Interference due to Null Space Mismatch in Cooperative Multipoint MIMO Cellular Networks

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Abstract—Cooperative Multi-Point (CoMP) has emerged as a new paradigm to improve both average cell and cell edge throughput in cellular networks. However, the performance is significantly degraded due to Out-of-Group Interference (OGI). One way to mitigate OGI is to restrict the interfering signal to lie inside the null space of the unintended receiver. Yet, accurately tracking this null space is a challenge in time-varying channels. In this work, we address the effect of null space variations on the OGI mitigation. A measure for accuracy of null space estimates is proposed and bounds are derived on the residual interference. We define the Null Space Update Rate as the inverse of the Null Space Coherence Time; i.e., the time it takes a null space estimate to become outdated relative to an interference threshold on the unintended receiver. In the case of Rayleigh fading channels, we derive a bound on the Null Space Coherence Time in closed form, and compare the performance of a given null space tracking algorithm against this bound. Both Monte Carlo simulations and the analytical bounds show that the worst-case interference levels are sensitive to null space variations, independent of the null space learning algorithm employed.

Index Terms—MIMO, Cooperative Multi-point communications, Out of Group interference mitigation, Null Space Mismatch, Null Space Coherence Time

I. INTRODUCTION

Space-time precoding, in short precoding, is an important tool in Multi-Input Multi-Output (MIMO) wireless communications (see [1] for a survey on MIMO linear precoding). Precoding can be used for interference suppression between wireless systems. To completely avoid interference, each transmitter must learn the null space of the interference channel to the unintended receiver and use a precoding matrix which restricts its transmitted signal to lie in that null space. This approach could be very useful in Coordinated Multi-Point (CoMP) technology [2] where out-of-group interference (OGI) limits CoMP’s performance gain. Restricting the interfering signal to the null space is attractive in CoMP because it is very simple from the User Equipment (UE) point of view. In particular, once the null space is estimated, the UE does not have to take any role in the interference suppression. In [3] we introduced the Energy Null Space Learning (ENSL) problem formulation, in which the Base Station Groups (BSGs) exchange only the noise plus interference experienced by their UEs, from which the interfering BSGs learn the null space of the corresponding interference channels. One of the advantages of this formulation is that the UEs do not play any active role in the OGI mitigation process since they treat interference as noise; the null space is obtained based only on cooperation between adjacent BSGs. Numerical results showed that the Rayleigh Energy Null Space Tracking (RA-ENST) algorithm, proposed in [3] to address the ENSL problem via a variation of the Rayleigh Quotient minimization [4], is sensitive to channel variations. An open question from that investigation was to understand whether this was a limitation of the RA-ENST algorithm, or rather a fundamental sensitivity in null-space estimation for any tracking algorithm.

The aim of this work is to address this question. To do so, we first propose a distance metric to characterize null space mismatch. We then analytically characterize how fast the null space of a time varying channel changes relative to a bound on this distance metric. Finally, we analyze and compare the performance of the RA-ENST algorithm with that of the derived analytical expressions and bound. It should be noted that, even though we present our work inside the CoMP framework, our results are applicable to any multiuser MIMO system with distributed antennas that employs null space learning for interference mitigation [5].

In our analysis, we first define Null Space Mismatch as the distance between the current and an outdated null space of a time varying channel. This measure is then used to derive an upper bound on the average interference to a UE due to the null space variations as a function of the null space tracking frequency. This leads to the notions of Null Space Coherence Time and Null Space Update Rate; the latter is defined as the maximum update rate of a time varying null space such that the average induced interference levels to the UE remain lower than a predefined threshold, independently of the employed null space learning algorithm. We derive a closed form expression for that rate for a broad class of channel matrices which includes the Rayleigh fading channel following Jakes’ model [6]. The performance of the RA-ENST algorithm is shown through numerical simulations to be close to the derived analytical quantities.

A similar question was addressed in [7] where the authors...
investigate the robustness of transmit precoding in the case of imperfect CSIT; this differs from our work in that the mismatch is due to channel estimation error rather than variations. Also, general subspace comparison results are not applicable to our null space mismatch problem since our focus is on interference effects of null space mismatch. For example, in linear algebra the notion of principal angles [8], [9], has been used to quantify the distance between two subspaces. In [10], [11] the authors used a similar idea in order to address the optimal beamforming problem. In the machine learning community, a collection of “subspace comparison metrics” can be found in [12], but none of this is useful for nullspace-based interference mitigation in the CoMP framework.

The rest of the paper is organized as follows. Section II presents the system model and reviews the RA-ENST algorithm. In Section III we define the Null Space Mismatch and present several interesting properties. In addition, we define the notion of Null Space Update Rate and Null Space Coherence Time and derive an analytic upper bound on the average Null Space Mismatch for a general class of channel matrices. Lastly, Section IV shows numerical results which corroborate our analysis and compare the RA-ENST performance against the analytical bound derived for Rayleigh fading. Section V concludes the paper and describes future research directions.

Notation: All vectors are column vectors, $\mathbf{H}^*$ denotes the complex conjugate transpose of a matrix $\mathbf{H}$, $\mathcal{C}(\mathbf{H})$ denotes the column space, i.e. the span of its column vectors, and $\mathcal{I}(\mathbf{H})$ denotes the real and imaginary part of the matrix $\mathbf{H}$. The null space of $\mathbf{H}$ is defined as $\mathcal{N}(\mathbf{H}) = \{ \mathbf{x} \in \mathbb{C}^{N_T \times 1} : \mathbf{H} \mathbf{x} = 0 \}$. Also, $\mathcal{C}(\mathbf{N}(0, \sigma^2/2))$ denotes the circular complex Gaussian random variable where the real and imaginary part are independent Gaussian random variables with variance $\sigma^2/2$. Denote as $\| \mathbf{x} \|_2$ the Euclidean norm of a vector $\mathbf{x}$. $\| \mathbf{H} \|$ represents the spectral norm of $\mathbf{H}$; i.e., $\| \mathbf{H} \| = \max_{\| \mathbf{x} \|_2 = 1} \| \mathbf{H} \mathbf{x} \|_2$.

II. PROBLEM FORMULATION

The CoMP cellular system is modeled as a distributed joint MIMO system where a BSG is composed of cooperative BSs. Each BSG can be seen as one virtual BS with $N_T$ antennas jointly transmitting $\mathbf{s} \in \mathbb{C}^{N_T \times 1}$. Consider a UE belonging to BSG$_1$ and assume that the dominant source of interference is coming from BSG$_2$ as shown in Fig. 1, which transmits in the same downlink channel as BSG$_1$. Then, the UE’s received signal in the allocated resource element can be written as

$$y_1(t) = \mathbf{H}_1 \mathbf{x}_1(t) + \mathbf{H}_2 \mathbf{x}_2(t) + v(t), \quad t \in \mathbb{N},$$

where $\mathbf{H}_q \in \mathbb{C}^{N_T \times N_T}$, $q = 1, 2$, is the channel matrix between BSG$_q$ and the UE, $N_R$ is the UE’s number of antennas, and $v(t)$ is a zero mean noise that includes interference from the rest of the neighboring BSGs and from the remaining UEs inside BSG$_1$. In general, after the UE receives $y_1(t)$, it uses a receive beamforming matrix $\mathbf{Q}$, i.e., $\tilde{y}_1(t) = \mathbf{Q}^* y_1(t)$, and decodes $\tilde{y}_1(t)$. In this work, we choose to decouple the problem of null space learning, which is the responsibility of BSG$_2$, from the receive beamforming problem, which is the responsibility of BSG$_1$ and the UE, and focus only on the former problem. Note that the latter problem, that of transmit-receive beamforming, is widely studied in the literature [10], whereas the former problem arises only in systems utilizing the relatively new technique of null space learning.

The objective of BSG$_2$ is to learn $\mathcal{N}(\mathbf{H}_2)$ in order to mitigate interference to the UE; i.e., to have $\mathbf{H}_2 \mathbf{x}_2(t) = 0$. For simplicity, since our focus is the interference channel $\mathbf{H}_2$, we denote $\mathbf{H}_2$ as $\mathbf{H}$. We also assume that $N_T > N_R$ so $\mathbf{H}$ would have a non-zero null space of rank $R \leq N_T-N_R$. This is justified since the number of antennas of BSG$_1$ is usually much larger than that of the UE. Similarly, we denote $\mathbf{x}_2(t)$ as $\mathbf{x}(t)$.

To make the paper self-contained, we briefly describe the Energy Null Space Learning problem and the Rayleigh Null Space Tracking algorithm (RA-ENST) [3].

A. The Energy Null Space Learning (ENSL) problem

In the ENSL problem, the time line is divided in Transmission Cycles (TCs), indexed by $n$, of length $T_{FB}$ slots, where $T_s$ is the slot duration. BSG$_2$ transmits a sequence of $N$ learning signals $\{ \tilde{x}(n) \}_{n=1}^N$, each for $T_{FB}$ slots, i.e., $x((n−1)T_{FB}) = \cdots = x(nT_{FB}−1) = \tilde{x}(n)$, and receives a scalar feedback $\{ q(n) \}_{n=1}^N$, $q(n) \triangleq \alpha \| \mathbf{H} \tilde{x}(n) \|_2^2 + c$, from BSG$_1$, which indicates the level of interference it inflicts on the UE, where $\alpha$, $c$, are unknown constants. BSG$_2$’s objective is to estimate $\mathcal{N}(\mathbf{H})$ based on $\{ \tilde{x}(n), q(n) \}_{n=1}^N$. The number of TCs required to obtain $\mathbf{N}_\mathbf{H}$ is referred to as TC Complexity and is denoted as $TC(N_T, N_R)$. In [13] we provide several examples of how $q(n)$ can be extracted in practice and discuss the effect of noise on it.

B. Rayleigh ENST (RA-ENST) Algorithm

The Rayleigh Energy Null Space Tracking algorithm, RA-ENST, is an efficient null space tracking algorithm proposed in [3] based on the well-known Rayleigh Quotient minimization algorithm [4]. Consider a block fading channel, in which $\mathbf{H}$ changes every coherence time $T_c$ to $\tilde{\mathbf{H}}$. Given $\mathbf{U} = [\mathbf{V} \mathbf{P}]$, where $\mathbf{V}, \mathbf{P}$ contain column vectors that span $\mathcal{N}(\mathbf{H})$ and the complement space respectively, BSG$_2$ obtains a set of
vectors \( \{ \tilde{v}_i \}_{i=1}^{N_T-N_R} \) that span \( \mathcal{N}(H) \). To learn \( \mathbf{v}_1 \), BSG2 transmits \((N_R+1)^2+1\) learning signals \( \tilde{x}_i(n) \), which are linear combinations of the columns of \( U \) [3] and, after receiving the corresponding feedback \( q(n) \), it solves a system of linear equations. Similarly, for the remaining \( N_T-N_R-1 \) vectors, BSG2 transmits \( 1+2N_R \) learning signals, and solves another linear system. Thus, after \( TC_R(N_T,N_R) = 2N_T-N_R + N_T-N_R+1 \) TCs, BSG2 learns \( \mathcal{N}(H) \).

### III. Channel variations and null space mismatch

Because channel variations affect the null space, a generic metric is required to assess the null space update rate; i.e., how often should a null space estimate be updated, so that BSG2 keeps the interference sufficiently low? We now present a general null space mismatch metric.

#### A. Null Space Mismatch

Two precoding matrices \( N_1, N_2 \in \mathbb{S} \) are equivalent if BSG2 could use either one and inflict the same interference on the UE. To make this statement rigorous, we propose two possible definitions of equivalence. Each definition captures a different meaning of equivalence between two precoding matrices and leads to two different metric spaces. Yet, we show that either one naturally leads to the same notion of null space mismatch in Definition 4. To be precise, define the following two equivalence relations:

**Definition 1.** \( N_1 \sim N_2 \) iff for every \( x \in \mathbb{C}^{m \times 1} \) transmitted by BSG2, the norm of the interference to the UE is the same; i.e.,

\[
N_1 \sim N_2 \iff \| HN_1x \|_2 = \| HN_2x \|_2, \quad \forall x \in \mathbb{C}^{m \times 1}, \| x \|_2 = 1.
\]

**Definition 2.** \( N_1 \sim_N N_2 \) iff the maximum norm of the interference to the UE over all \( x \in \mathbb{C}^{m \times 1} \) is the same; i.e.,

\[
N_1 \sim_N N_2 \iff \max_{x \in \mathbb{C}^{m \times 1}} \| HN_1x \|_2 = \max_{x \in \mathbb{C}^{m \times 1}} \| HN_2x \|_2.
\]

The differences between the two definitions are now discussed. The quantity \( \| HN_1x \|_2 \) denotes the interference caused to the UE when BSG2 uses \( N_1 \) as a precoding matrix and transmits the signal \( x \). The first definition uses a stricter constraint in order to classify two elements of \( \mathbb{S} \) as equivalent since it demands that, for every vector \( x \), the inflicted interference is the same. On the other hand, the second definition classifies two matrices \( N_1 \) and \( N_2 \) as equivalent whenever the maximum interference is the same. It is easy to show that both definitions lead to well defined equivalence relations, i.e., they are reflexive, symmetric and transitive on the set \( \mathbb{S} \). Depending on the definition we are interested in, one can define a different distance metric as follows:

**Definition 3.** Let \( N_1,N_2 \in \mathbb{S} \) and a non-zero matrix \( H \in \mathbb{C}^{N_R \times 2} \). Define the following two functions \( d^{(i)}_H(\cdot,\cdot), i \in \{1,2\} \), parametrized by the matrix \( H \), as

\[
d^{(1)}_H(N_1,N_2) = \max_{\| x \|_2^2 = 1} \frac{\| HN_1x \|_2^2 - \| HN_2x \|_2^2}{\| H \|_F^2}, \quad \text{(2)}
\]

\[
d^{(2)}_H(N_1,N_2) = \frac{\| HN_1x \|_2^2 - \| HN_2x \|_2^2}{\| H \|_F^2}, \quad \text{(3)}
\]

where \( \| H \|_F^2 \) is used for normalization purposes. Similarly to [14], [15], we refer to \( \| H \|_F^2 \) as Interference Temperature. It can be shown that the spaces defined by the couple \( (S,d^{(i)}_H) \), \( i = 1,2 \), are pseudo-metric spaces, since \( d^{(i)}_H(N_1,N_2) \geq 0 \), \( d^{(i)}_H(N_1,N_2) = d^{(i)}_H(N_2,N_1) \), \( d^{(i)}_H(N_1,N_1) = 0 \), \( d^{(i)}_H(N_1,N_3) \leq d^{(i)}_H(N_1,N_2) + d^{(i)}_H(N_2,N_3) \), and therefore metric spaces can be defined on the quotient spaces \( \hat{S} = S/\sim \), as is done in Chapter 1.5 of [16].

**Proposition 1.** Let \( N \in \mathbb{S} \) and \( N_H \in \mathbb{S} \) be an orthonormal basis for \( \mathbb{N}(H) \). Then,

\[
d^{(1)}_H(N,N_H) = d^{(2)}_H(N,N_H) = \| HN \|_F^2. \quad \text{(4)}
\]

Note that the value of both functions is independent of the particular choice of \( N_H \); i.e., the basis that spans \( \mathcal{N}(H) \).

**Proof:** By definition,

\[
d^{(1)}_H(N,N_H) = \frac{1}{\| H \|_F^2} \cdot \max_{\| x \|_2^2 = 1} \left( \| HN_Hx \|_2^2 - \| HN_Hx \|_2^2 \right)
\]

\[
\stackrel{(a)}{=} \frac{1}{\| H \|_F^2} \cdot \max_{\| x \|_2^2 = 1} \| HN_x \|_2^2 = \| HN \|_F^2,
\]

where (a) follows because \( HN_Hx = 0, \forall x \) by definition. Similarly for \( d^{(2)}_H(N,N_H) \). \( \square \)

Therefore both pseudo-metric spaces lead to the same result when comparing the “distance”, defined with respect to the equivalent relations in Definitions 1 and 2, of \( N \) from \( N_H \). Because of the importance of this proposition we define a new function \( d : \mathbb{S} \rightarrow \mathbb{R} \) with parameter \( H \) as follows:

**Definition 4.** Let \( N \in \mathbb{S} \), then the null space mismatch is the distance of \( N \) from \( \mathbb{N}(H) \) given by

\[
d_H(N) = \| HN \|_F^2/\| H \|_F^2. \quad \text{(5)}
\]

In words, the null space mismatch \( d_H(N) \) is the ratio of the worst inflicted interference if BSG2 uses \( N \) over the interference temperature in the UE. The following proposition shows that \( d_H(N) \) is bounded between 0 and 1.

**Proposition 2.** For any \( N \in \mathbb{S} \), \( 0 \leq d_H(N) \leq 1 \).

**Proof:** See Appendix A. \( \square \)

If the distance of \( N \) from \( \mathcal{N}(H) \) is 0, then the span of \( N \) is a subset of the null space; i.e.,

**Proposition 3.** For any \( N \in \mathbb{S} \), \( d_H(N) = 0 \iff \mathcal{C}(N) \subseteq \mathbb{N}(H) \).
Proof: It follows that \( d_H(N) = 0 \iff ||HN|| = 0 \iff ||HNx||_2 = 0, \forall x \iff C\{N\} \subseteq \mathbb{N}(H). \)

Proposition 4 provides a useful upper bound, which in some cases can be evaluated in closed form.

**Proposition 4.** Let \( H, \tilde{H} \in \mathbb{C}^{N_T \times N_T} \) and \( \Delta H = H - \tilde{H} \). Then,

\[
d_H(N_{\tilde{H}}) \leq \min \left\{ \frac{||\Delta H||^2}{||\tilde{H}||_F^2}, 1 \right\}.
\]

\[ (6) \]

**Proof:** See Appendix B.

**B. Null Space Coherence Time and Update Rate**

Consider a time varying \( H(t) \) and assume that BSG\(_2\) has an error-free estimate of \( \mathcal{N}(H(t)) \) at time \( t \), and a precoding matrix \( N \) that spans it. BSG\(_2\) uses \( N \) until \( t + \tau \) and then it re-estimates instantaneously and perfectly \( \mathcal{N}(H(t + \tau)) \). How long should \( \tau \) be before an update to the null space estimate is needed? For simplicity denote as \( \tilde{H} = H(t) \) and \( H = H(t + \tau) \). The Null Space Coherence Time and Null Space Update Rate are defined as follows:

**Definition 5.** Denote as \( P_T \) the maximum average tolerable induced interference to the UE, The Null Space Update Rate \( R_{NS} \) is defined as \( R_{NS} = T_{NS} - \tau \), where

\[
T_{NS} = \arg \max_{\tau \geq 0} \left( \mathbb{E}(d_H(N_{\tilde{H}})) \leq P_T \right)
\]

is the Null Space Coherence Time. The expectation is with respect to the joint probability distribution of \( H \) and \( \tilde{H} \).

Next we derive a lower bound on \( T_{NS} \) for the important case of Rayleigh fading channels.

**C. Independent Rayleigh fading MIMO channel**

Consider an independent Rayleigh fading MIMO channel; i.e., the entries of \( H \) are independent random processes, where the real (and the imaginary) part is a Gaussian Wide Sense (WSS) Stationary process with known autocorrelation function. An interesting special case is the well-known Jakes’ model [6] in which the autocorrelation of the real and imaginary parts follows the zero-order Bessel function.

**Theorem 1.** Consider a channel matrix \( H(t) \in \mathbb{C}^{N_T \times N_T} \) whose entries \( H_{ij}(t) \) are distributed according to \( \mathcal{CN}(0, \sigma^2/2) \). Denote \( H = H(t) \), \( \tilde{H} = H(t + \tau) \) and \( \Delta H = H - \tilde{H} \). Assume \( \{R \{H_{ij}\}, \mathcal{R} \{H_{ij}\}\} \) (and \( \{I \{H_{ij}\}, \mathcal{I} \{H_{ij}\}\} \)) are jointly Gaussian random variables with autocorrelation \( \sigma_i^2 \tau_R \). Denote as \( \tilde{\tau}_R = 1 - R_{\tau} \). Then,

\[
\mathbb{E}(d_H(N_{\tilde{H}})) \leq \min \left\{ \frac{2N_T R_{\tau}}{N_T R_{\tau} - 1} \tilde{\tau}_R - \frac{1}{N_T R_{\tau} - 1} \tilde{\tau}_R^2, 1 \right\}.
\]

**Proof:** From Proposition 4:

\[
\mathbb{E}(d_H(N_{\tilde{H}})) \leq \mathbb{E} \left( \min \left\{ \frac{||\Delta H||^2}{||H||_F^2}, 1 \right\} \right)
\]

\[ (a) \]

\[
\leq \min \left\{ \mathbb{E} \left( \frac{||\Delta H||^2}{||H||_F^2} \right), 1 \right\}
\]

\[ (b) \]

\[
= \min \left\{ \mathbb{E} \left( \frac{||\Delta H||^2}{||H||_F^2} \right), 1 \right\}
\]

\[ (c) \]

\[
\leq \min \left\{ \mathbb{E} \left( \frac{||\Delta H||^2}{||H||_F^2} \right), 1 \right\}, \quad \text{where (a) follows from Property 4, (b) holds because we interchange the min operator with the expectation, (c) follows from } \frac{||\Delta H||^2}{||H||_F^2} \leq \frac{||\Delta H||^2}{||\tilde{H}||_F^2} \text{ and from properties of conditional expectation. In Appendix C we prove that}
\]

\[
\mathbb{E}(\frac{||\Delta H||^2}{||H||_F^2}) = 2N_T R_{\tau} \left( \sigma^2 \tilde{\tau}_R - \frac{\sigma^2}{2} \tilde{\tau}_R^2 \right) + \tilde{\tau}_R^2 \frac{||H||_F^2}{||\tilde{H}||_F^2}. \quad (9)
\]

Finally, the proof is completed by substituting (9) into (8) (details in Appendix D).

Using Theorem 1, a lower bound on \( T_{NS} \) is given by:

\[
U_{T_{NS}} = \arg \max_{\tau \geq 0} \left( \frac{2N_T R_{\tau}}{N_T R_{\tau} - 1} \tilde{\tau}_R - \frac{1}{N_T R_{\tau} - 1} \tilde{\tau}_R^2 \leq P_T \right).
\]

\[ (10) \]

\( U_{T_{NS}} \) can be easily calculated for any correlation model and Doppler Frequency \( F_d \). This result provides BSG\(_2\) a non-trivial minimum amount of time that its null space estimate remain relatively constant, as the interference caused by the mismatch inflicted corresponds to an average interference level acceptable by the UE. In Jakes’ model, \( \tilde{\tau}_R = 1 - J_0(2\pi F_d) \), and thus maximizing over \( \tau \), is equivalent to maximizing over the normalized quantity \( \tilde{\tau}_R \).

**IV. NUMERICAL RESULTS**

In the first simulation we compare the performance of the RA-ENST algorithm against the derived bounds in Section III-C. Specifically, Fig 2 presents Monte Carlo simulation results of

\[
\tilde{T}^R_{NS} = T_{NS} - \arg \max_{\tau \geq 0} \left( \mathbb{E}(d_H(N_{\tilde{H}})) \leq P_T \right),
\]

\[ (11) \]

where \( \tilde{N}_{\tilde{H}} \) is the estimate of the null space at time \( t \) via the RA-ENST algorithm for \( T_{FB} = 16 \) and \( T_s = 66.7 \) usec. i.e., BSG\(_2\) receives \( q(n) \) every 1 usec. \( T_{NS} \) is compared to \( T_{NS} = T_{NS F_d} \) and \( \tilde{T}_{NS} = U_{T_{NS}} F_d \) as a function of \( P_T \) (dB). The number of antennas is \( (N_T, N_R) = (6, 2) \), referred to as a \((6, 2)\)-channel, and \( H(t) \) is a Rayleigh fading channel with maximum Doppler Frequency \( F_d = 1, 3, 5, 8 \) Hz, following Jakes’ model. The Monte Carlo Estimate of \( T_{NS} \) is estimated using the following experiment: Simulate a Rayleigh fading channel matrix \( H(t) \) with Doppler Frequency \( F_d \) from \( t = 0 \) to \( t = \tau \) using Jakes’ model and obtain the null space at time \( \tilde{H} = H(0) \), i.e., \( \tilde{N}_{\tilde{H}} \), and then calculate \( d_H(N_{\tilde{H}}) \). The maximum \( \tau \) such that \( \mathbb{E}(d_H(N_{\tilde{H}})) \leq P_T \) is the estimate of \( T_{NS} \). We observe that \( \tilde{T}_{NS}^R \) is close to \( \tilde{T}_{NS} \) for \( F_d = 1 \) Hz, and that the derived bound provides an informative approximation of \( T_{NS}^R \), i.e., the bound predicts the behavior of the algorithm as a function of \( P_T \). Also, as \( F_d \) increases, channel variations during a RA-ENST sweep introduce errors which lead to shorter Null Space Coherence Times. Fig. 3 presents the same quantities for a \((8, 1)\)-channel and demonstrates the same phenomenon as in Fig. 2. Fig. 4 presents \( U_{T_{NS}} \) for different \((N_T, N_R)\)-channel combinations. It can be easily seen from the expression in Theorem 1 that, for a fixed \( P_T \), \( U_{T_{NS}} \) is an increasing function of \( N_T N_R \). Similarly, Fig. 5, 6 present the Monte Carlo estimate of \( T_{NS} \) for \((6, 2)\)-channels and \((8, 1)\)-channels following Rician fading with \( K = 0, 1, 3, 6 \) dB, where \( K = 0 \) coincides with Rayleigh fading, along with \( T_{NS}^R \) for \( F_d = 3 \) Hz. As expected, the
Null Space Coherence Time in Rician fading is an increasing function of $K$. We consider the problem of characterizing this dependence as an important problem for future research.

Therefore, $\tilde{U}_{TNS}$ can be used as a pessimistic but not-trivial approximation of $T_{NS}$, applicable to the RA-ENST algorithm and any other practical null space learning algorithm. An interesting observation is that even though in the wireless communication literature (e.g. [17]) the coherence time of a SISO channel with Rayleigh fading following Jakes’ model is equal to $0.4/f_d$, the derived bound, along with the Monte Carlo simulation results, indicate that $T_{NS}$ is significantly smaller. For example, for $P_T = -5$ dB, $(N_T, N_R) = (6, 2)$ and Rayleigh fading, BSG$_2$ needs to update the null space estimate every $0.2/F_d$ seconds. Moreover, our results show that the ENSL problem is fundamentally limited to fading environments with low Doppler frequencies considering that $T_{FB}$ is of the order of milliseconds in current wireless systems. For example, for $F_d = 3$ Hz, any null space learning algorithm would need to be repeated every 65 msec to ensure a $P_T = -3$ dB. Considering a value of $T_{FB} = 1$ msec, and that $T_{CR}(6,2) = 23$, BSG$_2$ spends an impractically large amount of time learning a null space estimate without transmitting information. These results motivate our current research direction of null space learning and simultaneous information transmission which is omitted in this paper due to space limitations.

V. CONCLUSIONS AND FUTURE WORK

We have formulated a novel framework to study the robustness of MIMO channel null space estimates to time varying environments. We introduced a Null Space Mismatch metric, from which we defined the Null Space Update Rate and Null Space Coherence Time. We have also derived an analytical bound on the Null Space Coherence Time, which was evaluated in closed form in the case of Rayleigh fading. This bound indicates how fast the null space must be updated to maintain a given average level of worst-case interference, irrespective of the update algorithm. Our analysis shows that the null space becomes outdated at a fraction of the coherence time of the channel, e.g. $0.2/F_d$ seconds in Rayleigh fading, which means that the Energy based Null Space Learning problem in the CoMP framework is fundamentally limited to slow time-varying channels, due to the slow feedback acquisition in current wireless systems. Note that the bound applies for any null space learning and tracking algorithm; i.e., every algorithm must update the null space as indicated by the bound. Lastly, simulation results show that the derived interference bound predicts well the performance of the RA-ENST algorithm.

In terms of future work, due to the fast null space variations even in slow time varying scenarios, the RA-ENST algorithm must be modified to allow BSG$_2$ to simultaneously transmit information to its intended receiver and learn the null space towards the unintended receiver. Furthermore, the derived bound can be loose when $N_R$ is close to $N_T$. Lastly, more work is needed to find an analytic formula for the Null Space Coherence Time.

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**Fig. 2:** Monte Carlo estimates of $\tilde{T}_{NS}$, $\tilde{T}_{NS}^R$ and $\tilde{U}_{TNS}$ for $(N_T, N_R) = (6, 2)$ for Rayleigh fading with $F_d = 1, 3, 5, 8$ Hz.

**Fig. 3:** Monte Carlo estimates of $\tilde{T}_{NS}^R$, $\tilde{T}_{NS}$ and $\tilde{U}_{TNS}$ for $(N_T, N_R) = (8, 1)$ for Rayleigh fading with $F_d = 1, 3, 5, 8$ Hz.

**Fig. 4:** $\tilde{U}_{TNS}$ for different $(N_T, N_R)$-channels for Rayleigh fading with $F_d = 1$ Hz.

**Fig. 5:** Monte Carlo estimates of $\tilde{T}_{NS}$, $\tilde{T}_{NS}^R$ for $(N_T, N_R) = (6, 2)$, Rician fading with $F_d = 3$ Hz and $K = 0, 1, 3$ dB.
Fig. 6: Monte Carlo estimates of $\hat{T}_{NS}$, $\hat{T}_{RS}$ for ($N_T, N_R$) = (8, 1), Rician fading with $F_d = 3$ Hz and $K = 0.1, 3$ dB.

APPENDIX A

By definition,

$$d_H(N) = \frac{||HN||^2}{||H||^2} = \max_{x \in \mathbb{C}^{N \times 1}} x^* \bar{N}^* H^* H N x$$

$$= \max_{y \in \mathbb{C}^{N \times 1}} \frac{y^* H^* H y}{||H||^2} \leq 1,$$

where (a) is because $N^* H = I$, (b) because we relax a constraint in the maximization and (c) because $||H||^2 \geq ||H||^2$.

APPENDIX B

To show (6) we use

$$d_H(N) = \frac{||HN||^2}{||H||^2} = \frac{||\Delta H + \bar{H} N H||^2}{||H||^2}$$

$$= \max_{y \in \mathbb{C}^{N \times 1}} \frac{y^* \Delta H^* \Delta H y}{||H||^2},$$

where (a) is because $N^* H N = I$ and (b) is because a constraint in the maximization is relaxed.

APPENDIX C

Denote as $R = R(H)$, $R = R(H)$ and $I = I(H)$, $\tilde{I} = I(\bar{H})$. Then,

$$E(||\Delta H||^2_F | H) = \sum_{j=1}^{N_T} \sum_{i=1}^{N_R} \left( E(\{\tilde{R}_{i,j} - R_{i,j}\}^2 | R_{i,j}) + E(\{(\tilde{I}_{i,j} - I_{i,j})^2 | I_{i,j}) \right),$$

where $R_{i,j}$ denotes the $i, j$-th entry of $R$ (and the same applies for the other matrices). The latter equality is because the real and imaginary part of $H_{i,j}$ are independent. The first term of the above summation is

$$E(\{\tilde{R}_{i,j} - R_{i,j}\}^2 | R_{i,j}) = R_{i,j}^2 + E(\tilde{R}_{i,j} | R_{i,j}) - 2R_{i,j} E(\tilde{R}_{i,j} | R_{i,j})$$

and for jointly Gaussian random variables it holds that

$$E(\tilde{R}_{i,j} | R_{i,j}) = \frac{E(R_{i,j} | \tilde{R}_{i,j})}{E(\tilde{R}_{i,j})} = R_{i,j}.$$

Also,

$$E(\tilde{R}_{i,j} | R_{i,j}) = Var(\tilde{R}_{i,j}) + (E(\tilde{R}_{i,j} | R_{i,j})^2)^2$$

where

$$Var(\tilde{R}_{i,j}) = \mathbb{E}(\tilde{R}_{i,j}^2) - \mathbb{E}(\tilde{R}_{i,j})^2 = \frac{\sigma^2}{2} - \frac{\sigma^2}{2} R_{i,j}^2.$$

Therefore,

$$E(\{\tilde{R}_{i,j} - R_{i,j}\}^2 | R_{i,j}) = \frac{\sigma^2}{2} R_{i,j}^2 + \frac{\sigma^2}{2} (1 - R_{i,j}^2) + R_{i,j}^2 \tilde{R}_{i,j}^2 + 2R_{i,j} \tilde{R}_{i,j}$$

Substituting (13) to (12), it follows that

$$E(||\Delta H||^2_F | H) = \sum_{j=1}^{N_T} \sum_{i=1}^{N_R} \left( 2(\sigma^2 \tilde{R}_{i,j} - \frac{\sigma^2}{2} R_{i,j}^2) + \tilde{R}_{i,j}^2 (R_{i,j}^2 + I_{i,j}^2) \right)$$

$$= 2N_T N_R (\sigma^2 \tilde{R}_{i,j} - \frac{\sigma^2}{2} R_{i,j}^2) + \tilde{R}_{i,j}^2 ||H||^2_F.$$ (14)

APPENDIX D

From (9) and (8) it follows that

$$E(d_H(N)) \leq \min \left\{ \frac{\sigma^2}{2} N_T N_R \left( 2N_T N_R - 1 \right) \right\}$$

where

- equality (a) follows because $\frac{\sigma^2}{2} N_T N_R$ is inv-$\frac{\sigma^2}{2} N_T N_R$ distributed, and therefore $E \left( \frac{1}{||H||^2_F} \right) = \frac{\sigma^2}{2(2N_T N_R - 2)}$.

REFERENCES