MP-OWA: The most preferred OWA operator

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A R T I C L E   I N F O

Article history:
Received 29 September 2007
Accepted 28 March 2008
Available online 11 April 2008

Keywords:
OWA operator
Aggregate weighting
Decision making
Preference of alternatives
Most preferred OWA
Meta-search engine

A B S T R A C T

In practical term any result obtained using an ordered weighted averaging (OWA) operator heavily depends upon the method to determine the weighting vector. Several approaches for obtaining the associated weights have been suggested in the literature, in which none of them took into account the preference of alternatives. This paper presents a method for determining the OWA weights when the preferences of alternatives across all the criteria are considered. An example is given to illustrate this method and an application in internet search engine shows the use of this new OWA operator.

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1. Introduction

In decision making group a set of experts work in a decision process to obtain a fair value which is representative of the group. The process usually starts by the individual evaluations of the experts; each decision maker rates each alternative on the basis of an adopted evaluation scheme [5,9]. The result of this performance judgment is a set of numerical scales that is given to each alternative. It is important to combine these numerical scales for each alternative to give a final individual evaluation or used them for ranking individual alternatives. This value must be representative of a collective estimation and is obtained by the aggregation of the opinions of the experts [18,3,17,4,20]. Finally, the process concludes with the selection of the best alternative/s as the most representative value of solution of the problem.

Recently several authors used ordered weighted averaging (OWA) operator that is defined by Yager [22] for ranking alternatives in group decision making problem. The OWA operators provides a unified framework for decision making under uncertainty, in which different decision criteria such as maximax (optimistic), maximin (pessimistic), equally likely (Laplace) and Hurwicz criteria are characterized by different OWA operator weights.

However to apply the OWA operator for decision making, a very crucial issue is to determine its weights. O'Hagan [16] suggested a maximum entropy method as the first approach to determine OWA operator weights in which he formulated the OWA operator weight problem to a constrained nonlinear optimization model with a predefined degree of orness. Fuller and Majlender [6] transformed the maximum entropy method into a polynomial equation that can be solved analytically. Recently, Majlender [15] extended the maximum entropy method to Rényi entropy and proposed a maximal Rényi entropy method that produces maximal Rényi entropy OWA weights for a given level of orness. Liu and Chen [14] put forward a parametric geometric method that could be used to obtain the maximum entropy weights. Liu [11] also brought forward an equidifferent OWA operator whose weights vary in a manner of arithmetical progression.

Wang and Parkan [19] introduced a minimax disparity approach which determines the OWA operator weights by minimizing the maximum difference between two adjacent weights under a given level of orness. The minimax disparity approach was also extended by Amin and Emrouznejad [1] and Amin [2]. It is observed that most of the above methods produce regular weight distributions, which vary either in the form of exponential (geometric progression) or in the form of arithmetical progression. For example, the maximum entropy weights vary in the form of exponential, while the minimal variability weights [7] and the minimax disparity weights vary in the form of equidistance. Although regular weight distributions make sense, there is no reason to believe that OWA operator weights can vary regularly [12].

One of the main problems in decision making is how to define a fusion method which considers the majority opinions from the individual opinions [10,17] and recently Xinwang Liu and Shilian Han [21] introduced a method for calculating OWA operator to consider priori preference of alternatives. The center of this paper
is to introduce an alternative OWA operator to consider majority opinion using experts choices and preference of alternatives.

The paper continues with the definition of OWA operators in Section 2. Section 3 briefly reviews recent disparity models of the OWA operator and the models for determining its weights. Section 4 develops a new model, which are examined with a numerical example in Section 5. An application in search engine rating illustrates the use of new approach in Section 6. The paper is concluded in Section 7.

2. Preliminaries

An OWA operator of dimension $n$ is a mapping $F: \mathbb{R}^n \rightarrow \mathbb{R}$ that has an associated with weighting vector $w = [w_1, w_2, \ldots, w_n]^T$ such that $\sum_{i=1}^{n} w_i = 1; w_i \in [0, 1]$ and $F(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i b_i$ where $b_i$ is the $i$th largest element of the collection of the aggregated objects $a_0, \ldots, a_n$. The function value $F(a_1, \ldots, a_n)$ determines the aggregated value of arguments $a_0, \ldots, a_n$. A fundamental aspect of the OWA operator is the re-ordering step, in particular an argument $a_i$ is not associated with a particular weight $w_i$ but rather a weight $w_i$ is associated with a particular ordered position $i$ of the arguments. A known property of the OWA operators is that they include the Max, Min and arithmetic mean operators for the appropriate selection of the vector $W$:

For $w = [1, 0, 0, 0]^T$, $F(a_1, a_2, a_3) = \text{Max}(a_i)$

For $w = [0, 1, 0, 0]^T$, $F(a_1, a_2, a_3) = \text{Min}(a_i)$

For $w = \frac{1}{n} \frac{1}{n} \frac{1}{n} \frac{1}{n} \frac{1}{n} \frac{1}{n} \frac{1}{n} \frac{1}{n} \frac{1}{n} ^T$, $F(a_1, a_2, a_3) = \frac{1}{n} \sum_{i=1}^{n} a_i$

OWA operators are aggregation operators, satisfying the commutative, monotonicity and idempotency properties and that they are bounded by the Max and Min operators [13];

$\text{Min}(a_i) \leq F(a_1, a_2, a_3) \leq \text{Max}(a_i)$.

However, one important issue in the theory of OWA aggregation is the determination of the associated weights. A number approaches have been suggested in the literature for obtaining the weights [21,19,1]. None of these approaches considers the preference of alternatives across the criteria. For decision making it does not make sense to define such operator without any consideration of the view of alternatives.

3. Recent development of OWA operator

Wang et al. (2007) have recently introduced a new OWA operator weights called it least squares deviation (LSD) as follows.

$$\text{Min} \sum_{i=1}^{n} (w_i - w_{i+1})^2$$

subject to :

$$\text{Orness}(W) = x = \frac{1}{n-1} \sum_{i=1}^{n} (n-i)w_i \quad 0 \leq x \leq 1$$

$$\sum_{i=1}^{n} w_i = 1$$

$$0 \leq w_i \leq 1, \quad i = 1, \ldots, n$$

In this model the sum square of deviation between $w_i$ and $w_{i+1}$ are minimized.

Recently Amin and Emrouznejad [1] introduced an extended minimax disparity model to determine the OWA operator weights as follows:

$$\text{Min} \delta$$

subject to :

$$\text{Orness}(W) = x = \frac{1}{n-1} \sum_{i=1}^{n} (n-i)w_i \quad 0 \leq x \leq 1$$

$$w_j - w_i + \delta \geq 0 \quad i = 1, \ldots, n - 1 \quad j = i + 1, \ldots, n \quad (2)$$

$$w_i - w_j + \delta \geq 0 \quad i = 1, \ldots, n - 1 \quad j = i + 1, \ldots, n$$

$$\sum_{i=1}^{n} w_i = 1$$

$$0 \leq w_i \leq 1, \quad i = 1, \ldots, n$$

In this model it is assume that the deviation of $w_i$ to $w_j$ is always equal to $\delta$ (for all $i$ and $j$, $i \neq j$).

4. The most preferred OWA operator

Consider a set of $n$ alternatives $A = \{a_1, a_2, \ldots, a_n\}$, $n \geq 2$. The aim is to classify the alternatives from best to worst (ordinal ranking), using the information known according to a finite set of general criteria or experts $C = \{C_1, C_2, \ldots, C_m\}$, $m \geq 2$. We assume that the experts’ preferences over the set of alternatives, $A$, are represented by means of the multiplicative preference relations. So we have a multi expert-multi criteria decision making problem. The ranking system described in the following is a four stage process as described below.

In the first stages, individual experts are asked to provide an evaluation of the alternatives. This evaluation consists of giving each alternative a rating on each of the criteria, where the ratings are chosen from the scale $S = \{s_1, \ldots, s_r\}$ (that is $s_1 < \ldots < s_r$). An example of this is using the scale of $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, where the number $1–9$ are ordinal scale (for example stand for $1$ = extremely poor, $2$ = very poor, $3$ = poor, $4$ = moderately poor, $5$ = fair, $6$ = moderately good, $7$ = good, $8$ = very good, $9$ = excellent). Let’s follow the equation $s_r = \sum s_r$. The following table shows the matrix of alternatives and scales given to each criteria where $S_{ij} \in S$.

Matrix of preference rating of $m$ criteria with $n$ alternatives

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_1$</th>
<th>$C_j$</th>
<th>$C_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternatives</td>
<td>$A_1$</td>
<td>$S_{12}$</td>
<td>$S_{1j}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$A_i$</td>
<td>$S_{i1}$</td>
<td>$S_{ij}$</td>
<td>$S_{im}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$A_n$</td>
<td>$S_{n1}$</td>
<td>$S_{nj}$</td>
<td>$S_{nm}$</td>
</tr>
</tbody>
</table>

In the second stage and for each criteria we count number of each scale that is given by alternatives and we summarized this in the following matrix.

Matrix of criteria and scales

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_1$</th>
<th>$C_j$</th>
<th>$C_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scales</td>
<td>$1$</td>
<td>$N_{11}$</td>
<td>$N_{1j}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$k$</td>
<td>$N_{k1}$</td>
<td>$N_{kj}$</td>
<td>$N_{km}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$9$</td>
<td>$N_{91}$</td>
<td>$N_{9j}$</td>
<td>$N_{9m}$</td>
</tr>
</tbody>
</table>
For example, \( N_k \) is the number of scale \( s_k \) given to criteria \( C_j \) by all alternatives.

In the third stage, we create the following vector in which its elements are the most popular scale for each criterion. As a result, we define vector \( V \) as follows.

\[
V = [V_1, \ldots, V_k, \ldots, V_m]^T
\]

and we name this as a vector of most preferred scale.

Finally, we define the OWA operator according to this vector as follows; hence we name this operator "MP-OWA" (Most Preferred Ordered Weighted Averaging).

\[
w = [w_1, \ldots, w_k, \ldots, w_m]^T = \left[ \frac{V_1}{\sum V_k}, \ldots, \frac{V_k}{\sum V_k}, \ldots, \frac{V_m}{\sum V_k} \right]^T
\]

It is obvious that \( \sum w_k = 1 \).

5. A numerical illustration

5.1. Finding MP-OWA with 10 alternatives and 6 criteria

Consider a set of 10 alternatives that rate 6 criteria as follows. For illustration purpose, we assume that the scales belong to \( S = \{1, 2, 3\} \).

Accordingly, the matrix of criteria and scale is as follows:

\[
\begin{array}{cccccc}
\text{Alternatives} & \text{Criteria} & c1 & c2 & c3 & c4 & c5 & c6 \\
A1 & 3 & 2 & 3 & 2 & 3 & 1 \\
A2 & 2 & 3 & 2 & 3 & 2 & 1 \\
A3 & 2 & 2 & 3 & 2 & 2 & 1 \\
A4 & 3 & 2 & 3 & 3 & 3 & 2 \\
A5 & 2 & 2 & 3 & 2 & 3 & 1 \\
A6 & 3 & 2 & 3 & 2 & 3 & 1 \\
A7 & 1 & 2 & 3 & 2 & 3 & 2 \\
A8 & 1 & 2 & 3 & 2 & 3 & 1 \\
A9 & 1 & 2 & 2 & 2 & 3 & 2 \\
A10 & 1 & 2 & 2 & 3 & 3 & 1 \\
\end{array}
\]

Hence, the matrix of criteria and scale is as follows:

\[
\begin{array}{cccccc}
\text{Scales} & \text{Criteria} & c1 & c2 & c3 & c4 & c5 & c6 \\
1 & 4 & 0 & 0 & 0 & 0 & 6 \\
2 & 3 & 9 & 2 & 8 & 1 & 4 \\
3 & 3 & 1 & 8 & 2 & 9 & 0 \\
\end{array}
\]

Accordingly, the vector of most preferred scale is \( V = [4, 9, 3, 8, 9, 6]^T \). Hence, the OWA operator can be obtained as follows.

\[
w = [0.0909091, 0.2045455, 0.1818182, 0.1818182, 0.2045455, 0.1363636]^T
\]

### Table 1

Comparison of the OWA operator weights (\( \alpha = 0.3 \))

<table>
<thead>
<tr>
<th>Criteria</th>
<th>MP-OWA</th>
<th>Model 2</th>
<th>MIN-OWA (orness)</th>
<th>Arithmetic average OWA</th>
<th>MAX-OWA (andness)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.0909091</td>
<td>0.05556</td>
<td>0.166667</td>
<td>0.166667</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>0.2045455</td>
<td>0.05556</td>
<td>0.166667</td>
<td>0.166667</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>0.1818182</td>
<td>0.05556</td>
<td>0.166667</td>
<td>0.166667</td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>0.1818182</td>
<td>0.27778</td>
<td>0.166667</td>
<td>0.166667</td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td>0.1363636</td>
<td>0.27778</td>
<td>0.166667</td>
<td>0.166667</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2

Comparison of the alternatives using different OWA operator

<table>
<thead>
<tr>
<th>Criteria</th>
<th>MP-OWA</th>
<th>Model 2</th>
<th>MIN-OWA (orness)</th>
<th>Arithmetic average OWA</th>
<th>MAX-OWA (andness)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>2.340909</td>
<td>1.88889</td>
<td>8.4</td>
<td>2.448717</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>2.477273</td>
<td>2.16667</td>
<td>8.4</td>
<td>2.602563</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>1.954545</td>
<td>1.77778</td>
<td>6.8</td>
<td>2.141025</td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>2.659091</td>
<td>2.44444</td>
<td>8.8</td>
<td>2.807691</td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td>2.159091</td>
<td>1.83333</td>
<td>7.4</td>
<td>2.371794</td>
<td></td>
</tr>
<tr>
<td>A6</td>
<td>1.954545</td>
<td>1.83333</td>
<td>7.4</td>
<td>2.371794</td>
<td></td>
</tr>
<tr>
<td>A7</td>
<td>1.954545</td>
<td>1.55556</td>
<td>7.2</td>
<td>2.141025</td>
<td></td>
</tr>
<tr>
<td>A8</td>
<td>1.954545</td>
<td>1.55556</td>
<td>7.2</td>
<td>2.141025</td>
<td></td>
</tr>
</tbody>
</table>

5.2. A comparison study

Here the paper presents a numerical illustration to compare the weights generated with those obtained by models (2) and common OWA operators of Max, Min, and Arithmetic mean. Assume \( n = 6 \) and \( \alpha = 0.3 \).

An optimal solution for all four models is given below: (see Table 1)

Table 2 shows the OWA operator weights determined by the most preference OWA operator introduced in this paper and by other methods.

Table 3 shows the ranking for each alternative using different OWA operator.

From this table, it is clear that the rank is given to each alternative is different using different OWA operator, however the advantage of using MP-OWA is that it is the most preferred by alternatives.

6. An application of MP-OWA in internet search engine

Keyhanipour et al. [10] used OWA operator to measure the performance of the search engines by factors such as average click rate, total relevancy of the returned results and the variance of table 1.
For ranking purpose it is important to choose an appropriate OWA operator. This paper shows different OWA operators produce different ranks for alternatives. The paper introduced an approach to obtain an alternative OWA operator that is based on the most popular criteria for within all alternatives. Using this OWA for ranking purpose guarantees that the preferences of alternatives across all the criteria are considered. An application in the search engine has shown how the new OWA operator can be used for ranking search engines. Further research is needed to explore the properties of MP-OWA and to compare its properties with other OWA operators in the literature.

7. Conclusions

In their method they used optimistic OWA operator as introduced in Yager [22] to combine the decisions of underlying search engines which have different degrees of importance in a certain category.

Table 4 shows the score is given to each search engine using MP-OWA as developed in this paper. The rank of alternatives using MP-OWA is reported in column 3 as opposed to the rank calculated by Keyhanipour et al. [10] and reported in the last column of Table

4. As it can be seen, weighted average clicks of ProFusion is 1.73 while it is 1.59 and 1.5 for Max fusion and Optimistic OWA methods, respectively.

The table clearly shows that the rank of many of the search engines on the both methods are the same, specially the WebFusion-OWA, WebFusion-Max and ProFusion are top ranked in both method. This also confirms the results obtained by Keyhanipour et al. [10] that the WebFusion-OWA is a better search engine.

**References**


