Abstracts

Abstracts of invited talks

A moving balls approximation method for a class of smooth constrained minimization

Alfred Auslender
Université de Lyon, France

We introduce a new algorithm for a class of smooth constrained minimization problems which is an iterative scheme that generates a sequence of feasible points that approximates the constraints set by a sequence of balls, and is accordingly called the Moving Balls Approximation Algorithm (MBA). The computational simplicity of MBA, which uses first order data information makes it suitable for large scale problems. Theoretical and computational properties of MBA and of some variant, in its primal and dual forms are studied, and convergence and global rate of convergence results are established for nonconvex and convex problems. Extension of MBA is also developed for a class of variational inequalities. Initial numerical experiments on quadratically constrained problems demonstrate the viability and performance when compared to some existing state-of-the-art optimization methods/software such as an SQP solver from the IMSL Library, and the CVXOPT software package for convex optimization.

Based on a common work with Ron Shefi and Marc Teboulle, School of Mathematical Sciences Tel-Aviv University, Israel.
On the distance to a geometrical property. Problems and algorithmic approaches

Emilio Carrizosa

Universidad de Sevilla, Spain

Let $\mathcal{F}_{k,n}$ be a non-empty subset of $\mathbb{R}^n \times \binom{k}{\mathbb{k}} \times \mathbb{R}^n$. We are interested in sets $\mathcal{F}_{k,n}$ defining a geometrical property, such as $r$-coincidence, collinearity, co-circularity, co-hyperplanarity, linear separability, etc. Given $x = (x_1, \ldots, x_k)$ in $\mathbb{R}^n \times \binom{k}{\mathbb{k}} \times \mathbb{R}^n$, we seek a perturbation vector $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_k) \in \mathbb{R}^n \times \binom{k}{\mathbb{k}} \times \mathbb{R}^n$ of $x$ such that $x + \varepsilon \in \mathcal{F}_{k,n}$ and the intensity of the perturbation $\varepsilon$ is minimized. The intensity of the perturbation is measured by a nondecreasing function $\phi$ of the vector of norms $\|\varepsilon_j\|$.

Different sets $\mathcal{F}_{k,n}$ as well as different choices of $\phi$ and $\|\cdot\|$ yield optimization problems with rather different properties. We review the state-of-the-art and present new results for some particular instances.

Second-Order Optimality Conditions for a Control Problem with a Non Differentiable Cost Functional

Eduardo Casas

Universidad de Cantabria, Spain

In this talk we consider an optimal control problem governed by a semilinear elliptic partial differential equation. The cost functional involves the $L^1$ norm of the control. To solve this problem we have to discretize it, which is typically done by using a finite element method. An important point in the numerical analysis of this control problem is the proof of error estimates for the discretizations. To achieve this goal we need to provide some sufficient second-order optimality conditions for the infinity dimensional control problem. This is done in an abstract framework well adapted to the control problem. Finally, we show how the use of these sufficient second-order optimality conditions allows us to deduce the error estimates for the optimal controls.
The Bishop-Phelps-Bollobas theorem and Asplund operators
Bernardo Cascales
Universidad de Murcia, Spain

In this lecture we will present a strengthening of the Bishop-Phelps property for operators that in the literature is called the Bishop-Phelps-Bollobás property. Let $X$ be a Banach space and $L$ a locally compact Hausdorff space. We prove that if $T : X \to C_0(L)$ is an Asplund operator and $\|T(x_0)\| \cong \|T\|$ for some $\|x_0\| = 1$, then there is an norm attaining Asplund operator $S : X \to C_0(L)$ and $\|u_0\| = 1$ with $\|S(u_0)\| = \|S\| = \|T\|$ such that $u_0 \cong x_0$ and $S \cong T$. As particular cases we obtain: (A) if $T$ is weakly compact, then $S$ can also be taken being weakly compact; (B) if $X$ is Asplund (for instance, $X = c_0$), the pair $(X, C_0(L))$ has the Bishop-Phelps-Bollobás property for all $L$; (C) if $L$ is scattered, the pair $(X, C_0(L))$ has the Bishop-Phelps-Bollobás property for all Banach spaces $X$.

Coauthors: R. M. Aron and O. Kozhushkina, Kent State University, Ohio, USA.

Various Lipschitz Like Properties for Functions and Sets: Subdifferential and Normal Characterizations
Rafael Correa
Centro de Modelamiento Matemático, Chile

In a recent paper (SIAM J. OPT Vol 20, N°4, 2010) RC, P. Gajardo and L. Thibault, obtained in a unified way, equivalences for various Lipschitz Like properties of functions and sets. With this purpose we introduced the notion of K directionally Lipschitzian properties which covers, for functions, the concepts of locally Lipschitzian, directionally Lipschitzian, and compactly epi-Lipschitzian properties. The characterizations of this new notion are provided in terms of lower Dini directional derivatives. In this work we give characterizations of K Lipschitz-like properties of functions in terms of subdifferentials which are not (necessarily) generated by generalized directional derivatives as support functions.

Coauthors: P. Gajardo, Departamento de Matemática, Universidad Técnica Federico Santa María, Chile, L. Thibault, Département des Sciences Mathématiques, Université de Montpellier II, France.
Maximality is nothing else but continuity
Jean-Pierre Crouzeix
Université Blaise Pascal, Clermont-Ferrand, France

Variational Inequality Problems are of the form

Find \( x \in C \subset X \) and \( x^* \in F(x) \) such that \( \langle x^*, y - x \rangle \geq 0 \) \( \forall y \in C. \)

This problem is called monotone if \( C \) is convex and the point-to-set mapping \( F \) is monotone on \( C \). These problems contain optimization problems of the form

Find \( x \in C \subset X \) such that \( f(y) \geq f(x) \) \( \forall y \in C \)

where \( f : C \to \mathbb{R} \) is lsc convex: set \( F = \partial f \). \( F \) is then maximal monotone and cyclically monotone. The cyclic monotonicity of \( F \) corresponds to the convexity of \( f \) and its maximality to the lower semi-continuity of \( f \).

Maximality is defined by an inclusion condition on the graph. It is very often characterized thanks to the celebrated Minty’s theorem which needs for its proof advanced mathematical results. We show that in the case where \( X \) is a finite dimensional space, maximality is thoroughly characterized by some continuity properties both on the mapping and its domain. The proofs use very elementary arguments of linear programming and topology. As an corollary we obtain the Minty’s characterization.

Metric Regularity
Assen L. Dontchev
Mathematical Reviews-Michigan University, USA

Metric regularity has its roots in the Banach open mapping principle and in the subsequent works of Lyusternik and Graves, and has been recognized as a basic property in the general area of the theory of extremum. It serves as a constraint qualification condition in deriving optimality conditions, and more importantly, is very instrumental in obtaining error bounds for perturbed minima and proving convergence of algorithms for solving optimization problems and beyond. In this talk we move into a wider territory in establishing a link between metric regularity and set-valued contractions. As applications we discuss what metric regularity means for Newton’s iteration and for approximations in optimal control.
On solving large-scale stochastic mixed 0-1 linear problems

Laureano Escudero
Univrsidad Rey Juan Carlos, Spain

A Branch-and-Fix Coordination (BFC) framework for solving large-scale multistage mixed 0-1 stochastic problems is presented. A scenario tree based scheme is used to represent the Deterministic Equivalent Model of the stochastic mixed 0-1 program with complete recourse. A mixed 0-1 model for each scenario cluster is considered plus the non-anticipativity constraints that equate the 0-1 and continuous so-called common variables from the same group of scenarios in each stage. Given the high dimensions of the stochastic instances in the real world, it is not realistic to obtain in some cases the optimal solution for the problem. Instead, the heuristic extension of BFC, so-called Fix-and-Relax Coordination (FRC) algorithm is proposed to exploit the characteristics of the non-anticipativity constraints of the stochastic model in the BFC framework for solving large-scale instances. An exact two-stage BFC algorithm is also presented as a by-product.

Coauthors: Antonio Alonso-Ayuso, Pablo Olaso and Celeste Pizarro

Cost sharing in inventory transportation systems

Ignacio García Jurado
Universidad de La Coruña, Spain

In the first part of this lecture we make an introduction to inventory games. We start describing a centralized economic order quantity problem, then we introduce and analyse the class of inventory games, and finally we present the SOC-rule, a cost sharing rule in this context. In the second part, we deal with a more general model in which part of the fixed costs of an order are transportation costs and, thus, possibly different for each coalition. In this new framework we analyze under what conditions is it profitable for the agents to cooperate and we propose and study some cost allocation rules.

Coauthors: M.G. Fiestras-Janeiro, A. Meca, and M.A. Mosquera.
**Voronoi cells of arbitrary sets via linear inequality systems**

Miguel Ángel Goberna  
*Universidad de Alicante, Spain*

Given a set $T$ of the Euclidean space and a point $s$ in $T$, the Voronoi cell of $s$ is the set of points whose distance to $s$ is not greater than its distance to any other point of $T$. The Voronoi diagram of $T$ is the family of Voronoi cells of all the elements of $T$. There exists a wide literature on Voronoi diagrams of finite sets, tessellations of the space which constitute a basic tool of computational geometry. In this talk we analyze the properties of the Voronoi cells when $T$ is an infinite set taking into account that they can be expressed as solution sets of linear inequality systems indexed by $T$.

**Coauthors:** Margarita Rodriguez, *University of Alicante*, and Virginia N. Vera de Serio, *National University of Cuyo, Mendoza, Argentina*.

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**Relax ! This is just a question of rank...**

Jean-Baptiste Hiriart-Urruty  
*Université Paul Sabatier de Toulouse, France*

One of the approaches used by mathematicians when they are faced with too difficult to tackle problems, is to “relax” them; this term has several different meanings in mathematics, depending of the fields of application: softening the constraints (when they are too hard), enlarging the underlying (functional) spaces where the solutions will be searched, substituting a more amenable objective function for the original one, etc. In the present communication, we consider the relaxation of a very bumpy function, occurring often in numerical matricial analysis as well as in a class of specific optimization problems (the so-called “matrix rank optimization problems”), that is: the rank of a matrix. A function which is naturally associated with it is

$$x = (x_1, ..., x_d) \in \mathbb{R}^d \rightarrow c(x) := \text{number of } x_i \text{ which are } \neq 0$$

(apparating in constraints or as an objective function in transmitting data for example). There is an intense research activity (*cf.* the references for
some recent entries) around the relaxation of such functions and the use of resulting convex optimization techniques. Our aim here is to explain the convex relaxation forms of functions like rank, cf. [3] (they came to us as a surprise), to give new proofs of them, and to suggest further developments (for other types of relaxation arising in variational calculus).

We have done that with LE HAI YEN, a student in Master.

References.


Metric regularity theory
Alexander Ioffe
Technion Israel Institute of Technology, Israel

The fact that the regularity concepts are basically metric in nature was recognized already in early eighties. Still, up to very recently the main effort in studies of (metric) regularity was applied to mappings between Banach spaces. There was a good reason for that: availability of estimates for regularity moduli based on the calculus of subdifferentials and normal and tangent cones. But during last 8-10 years powerful metric space mechanisms to verify regularity and to estimate regularity moduli have also been discovered.

In the talk we shall be concerned with
(a) discussions of some basic principles of regularity theory in metric spaces (both known for a few years and new);
(b) demonstration how some central results of the regularity theory in Banach spaces whose very statements are tightly bound up with specific properties of linear spaces (e.g. theorem of Robinson-Ursescu, tangential regularity criterion, Milyutin’s additive perturbation theorem) can be obtained, sometimes with an extreme easiness, from the general principles of the metric theory.

General Semi-Infinite Optimization
Hubertus Th. Jongen
RWTH Aachen University, Germany

In this lecture we will present a survey of recent generic structural results in GSIP (General Semi-Infinite Programming/Optimization). The focus will be on the following topics: description of the closure of the feasible set, the Symmetric Mangasarian Fromovitz Constraint Qualification, the Nonsmooth Symmetric Reduction Ansatz, appropriate notion of Karush-Kuhn-Tucker points and critical point theory. The survey is based on joint work with D.Dorsch, H. Guenzel, F. Guerra-Vazquez, J.-J. Rueckmann, O. Stein and V. Shikhman.
Parametric optimization and nonsmooth Newton schemes for generalized semi-infinite programs
Diethard Klatte
Universität Zürich, Switzerland

We adapt Kummer’s inexact local Newton method for solving Lipschitz equations as well as a globalization of it to the solution of (generalized) semi-infinite programs. This is done under a reduction approach, but without strict complementarity. Using results from parametric optimization and variational analysis, we work out in detail the concrete Newton schemes and the construction of a path for the globalized method. We discuss a series of numerical results for semi-infinite and generalized semi-infinite optimization problems. The results were obtained in collaboration with Stephan Buetikofer, ZHAW Winterthur, and Bernd Kummer, Humboldt University Berlin.

Stability in Infinitely Constrained Optimization: A Personal Tour
Marco A. López
Universidad de Alicante, Spain

Expertly guided by my coauthors, I started, fifteen years ago, a tour in the challenging field of stability analysis applied to infinitely constrained optimization. In this talk I am presenting some results which can be considered as the most representative of our contributions in this field. They are focused on the qualitative and the quantitative approaches to stability, and a number of them are stated in an infinite-dimensional setting. In particular the talk deals with the lower/upper semicontinuity of the feasible/optimal set mappings, different types of ill-posedness, distance to ill-posedness, Lipschitz properties of these mappings under different types of perturbations, and estimates of the associated Lipschitz bounds.
Generic well posedness in linear programming
Roberto Lucchetti
Politecnico di Milano, Italy

The paper “A generic result in linear semi-infinite optimization”, one of the many valuable results obtained by M. A. Lopez, joint work with M.A. Gob-erna and M.I. Todorov, Applied Mathematics and Optimization, (48), 2003, p. 181-193, deals with the question of “how many” linear programming problems have unique solution. The papers I present study the same topic, the main difference being that not all data of the problems are under pertur- bation. This requires a modification of the main techniques for proving the results. We study in particular the case when the right hand side and the cost function are fixed, while the matrix of the constraint can vary. Our approach is motivated by the fact that a zero sum game in matrix form can be solved, in an equivalent way, by considering a pair of linear programming problems in duality, where the matrix describing the game appears as constraint matrix, while the cost function and the right hand side are fixed for all games. Our results extend previous ones in the setting of the games, and are analogous to those of the quoted paper of M.A. Lopez et al., also if our requirement that only the matrix can vary forces to add some (unavoidable) extra-conditions in order to guarantee the generic nature of uniqueness in this setting.

On a sufficient condition for equality of two maximal monotone operators
Juan Enrique Martínez Legaz
Universidad Autónoma de Barcelona, Spain

In this joint work with Regina S. Burachik and Marco Rocco, we establish conditions under which two maximal monotone operators coincide. Our first result is inspired by an analogous result for subdifferentials of convex functions. In particular, we prove that two maximal monotone operators T, S which share the same convex-like domain D coincide whenever T(x) and S(x) have a nonempty intersection for every x in D. We extend our result to the setting of enlargements of maximal monotone operators. More precisely, we prove that two operators coincide as long as the enlargements have nonempty
intersection at each point of their common domain, assumed to be convex-like and open. We then use this to obtain new facts for convex functions: We show that the difference of two convex functions whose subdifferentials have a common open convex-like domain is constant if and only if their epsilon-subdifferentials intersect at every point of that domain.

**Metric Regularity and Conditioning for Matrix Games**

*Boris S. Mordukhovich*

*Wayne State University, Detroit, Michigan, USA*

This talk, based on the joint paper with Javier Pena (Carnegie Mellon) and Vera Roshchina (Evora), concerns computing a crucial condition measure for a first-order numerical algorithm of solving two-person zero-sum matrix games. This algorithm is a modification and improvement for matrix games of the celebrated Nesterov first-order smoothing method in nonsmooth convex optimization. We establish precise relationships between the aforementioned condition measure and the exact bound of metric regularity for an associated set-valued mapping. In this way, using appropriate tools of variational analysis and generalized differentiation, we compute the condition measure in terms of the initial matrix game data.

**Some new results on approximate solutions of vector optimization problems**

*Vicente Novo*

*Universidad Nacional de Educación a Distancia, Spain*

This talk concerns with approximate (epsilon-efficient) solutions of vector optimization problems. Since the publication of the first approximate efficiency concept in 1973 by Kutateladze, a lot of results and procedures based on this kind of solutions have been published in the literature. Our aim here is to show some new ones about the existence, the characterization via scalarization, the limit behaviour when the error tends to zero and the Ekeland variational principle. These results draw a unified approach since they are based on a very general approximate efficiency notion that encompasses the more popular epsilon-efficiency concepts of the literature.

**Coauthors:** C. Gutiérrez, *Universidad de Valladolid*, and B. Jiménez, *UNED.*
On Minimax Regret Location on Networks

Justo Puerto

Universidad de Sevilla, Spain

The talk studies the problem of finding optimal paths on networks with respect to the center, median and centdian objective functions, where vertex weights are uncertain and the uncertainty in weights is given by intervals. Our approach looks for minimax regret paths which minimize the worst-case opportunity loss in the corresponding objective function. These problems are NP-hard on general graphs; therefore we study the minimax regret path location on trees and provide polynomial algorithms for the three versions of the problem with respect to the center, median and double weighted centdian objective functions.

Traditionally, location problems have been concerned with frameworks where the data describing an instance of a problem are known precisely. However, in real world situations, most of the times it is impossible to make accurate estimations of those data that, very often, involve an important portion of uncertainty. This fact has motivated the study of location problems under uncertainty that in the last years attracted an increasing interest of the researchers (see, for example, and the references therein).

The definition of an instance of an optimization problem requires the specification of the problem parameters, like resource limitations and coefficients of the objective function in a linear program, or edge capacities in network flow problems, which may be uncertain or imprecise. One way to represent uncertainty is through the concept of a scenario that corresponds to an assignment of plausible values to the model parameters.

There are different techniques to handle uncertainty. Among the most important ones we cite stochastic programming and the minimax regret approach. In the former, one assumes some knowledge on the probability distribution of the unknown parameters so that a deterministic equivalent model is built via some expectation operators. In the latter, one assumes to be under the complete uncertainty paradigm, so that no information is known on the probability distributions over the parameters and it is required to minimize the worst-case opportunity loss in the objective function that is defined as the difference between the achieved objective function value and the optimal objective function value under the realized scenario.
Most of the published research on location problems under uncertainty has concentrated on single facility location models. Robust single-facility location problems, where uncertainty is present only on the weights of the vertices (clients), and represented by interval-data, are polynomially solvable on general networks. However, if uncertainty in edge lengths is present, both the single-facility median and center problems are strongly NP-hard on general networks.

Unlike the above studies, where the nature of the solution is to be a point, we were motivated by facility location problems where the solution is extensive and the demand of the clients is uncertain. The first studies on location of extensive facilities appeared in the early eighties. The paper by Hakimi et al. was a breakthrough in the research in this field since it improved the complexity of several of the above mentioned results and listed a number of open problems that became challenges for researchers over the years. Since its publication, several of their results have been improved while they have also contributed to start new interesting research lines.

The goal of this talk is to study discrete and continuous minimax regret path location problems on networks with uncertainty on vertex weights. We will restrict ourselves to the three most relevant objective functions in the literature: center, median and centdian. First of all, we note that the problem on general graphs is NP-hard since the corresponding standard optimization problems, namely, finding the center or the median or the centdian paths on general graphs, are already NP-hard. For this reason, we will concentrate on the study of minimax regret path location problems on trees. We prove that the problem is polynomially solvable in its three versions and provide algorithms for solving them.

**Regularization procedures for monotone operators**

Julian P. Revalski

*Bulgarian Academy of Sciences, Bulgaria*

In this talk we will focus our attention on certain regularization techniques related to two operations involving monotone operators: point-wise sums of maximal monotone operators and pre-compositions of such operators with linear continuous mappings. These techniques, whose underlying idea is to obtain a bigger operator as a result, lead to two concepts of generalized
operations—extended and variational sums of maximal monotone operators and, the corresponding to them, extended and variational compositions of monotone mappings with linear continuous operators. We will review some of the basic results concerning these generalized concepts, as well as will present some recent important advances.

Mathematical Programs with Complementarity Constraints: Critical Point Theory
Jan-J. Rückmann
University of Birmingham, UK

We study mathematical programs with complementarity constraints (MPCC) from a topological point of view. Under the linear independence constraint qualification (LICQ) we derive a Morse lemma at nondegenerate C-stationary points. Furthermore, two basic theorems from Morse theory (deformation theorem and cell-attachment theorem) are proved. As a consequence, when passing a level containing a C-stationary point, the topology of the lower level set changes via the attachment of a q-dimensional cell. The dimension q is called the C-index of the (nondegenerate) C-stationary point and depends on both the restricted Hessian of the Lagrangian and the Lagrange multipliers related to biactive complementarity constraints.

Nondegeneracy concept in conic programming
Alexander Shapiro
Georgia Institute of Technology, USA

In this talk we discuss an extension of the nondegeneracy concept to optimization problems subject to constraints formulated in a form of cone inclusions. A nontrivial example of a class of such problems are semidefinite programming problems. We discuss generic properties of this nondegeneracy condition and its application to first and second order optimality conditions and sensitivity analysis of parameterized optimization problems.
Adaptive convexification for semi-infinite programs with arbitrary index sets

Oliver Stein
Karlsruhe Institute of Technology, Germany

A numerical solution method for semi-infinite optimization problems with arbitrary, not necessarily box-shaped, index sets is presented. Convex relaxations of the lower level problem are adaptively constructed and then reformulated as MPCCs and solved. This approximation produces feasible iterates for the original problem. The convex relaxations and needed parameters are constructed with ideas of the alphaBB method of global optimization and interval methods. It is shown that after finitely many steps an approximately stationary point of the original semi-infinite problem is reached. Numerical examples illustrate the performance of the proposed method. The talk is based upon joint work with Paul Steuermann.

Nonsmooth Lyapunov pairs for infinite-dimensional first-order differential inclusions

Michel Thera
Université de Limnoges, France

With the ever-increasing influence of mathematical modeling and engineering in various branches of applications, the theory of differential inclusions plays a central role in the understanding of various applications, including for instance, physical problems, biological and physiological systems, population dynamics.

The stability analysis of the different systems involved in real applications and more precisely the characterization of the evolution of a point to its stable state has been studied by many authors since the pioneering work by Lyapunov at the end of the 19th century. The main feature of Lyapunov’s approach is that it does not require an explicit expression for the solutions of the dynamical system. In order to find good candidate (energy-like) functions which well-behave with the underlying trajectories, we only need to use the involved data. In the literature such functions are termed Lyapunov’s functions. This indirect method is very useful especially when dealing with complex real-world applications, where only the behavior of the system is
of major interest (such as, for instance, the behavior at infinite-time or the stability of the equilibrium sets).

The aim of this presentation that summarizes a recent joint work with S. Adly and A. Hantoute is to provide explicit primal and dual criteria for weakly lower semicontinuous (lsc, for short) Lyapunov functions, or, more generally, for Lyapunov pairs associated to a first-order differential inclusion of the type:

\[ \dot{x}(t; x_0) \in f(x(t; x_0)) - Ax(t; x_0) \quad a.e. \quad t \geq 0, \quad x_0 \in \text{cl(dom } A) \]. (0.0.1)

The resulting criteria in the approach we propose are given explicitly by means of the proximal subdifferential of the involved functions and therefore, an a priori knowledge of the semigroup generated by \(-A\) is not required. Our approach is essentially based on advanced tools of variational analysis and generalized differentiation.

**Subsmooth sets and functions**

**Lionel Thibault**

*Université Montpellier II, France*

I will revisit the concept of subsmooth functions first introduced by Spingarn in \(\mathbb{R}^n\) and then consider the subsmoothness property for sets and functions in the Banach setting. I will discuss in this context several recent results obtained with Aussel and Daniilidis.

**Criteria for efficiency in vector optimization**

**Maxim I. Todorov**

*Universidad de las Américas, Puebla, México*

We consider unconstrained finite dimensional multi-criteria optimization problems, where the objective functions are continuously differentiable. Motivated by previous work of Brosowski and da Silva, we suggest a number of tests to detect, whether a certain point is a locally (weakly) efficient solution for the underlying vector optimization problem or not. Our aim is to show that the points, at which none of the TESTs 1-4 can be applied, form a nowhere dense set in the state space. TESTs 1 and 2 are exactly those
proposed by Brosowski and da Silva. TEST 3 deals with a local constant behavior of at least one of the objective functions. TEST 4 includes some conditions on the gradients of objective functions satisfied locally around the point of interest. It is formulated as a Conjecture. It is proven under additional assumptions on the objective functions, such as linear independence of the gradients, convexity or directional monotonicity.

**Time series forecasting using exponential smoothing**

**Enriqueta Vercher**  
*Universidad de Valencia, Spain*

Exponential smoothing methods are extensively used forecasting procedures in industry and business planning. Their model structures allow incorporating mean level, trend and seasonal components of times series, simplifying both computational features and direct interpretation, and providing robust and accurate forecasts. The general innovations state space model provides a general framework for linear exponential smoothing. For the additive Holt-Winters model we propose a linear heteroscedastic model formulation that simplifies both obtaining maximum-likelihood estimates of all unknowns, smoothing parameters and initial conditions, and the computations of point forecasts and prediction intervals. Our optimization-based scheme unifies the stages of parameter estimation and model selection and it can be also applied to work with series with multiplicative seasonality. Moreover, for the generalized Holt-Winters scheme we develop a model selection strategy based on a full optimization framework (SIOPRED) which permits the simultaneous consideration of a wide range of exponential smoothing forecasting methods.

**A primal-dual operations on sets related with argmin-calculus and subdifferential calculus**

**Michel Volle**  
*Université d’Avignon, France*

We study some properties of a primal-dual operation on sets linked with some recent results obtained in [1], [2].


Duality results involving functions associated to nonempty subsets of locally convex spaces

Constantin Zalinescu

University “A.I. Cuza” Iasi, Romania

In many papers on consumer theory and production analysis duality results between profit, revenue, cost, input, output and shortage functions are established. This functions are associated to certain subsets of $\mathbb{R}^n$. The aim of this talk is to present a systematic study of such duality results in locally convex spaces and to derive them under minimal hypotheses.

Condition Number Theorems in Linear Quadratic Optimization

Tullio Zolezzi

Universita’ di Genova, Italy

Following the standard direct approach, the absolute condition number of a scalar optimization problem is defined as the sensitivity of the optimal solution with respect to small changes of problem’s data.

The condition number theorem states that (for many mathematical problems) the condition number of a given problem’s instance is equal, or proportional, to the reciprocal of the problem’s distance to ill-conditioning.

We present versions of such result for linear - quadratic optimization and minimum norm solutions of least-squares problems in the infinite - dimensional setting. Some counterexamples are also pointed out.