space–time data. Thus it is essential to know the space–time response from a point target. We use the point target model to write the received signal reflected from a point target at position \( r \) as a time delayed and scaled version of the transmitted signal \( s(t) \),

\[
\xi_i(t) = \frac{\sigma_i s(t - \tau_i(r))}{A_{i,r}},
\]

where \( \tau_i(r) \) is the total round-trip delay between the antenna and the target at \( r \) for the \( i \)th scan point, \( \sigma_i \) is the reflection coefficient of the target and \( A_{i,r} \) is a scaling factor used to account for any attenuation and spreading losses. In our modeling of the received GPR signal, the parameter \( \tau_i(r) \) is very important and its calculation requires knowledge of wave velocities in both air and ground, transmitter–receiver locations, and target positions as shown in Fig. 3. To generate the dictionary for the GPR data, the target space \( \kappa \) which lies in the product space \([x_i, x_f] \times [y_i, y_f] \times [z_i, z_f]\) is discretized. Here \( x_i, y_i, z_i \) and \( x_f, y_f, z_f \) denote the end points of the target space to be imaged along each axis. Discretization generates the set of target points \( B = \{\kappa_1, \kappa_2, \ldots, \kappa_{N_T}\} \), where \( N_T \) determines the resolution in the target space and each \( \kappa_j \) is a 3D vector \([x_j; y_j; z_j]\).
Using (8) and the time delay for each target space point, the signal at the GPR receiver can be calculated for a given element of \( B \) using \( \sigma_\tau = 1 \) in (8). For the \( i \)th scan point, the \( j \)th column of \( \Psi_i \) is the received signal that corresponds to a target at \( \kappa_j \). The \( n \)th index of the \( i \)th column can be written as

\[
[\Psi_i]_{n,j} = \frac{s(t_n - t_\tau(\kappa_j))}{\|s(t_\tau(\kappa_j))\|_2}
\]

where \( t_n = t_0 + n/F_s \), with \( 0 \leq n \leq N_t - 1 \), and the denominator is the energy in the time signal where \( s(t_\tau(\kappa_j)) \) represents the received signal at the \( i \)th scan point from a target at \( j \)th position. \( F_s \) is the sampling frequency, \( t_0 \) is the appropriate initial time, and \( N_t \) is the number of temporal samples. Thus each column has unit norm, is independent of the spreading factor in (8), and depends only on the travel time. Repeating (9) for each possible target point in \( B \) generates the dictionary \( \Psi_i \) of size \( N_t \times N \) when the GPR is at the \( i \)th scan point.

This allows us to write the received signal \( \zeta_i \) that consists of reflections from multiple targets as linear combinations of dictionary columns as

\[
\zeta_i = \Psi_i f
\]

where \( f \) is the target space image, i.e., a nonzero value at index \( j \) if \( f \) selects a target at \( \kappa_j \). Using (3) and (10) the measured data can be represented as \( \zeta_i = \Psi_i H p \) in terms of the sparse feature parameter vector \( p \). In the spirit of CS [17, 16], a small number of “random” measurements can carry enough information to reconstruct a signal. Rather than sampling \( \zeta_i \) at a high sampling rate, we measure linear projections of \( \zeta_i \) onto a second set of randomly selected basis vectors \( \phi_m \), \( m = 1, 2, \ldots, M \), which can be written in matrix form using \( \Phi = [\phi_1, \phi_2, \ldots, \phi_M] \) for the \( i \)th scan point as

\[
\beta_i = \Phi \zeta_i = \Phi \Psi_i H p
\]

Here the entries of \( \phi_m \) can be selected randomly from different distributions such as normal or Bernoulli. Using the data collected from \( L \) scan points, the sparse parameter space vector \( p \) can be recovered by solving

\[
\hat{p} = \arg\min \|p\|_1 \quad \text{s.t.} \quad \beta = \Phi \Psi_i H p
\]

The notations are \( \beta = [\beta_1^T, \ldots, \beta_L^T]^T, \Psi = [\Psi_1, \ldots, \Psi_L]^T, \) and \( \Phi = \text{diag}(\phi_1, \ldots, \phi_L) \). Since the GPR signal is generally noisy, we use the stable recovery procedure in [18] that obtains the sparsity pattern vector \( p \) by solving the following convex optimization problem

\[
\hat{p} = \arg\min \|p\|_1 \quad \text{s.t.} \quad \|A^T(\beta - Ap)\|_\infty < \epsilon
\]

where \( A = \Phi \Psi H \). Both (12) and (13) are linear programs that minimize convex functionals, so a global optimum is guaranteed. For the numerical solution of (13) a convex optimization package called \( \ell_1 \)-magic [39] is used.

The total number of measurements for (13) to construct a stable and correct parameter space depends on the restricted isometry property (RIP) [40] of \( A \) which is related to the coherence between \( \Phi \) and \( \Psi H \). As detailed in [41] use of redundant dictionaries require small coherence values. \( \Phi \) is selected randomly and will have small coherence with both \( \Psi \) and \( H \). The optimal construction of \( \Phi \) is an open waveform design problem which this paper doesn't focus on. For the GPR application we used a double differentiated Gaussian pulse which decorrelates quickly not leading to large cross correlation between too many columns. It also helps to handle non-ideal situations due to discretization in target and parameter spaces.

The other effect on the required number of measurements is the type of the random matrix \( \Phi \) used. Here three different types of random matrices are tested. The entries of the Type I random matrix are drawn from \( N(0,1) \). The Type II random matrix has random ±1 entries with probability of 1/2, and the Type III random matrix is constructed by randomly selecting some rows of an identity matrix which amounts to taking random time measurements. Each matrix is normalized to have unit norm rows. The average mutual coherence between the random matrices and the sparsity basis is \( \mu_1 = 5.2751, \mu_2 = 5.0059 \) and \( \mu_3 = 12.7500 \) for Types I, II and III random matrices respectively. This means that the required number of compressive measurements to detect a shape will be similar if Type I or II matrices are used. Using Type III matrix will require approximately 6.5 times more compressive measurements for the same detection capability.

4. Results

A test example will illustrate the ideas presented in the previous section. A 2-D slice of the target space containing two linear structures with line parameters \( (\rho_1, \theta_1) = (-5.2, 44) \) and \( (\rho_2, \theta_2) = (2.5, 116) \) is shown in Fig. 4(a). The GPR scans the region with 2 cm spatial resolution at 20 scan positions. Standard time domain GPRs would measure the noisy target space response shown in Fig. 4(b). The signal-to-noise ratio (SNR) is 10 dB. Targets are simulated as a combination of independent point reflectors and the GPR data is generated in MATLAB [42]. To detect the linear features existing methods invert the data to create an image of the target space, and then search the image for features using the HT. The target space image formed with time domain backprojection is shown in Fig. 4(c). The parameter space formed by applying the HT to Fig. 4(c) is shown in Fig. 4(d). Although two peaks corresponding the two targets can be seen, the target space is cluttered since the HT has limited resolution.
Our CS-based method doesn’t require sampling of all of the space–time domain data. Instead of measuring the space–time domain response at each scan position, 20 inner product measurements are formed at each scan position making 400 measurements in total for 20 scan positions. This is much less than the \(512 \times 20\) raw space–time domain measurements used to make the image in Fig. 4(c). The inner products can be written as the matrix product of the time-domain response with rows of a random matrix \(\Phi_i\) of size \(20 \times 512\) whose entries are drawn independently from \(\mathcal{N}(0, 1/\sqrt{512})\). These 400 measurements, shown in Fig. 5(a), are the only information used to sense the target space area. The number of linear structures is not assumed to be known. The parameter space image created with (13) from the compressive measurements is shown in Fig. 5(b). The CS-based method is able to find the line parameters correctly and directly using a very small number of compressive raw measurements without the image formation step. Also, since the algorithm favors sparse solutions, the resultant parameter space is much sparser than the HT image.

4.1. Experimental results

An experimental scenario has been investigated with scale models of linear structure in an experimental model sandbox [26] filled with nearly-homogeneous sand as shown in Fig. 6(a). In this case, a scale model for a tunnel is buried within a 1.8 m × 1.8 m region near the center of the tank. The tunnel is 10 cm in diameter and is buried approximately 58 cm deep to give a 20:1 scale model for a shallow tunnel just big enough for a man to slide through. The GPR sensor scans the region with 2 cm spatial resolution, collecting data in the frequency domain. The frequency-domain measured data is inverse
Fig. 6. (a) Model in the sand tank of a 10 cm tunnel buried approximately 58–60 cm deep from the surface. (b) Backprojected image using all the space–time raw GPR data. The line drawn is obtained from (d). (c) HT of the image shown in (b). (d) Parameter space obtained by the proposed CS method.

transformed to obtain equivalent time-domain measurements, and then at each scan position we create 20 compressive measurements as projections of the received time-domain responses onto a different random Gaussian measurement matrix.

Fig. 6(b) shows the 2-D slice of the backprojected image along the tunnel created using all the space–time domain samples. Solving for the line parameters by the proposed method using the twenty compressed measurements results in the parameter space shown in Fig. 6(d). The corresponding line is also plotted on the backprojected image. The tunnel could be detected and its parameters are found using a much smaller number of measurements, and without need to create the image itself.

5. Conclusion

A new framework based on compressive sensing to directly apply feature extraction from raw sensor data skipping the image formation step with minimal sensor measurements is presented. Applications to the line detection in images and linear structure extraction from GPR data are shown. We believe extension of this idea to different feature detection problems in varying sensing areas would be beneficial.

References

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Waymond R. Scott Jr. was awarded the B.E.E., M.S.E.E., and Ph.D. degrees from the Georgia Institute of Technology in 1980, 1982, and 1985, respectively. In 1986, he joined the School of Electrical and Computer Engineering at the Georgia Institute of Technology as an Assistant Professor, where he was subsequently promoted to the rank of Professor. His research involves the interaction of electromagnetic and elastic fields with materials. This research spans a broad range of topics, including the measurement of the properties of materials, experimental and numerical modeling, and systems for the detection of buried objects. Currently, his research is concentrated on investigating techniques for detecting objects buried in the earth. This work has many practical applications, for example, the detection of underground utilities, buried hazardous waste, buried structures, unexploded ordnance, and buried land mines.


