COMPARISON OF TWO IDENTIFICATION TECHNIQUES: THEORY AND APPLICATION

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Abstract: Parametric identification requires a good know-how and an accurate analysis. The most popular methods consist in using simply the least squares techniques because of their simplicity. However, these techniques are not intrinsically robust. An alternative consists in helping them with an appropriate data treatment. Another choice consists in applying a robust identification method. This paper focuses on a comparison of two techniques: a “helped” least squares technique and a robust method called “the simple refined instrumental variable method”. These methods will be applied to a single degree of freedom haptic interface developed by the CEA Interactive Robotics Unit.

1 INTRODUCTION

In most of cases, parametric identification is a hard task and requires a good know-how. A popular method consists in using the least squares technique (LS) because of its simplicity. However, it has been proven that the LS are sensitive to outliers, leverage points and noise models (Hampel, 1971) and (Hueber, 1981). In other words, the LS are not intrinsically robust and we must “help” them in order to obtain reliable results.

In the last decade, the IRCCyN robotic team has designed a identification method based on LS technique, inverse model, base parameters, exciting trajectories and appropriate data treatment (Gautier and Khalil, 1990), (Gautier and Khalil, 1991) and (Gautier, 1997). This technique has been successfully applied to identify inertial parameters of industrial robots (Gautier, Khalil and Restrepo, 1995) and (Gautier, 1997). Recently, it was used for identifying electrical parameters of a synchronous machine (Khatounian et al, 2006), inertial parameters of a haptic device (Janot et al, 2006) and the parameters of a compactor (Lemaire et al, 2006). The obtained results were interesting and reliable.

At the same time, robust methods have been successfully used (Mili, Phaniraj and Rousseeuw, 1990), (Mili et al, 1996), (Young 2006) and (Gilson et al, 2006). An interesting robust method is the simple refined instrumental variable (SRIV) because of its simplicity (Young 1979). Indeed, no noise model identification is needed and this method is consistent even in the coloured noise situation (Gilson et al, 2006). This method is implemented in the MATLAB CONTSID toolbox developed by the CRAN team (Garnier, Gilson and Huselstein, 2003) and (Garnier, Gilson and Cervellin, 2006). To our knowledge, this method has not been used to identify inertial parameters of robots.

Hence, it seems interesting to apply the SRIV method to a single degree of freedom haptic interface and to compare it with the LS method.

The paper is organized as follows: second section presents the general inverse dynamic model of robots, the LS technique and the SRIV method; experimental results are presented in section 3; finally, section 4 introduces a discussion.
2 THEORY

2.1 General Inverse Dynamic Model

The inverse dynamic model (commonly called dynamic model) calculates the joint torques as a function of joint positions, velocities and accelerations. It is usually represented by the following equation:

$$\Gamma = A(q)\ddot{q} + H(q, \dot{q}) + F_v \ddot{q} + F_c \text{sign}(\dot{q})$$  \hspace{1cm} (1)

Where, \(\Gamma\) is the torques vector, \(q, \dot{q}\) and \(\ddot{q}\) are respectively the joint positions, velocities and accelerations vector, \(A(q)\) is the inertia matrix, \(H(q, \dot{q})\) is the vector regrouping Coriolis, centrifugal and gravity torques, \(F_v\) and \(F_c\) are respectively the viscous and Coulomb friction matrices.

The classical parameters used in this model are the components \(XX, XY, XZ, YY, YZ, ZZ\) of the inertia tensor of link \(j\) denoted \(M_j\), the mass of the link \(j\) called \(M_j\), the first moments vector of link \(j\) around the origin of frame \(j\) denoted \(\mathbf{MS}_j = [M_{jX}, M_{jY}, M_{jZ}]^T\), and the friction coefficients \(F_{v_j}, F_{c_j}\).

The kinetic and potential energies being linear functions of joint positions, velocities and \(F\) and \(C\) are respectively the viscous and Coulomb friction matrices.

2.2 LS Method

2.2.1 General Theory

Generally, ordinary LS technique is used to estimate the base parameters solving an overdetermined linear system obtained from a sampling of the dynamic model, along a given trajectory \((q, \dot{q}, \ddot{q})\). (Gautier, Khalil and Restrepo, 1995), (Gautier, 1997). \(X\) being the \(b\) minimum parameters vector to be identified, \(Y\) the measurements vector, \(W\) the observation matrix and \(p\) the vector of errors, the system is described as follows:

$$Y(\Gamma) = W(q, \dot{q}, \ddot{q})X + p$$  \hspace{1cm} (4)

The L.S. solution \(\hat{X}\) minimizes the 2-norm of the vector of errors \(p\). \(W\) is a \(r \times b\) full rank and well conditioned matrix, obtained by tracking exciting trajectories and by considering the base parameters, \(r\) being the number of samplings along a trajectory. Hence, there is only one solution \(\hat{X}\) (Gautier, 1997). Standard deviations \(\sigma_{\hat{X}}\) are estimated using classical and simple results from statistics. The matrix \(W\) is supposed deterministic, and \(p\), a zero-mean additive independent noise, with a standard deviation such as:

$$C_p = E[pp^T] = \sigma_p^2 I_r$$  \hspace{1cm} (5)

where \(E\) is the expectation operator and \(I_r\) the \(r \times r\) identity matrix. An unbiased estimation of \(\sigma_p\) is:

$$\sigma_p^2 = \frac{\|Y - WX\|^2}{(r - b)}$$  \hspace{1cm} (6)

The covariance matrix of the standard deviation is calculated as follows:

$$C_{\hat{X}X} = \sigma_p^2 (W^T W)^{-1}$$  \hspace{1cm} (7)

$$\sigma_{\hat{X}_i} = C_{\hat{X}X_{ii}}$$ is the \(i\)th diagonal coefficient of \(C_{\hat{X}X}\).

The relative standard deviation \(\%\sigma_{\hat{X}_i}\) is given by:

$$\%\sigma_{\hat{X}_i} = \frac{\sigma_{\hat{X}_i}}{\hat{X}_i} \times 100$$  \hspace{1cm} (8)

However, in practice, \(W\) is not deterministic. This problem can be solved by filtering the measurement matrix \(Y\) and the columns of the observation matrix \(W\).
2.2.2 Data Filtering

Vectors $Y$ and $q$ are samples measured during an experimental test. We know that the LS method is sensitive to outliers and leverage points. A median filter is applied to $Y$ and $q$ in order to eliminate them.

The derivatives $\dot{q}$ and $\ddot{q}$ are obtained without phase shift using a centered finite difference of the vector $q$. Knowing that $q$ is perturbed by high frequency noises, which will be amplified by the numeric derivation, a lowpass filter, with an order greater than 2 is used. The choice of the cut-off frequency $\omega_0$ is very sensitive because the filtered data $(q_t, q_{t-1}, q_{t-2})$ must be equal to the vector $(q, q, \dot{q})$ in the range $[0, \omega_0]$, in order to avoid distortion in the dynamic regressor. The filter must have a flat amplitude characteristic without phase shift in the range $[0, \omega_0]$, with the rule of thumb $\omega_0 > 10^4 \omega_{syn}$ where $\omega_{syn}$ represents the dynamic frequency of the system. Considering an off-line identification, it is easily achieved with a non-causal zero-phase digital filter by processing the input data through an IIR lowpass butterworth filter in both the forward and reverse direction. The measurement vector $Y$ is also filtered, thus, a new filtered linear system is obtained:

$$Y_t = W_t X_t + \rho_t$$  \hspace{1cm} (9)

Because there is no more signal in the range $[\omega_0, \omega_0/2]$, where $\omega_0$ is the sampling frequency, vector $Y_t$ and matrix $W_t$ are resampled at a lower rate after lowpass filtering, keeping one sample over $n_d$ samples, in order to obtain the final system to be identified:

$$Y_{id} = W_{id} X_{id} + \rho_{id}$$  \hspace{1cm} (10)

with:

$$n_d = 0.8 \omega_s/2 \omega_t$$  \hspace{1cm} (11)

2.2.3 Exciting Trajectories

Knowing the base parameters, exciting trajectories must be designed. In fact, they represent the trajectories which excite well the parameters. If the trajectories are enough exciting, then the conditioning number of $W$ (denoted $\text{cond}(W)$) is close to 1. However, in practice, this conditioning number can reach 200 because of the high number of base parameters. Design and calculations of these trajectories can be found in (Gautier and Khalil, 1991).

If the trajectories are not enough exciting, then the results have no sense because the system is ill conditioned and some undesirable regroupings occur.

2.3 SRIV Method

From a theoretical point of view, the LS assumptions are violated in practical applications. In the equation (4), the observation matrix $W$ is built from the joint positions $q$ which are often measured and from $\dot{q}$, $\ddot{q}$ which are often computed numerically from $q$. Therefore the observation matrix is noisy. Moreover identification process takes place when the robot is controlled by feedback. It is well known that these violations of assumption imply that the LS solution is biased. Indeed, from (4), it comes:

$$W^T Y = W^T WX + W^T \rho$$  \hspace{1cm} (12)

As $\rho$ includes noises from the observation matrix $E(W^T \rho) \neq 0$.

The Refined Instrumental Variable approach deals with this problem of noisy observation matrix and can be statistically optimal (Young, 1979). It is the reason why we propose to enrich our identification methods by using concepts from this approach. In the following, we describe the application of this method in our field.

The Instrumental Variable Method proposes to remove the bias on the solution by building the instrument matrix $V$ such as $E(V^T \rho) = 0$ and $V^T W$ is invertible.

The previous equation becomes:

$$V^T Y = V^T WX + V^T \rho$$  \hspace{1cm} (13)

The instrumental variable solution is given by

$$\hat{X}_V = (V^T W)^{-1} V^T Y$$  \hspace{1cm} (14)

The main problem is to find the instrument matrix $V$. A classical solution is to build an observation matrix from simulated data instead of measured data. The following iterative algorithm describes this solution:

**Step 0**: a first set of parameters is given by using classical LS.

**Step k**: From a set of parameters $\hat{X}_{V(k-1)}$ given at the previous step, the following ordinary differential equation, describing the robot dynamic, is simulated:

$$\ddot{q} = A^{-1}(\Gamma + H(q, \dot{q}) + \Gamma_f)$$  \hspace{1cm} (15)
\( \Gamma \) is the motor vector which is generally given by a feedback built by comparison between the desired and the real movement.

\( \mathbf{A, H, \Gamma} \) are respectively the inertia matrix, the Coriolis-centrifugal vector and the friction torques. They are computed with the parameters identified at the step \( k-1 \): \( \hat{X}_{v_{k-1}} \)

By simulation of this differential equation, \( q_{k}, \dot{q}_{k}, \ddot{q}_{k} \) are obtained. Then the Instrument Matrix at the step \( k \) is computed: \( \mathbf{V}_{k} = \mathbf{W}(q_{k}, \dot{q}_{k}, \ddot{q}_{k}) \). The equation (14) gives the set of parameters \( \hat{X}_{v_{k}} \).

This iterative process is conducted until the convergence of the parameters. Practically, 4 or 5 steps are enough to decrease dramatically the correlation between the instrument matrix and the noise on the system.

### 3 APPLICATION

#### 3.1 Presentation and Modelling of the Interface

The interface to be identified is presented Figure 1. It consists of synchronous machine and a handle actuated thanks to a cable transmission. This type of transmission is a good compromise between friction, slippage and losses. The modelling and the identification are made under the rigid model assumption. Naturally, the authors have checked that this hypothesis is valid in the case of this study.

The modelling and the identification of the synchronous machine are given in (Khatounian et al., 2006).

The frames defined to model the interface are presented Figure 2. The inverse dynamic model is given by the following equation:

\[
\Gamma = J\ddot{q} - M_Xg\cos(q) + M_Yg\sin(q) + \Gamma_f \quad (16)
\]

where \( q \) and \( \ddot{q} \) are respectively the joint position and acceleration, \( J \) is the global inertia of the system.

The torque \( \Gamma \) is calculated through the current measurement. We have verified that we have \( \Gamma = NKcI \), where \( N \) is the gear ratio, \( K_c \) the torque constant and \( I \) the measured current. More details about the implemented control law can be found in (Khatounian et al., 2006).

The friction torque \( \Gamma_f \) was measured according to the method developed by (Spetch and Isermann, 1988), which consists in measuring the torque at different constant speeds and eliminating all transient modes. Figure 3 shows that the friction model can be considered at non-zero velocity as the following, where \( \text{sign}(\dot{q}) \) is the sign function of the velocity \( \dot{q} \), and \( f_c, f_v \) denote the Coulomb and viscous friction coefficients.

\[
\Gamma_f = f_c \dot{q} + f_v \text{sign}(\dot{q}) \quad (17)
\]

The dynamic model can be thus written as a sampled linear form:

\[
\Gamma = \mathbf{W} \mathbf{X} \quad (18)
\]

\[
\begin{bmatrix}
\ddot{q}_1 - g\cos(q_1) & g\sin(q_1) & \dot{q}_1 & \text{sign}(\dot{q}_1)
\end{bmatrix}^T
\]

\[
\begin{bmatrix}
\ddot{q}_r - g\cos(q_r) & g\sin(q_r) & \dot{q}_r & \text{sign}(\dot{q}_r)
\end{bmatrix}^T
\]

\[
\begin{bmatrix}
\Gamma_f^1 \\
\Gamma_f^r
\end{bmatrix}
\quad \text{and} \quad
\mathbf{X} = \begin{bmatrix}
J & M_X & M_X & f_v & f_c
\end{bmatrix}^T.
\]

In this simple case, the parameters \( J, M_X, M_Y, f_v \) and \( f_c \) constitute the set of base parameters (QR decomposition confirms that).

![Figure 2: Notations and frames used for modelling the interface](image)
3.2 Identification of the Base Parameters

3.2.1 LS Method

Current \(I\) and joint position \(q\) were measured, with a sampling period of 240\(\mu\)s. Exciting trajectory consists of trapezoidal speed reference. This trajectory was chosen because it is equivalent to a rectangular trajectory for the joint acceleration. It allows the estimation of the inertia term due to the change of the acceleration sign, of the gravitational and viscous friction terms because the velocity is constant over a period and of the Coulomb term due to the change of sign of the velocity. Vectors \(\dot{q}\) and \(\ddot{q}\) were derived from the position vector \(q\), and all data were filtered as described in section 2.2.2. Because of the friction model, the velocities close to zero are eliminated.

The cut-off frequency of the butterworth filter and of the decimate filter is close to 20Hz. This value can be calculated thanks to a spectral analysis of the data. In our case one has \(\omega_{\text{syn}}\) close to 2Hz. Thus with the rule of thumb \(\omega_{\text{r}} > 10 \times \omega_{\text{syn}}\), we retrieve the value given above.

The identified values of the mechanical parameters are summed up in Table 1.

The conditioning number of \(W\) is close to 30. Hence, the system is well conditioned and the designed trajectories are enough exciting.

Direct comparisons have been performed. These tests consist in comparing the measured and the estimated torques just after the identification procedure. An example is illustrated Figure 4. Notice that the estimated torque follows the measured torque closely.

Finally, we simulate the system by means of a SIMULINK model. We use the same speed references and the differential equation given by (15) is solved with the Euler’s method (ODE 1). The step time integrator is of 240\(\mu\)s. Figure 5 shows the measured and the simulated states. They are close to each others.

Table 1: Estimated mechanical parameter values

<table>
<thead>
<tr>
<th>Mechanical parameters</th>
<th>Estimated values</th>
<th>Relative deviation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J) (kg.m(^2))</td>
<td>1.45.10(^{-3})</td>
<td>0.5</td>
</tr>
<tr>
<td>(M_X) (kg.m)</td>
<td>2.2.10(^{-3})</td>
<td>5.0</td>
</tr>
<tr>
<td>(M_Y) (kg.m)</td>
<td>0.9.10(^{-3})</td>
<td>8.7</td>
</tr>
<tr>
<td>(f_v) (Nm.s.rad(^{-1}))</td>
<td>2.4.10(^{-3})</td>
<td>2.6</td>
</tr>
<tr>
<td>(f_c) (Nm)</td>
<td>5.9.10(^{-2})</td>
<td>0.8</td>
</tr>
</tbody>
</table>

\(|\text{Cond}(W)| = 30\)

![Figure 3: Friction torque measured](image1)

![Figure 4: Comparison between the measured and estimated torque calculated through the LS technique](image2)

![Figure 5: Comparison between the measured states (blue) and the simulated states (green)](image3)
It comes that the LS technique identification gives interesting and reliable results even if the assumptions on the noise model are violated.

### 3.2.2 SRIV Method

As explained section 2.3, the instrument matrix is built with the simulated states (see previous section for the simulation parameters) and the algorithm is initialized with the values given in Table 1. The parameters are estimated thanks to (14). These values do not vary consequently (only 2 steps are enough to decrease the correlation between the instrument matrix and the noise on the system). The results are summed up in Table 2.

We have also performed direct comparison (Figure 6). Once again, the estimated torque follows the measured torque closely. As an expected result, the SRIV method gives reliable and interesting results and can be used to identify inertial parameters of robots. The results given Table 2 are close to those exposed Table 1. That means that, in this case, the LS method detailed section 2.2 is as efficient as the SRIV technique detailed section 2.3.

Table 2: Estimated mechanical parameter values calculated through the SRIV method after 5 steps

<table>
<thead>
<tr>
<th>Mechanical parameters</th>
<th>Estimated values</th>
<th>Relative deviation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>J (kg.m²)</td>
<td>$1.45 \times 10^{-3}$</td>
<td>0.4</td>
</tr>
<tr>
<td>Mx (kg.m)</td>
<td>$2.2 \times 10^{-3}$</td>
<td>5.0</td>
</tr>
<tr>
<td>My (kg.m)</td>
<td>$0.9 \times 10^{-3}$</td>
<td>8.2</td>
</tr>
<tr>
<td>$f_v$ (Nm.s.rad⁻¹)</td>
<td>$1.9 \times 10^{-3}$</td>
<td>3.5</td>
</tr>
<tr>
<td>$f_c$ (Nm)</td>
<td>$6.3 \times 10^{-2}$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Cond($W$) = 30

| Figure 6: Comparison between the measured and estimated torque calculated through the SRIV |

### 4 Discussion

In the case of this study, the experimental results tend to prove that the identification technique developed by the IRCCyN robotic team is as efficient and consistent as the SRIV method. This mainly due to the data treatment described section 2.2.2. Indeed, it helps the LS estimator: outliers and leverage points are eliminated while the derivatives of $\mathbf{q}$ are calculated without magnitude and phase shift. In addition, the data filtering is well designed.

It is interesting to mix both methods. Indeed, with a simple analysis, we can detect which parameters are sensitive to noise measurement, noise modelling, high frequency disturbances...

To do so, we introduce an estimation relative error given by (19):

$$\%e = 100 \left| \frac{\hat{X}_{SRIV} - \hat{X}_{LS}}{\hat{X}_{LS}} \right|$$

Where $\hat{X}_{SRIV}$ is the $j^{th}$ parameter estimated by means of the SRIV method and $\hat{X}_{LS}$ is the $j^{th}$ parameter estimated thanks to the LS technique. The results are summed up in Table 3.

In our case, only the parameters of friction torque are quite sensitive. This is mainly due to the fact that they “see” all undesirable effects such as torque ripple, cable wear and imperfect contacts which are not modelled. Hence, that proves the interest of this approach and the use of the SRIV method.

Table 3: Estimation relative error

<table>
<thead>
<tr>
<th>Parameter</th>
<th>%e</th>
</tr>
</thead>
<tbody>
<tr>
<td>J (kg.m²)</td>
<td>0.1 %</td>
</tr>
<tr>
<td>Mx (kg.m)</td>
<td>0.2 %</td>
</tr>
<tr>
<td>My (kg.m)</td>
<td>1.3 %</td>
</tr>
<tr>
<td>$f_v$ (Nm.s.rad⁻¹)</td>
<td>20 %</td>
</tr>
<tr>
<td>$f_c$ (Nm)</td>
<td>7.3 %</td>
</tr>
</tbody>
</table>

During the experiments, it is appeared that the SRIV could be helpful when we are not familiar with the data filtering. We know that the LS estimators are sensitive to noises filtering and if the filtering is not well designed, the results could be controversial. These results will be published in a later publication.
5 Conclusion

In this paper, two identification methods have been tested and compared: the first based on the ordinary LS technique associated with an appropriate data treatment and the second based on the SRIV method. Both methods give interesting and reliable results. Hence, we can choose these techniques for a parametric identification.

In addition, the authors have introduced a simple calculation which enables us to know the parameters which are sensitive to noises and undesirable effects. Future works concern the use of both techniques to identify a 6 degrees of freedom robot.

REFERENCES


