ON APPLICABILITY OF OPTIMAL CONTROL THEORY TO ADAPTIVE SUPPLY CHAIN PLANNING AND SCHEDULING

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Abstract: Decisions in supply chain (SC) planning and control are interconnected and depend on tackling SC uncertainty and dynamics. In this paper, we investigate the applicability of optimal control theory to SC planning and scheduling. The methodology of this paper is based on the literature analysis, fundamentals of control and systems theory, and our own elaborations. We describe the important issues and perspectives that delineate dynamics in SCs, identify and systematize different streams in application of control theory to production, logistics, and SC management (SCM) in the period from 1960-2011. We derive some classifications, perform a critical analysis, and discuss future research avenues. Some drawbacks and missing links in the literature are pointed out. Several crucial application areas of control theory to SCM are discussed. Subsequently, we concentrate on optimal program control, challenges and advantages of its application in the SCM. We conclude that with the help of control theory, robustness, adaptability, and resilience of SCs can be investigated in their consistency with operations planning and execution control within a conceptually and mathematically integrated framework. However, although SCs resemble control systems, they have some peculiarities which do not allow a direct application of control theoretic methods. In this setting, we argue for further development of interdisciplinary approaches to SC optimization. We argue for an extended co-operation between control experts and SC managers that has the potential to introduce more realism to the dynamic planning and models and improve SC planning and control policies.

Keywords: supply chain; dynamics; planning; scheduling; control; optimal program control; adaptation; robustness.

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1. INTRODUCTION

The term “supply chain management” (SCM) was coined in the 1980-90s. A supply chain (SC) is a network of organizations, flows, and processes wherein suppliers, cooperate and coordinate along the entire value chain to acquire raw materials, to convert these raw materials into specified final products, and to deliver these final products to customers.

SCM studies human decisions on cross-enterprise collaboration and coordination processes to transform and use the SC resources in the most rational way along the entire value chain, from raw material suppliers up to customers, based on functional and structural integration, cooperation, and coordination. The impact of SCM on the changes in enterprise management paradigms can be compared with the developments of total quality management in 60-70s and computer-integrated manufacturing in 80-90s.

Since SCM deals a priori with physical movement of produced materials and products and integration and coordination of these processes with each other and with the production processes, dynamics exists in SC inevitably. Along with considerable advancements in (optimal) SC design, planning and scheduling [16], [18], [26], [79] achieved in recent two decades in operations research (OR), it has long been accepted in literature and practice that the effects of various sources of uncertainty should be considered during planning the supply chain (SC) performance and ensuring this performance during the execution control [51], [52], [55], [72]. Therefore, understanding the importance and the impact of dynamics and vulnerability on performance and resilience of SC systems is becoming a more and more important topic in literature and in practice.

In these settings and in view of new information technologies (IT), advanced investigations into SC dynamics and establishing adaptive feedbacks are becoming one of the major challenges in SCM [6], [10], [15], [21], [24], [25], [27], [44], [58], [69], [72], [82], see Figure 1.

Indeed, an important part of SCM issues is concerned with SC dynamics. Let us list a few of them. First, the issues of performance and uncertainty regarding balancing efficiency, complexity, flexibility and robustness are to be named [61], [83], [86].

Second, the problem of “modelled” optimality and real-life executability and adaptability is under consideration [15], [56]. Third, the problematic of sustainability regarding balancing...
economic output and environment-friendly resource- and energy consumption is of great importance [48]. In observing these problems it becomes evident that an important part of SCM problems is related to changes in the SC environment and reacting to these changes, that is, with SC dynamics.

Another challenge of modern SCM is the SCs with both continuous and discrete processes. Such SCs are typical in oil and gas industry, chemistry, etc. [23], [71], [76]. In optimizing the performance of these SCs, the methods are required to consider both continuous and discrete processes. In addition, even the SCs with only discrete processes frequently contain different technological feedbacks, re-manufacturing processes, etc. along their product lifecycle [34].

The achievement of the planned SC performance can be inhabited by changes and perturbation impacts in a real execution environment [51]. Therefore, SCs are to be reliable and flexible enough to be able to adapt their behaviour in the case of perturbations impacts in order to remain stable and resilient by recovering disruptions once disturbed.

Despite the fact that significant advancements have been made in the area of SC optimization, the domain of dynamics has as yet received little systematic or critical attention. Although it is intuitive that dynamics is likely to have impacts on profitability, there is little systematic analysis and documentation of the magnitude of these impacts in the literature both (1) at the planning stage while synthesizing SCs regarding uncertainty and analyzing these plans regarding different execution environments and (2) at the execution control stage while adapting SCs.

In these settings, the extensive development of approaches and models to tackling SC dynamics and considering SC planning and scheduling in terms of execution dynamics, adaptation, and robustness is becoming a timely and crucial topic in SCM. A possibility to address the above-mentioned challenges can open the control theoretic approach.

Control theory (CT) contains a rigor quantitative basis for planning optimal control policies including differential games and stochastic systems, stability of controlled processes and non-linear systems, controllability and observability, and adaptation [3], [7], [11], [13], [14], [21], [30], [57], [69], [70], [75], [77]. These tools can be applied for a wide range of systems, from discrete linear to stochastic non-linear systems with both stable and dynamically changing structures. CT can also be applied for analysis of equilibriums regarding resource consumption and system output [74], [75].

In this study, we analyse the applicability of CT approaches to the SCM domain based on recent literature on dynamics in SCM, recent literature on applications of CT to SCM, and our own elaborations. Special attention will be paid to optimal program control (OPC) and the domain of adaptive SC planning and scheduling. The purpose of this paper is to describe the important issues and perspectives that delineate dynamics and adaptation in adaptive SC planning and scheduling, comment on methodical issues, and conceptually describe one specific context of the OPC application to the adaptive SC planning and scheduling area.

The rest of this paper is organized as follows. We start with a state-of-the-art analysis. Section 2 analyses particular features of SCM problem regarding SC dynamics. Section 3 reveals general advantages and shortcomings of CT as applied to the SCM domain. In Section 4, we focus our discussion on OPC and its application to SCM and describe an OPC-based framework of interlinking SC synthesis and analysis domains. We conclude the paper by identifying future research avenues in Section 5 and summarizing the results of this study in Section 6.
2. ISSUES OF SUPPLY CHAIN DYNAMICS

Only a few years have passed since SCM has been considered just as an extension of logistics or procurement management. Nowadays, the understanding of SCM as a wider concept and, actually, as an independent scientific discipline and as one of the key management functions in enterprise is widely understood [17], [19], [36], [78].

SCs are characterized by a set of interrelated structures such as organizational, functional, informational, financial, etc. Decisions in all the structures are interrelated. Moreover, the structures are subject to changes; hence, SC structure dynamics [47] is frequently encountered. The dynamic SC characteristics are distributed upon different structures, e.g. organizational (i.e., agile supply structure, functional (i.e., flexible competencies), product-based (i.e., product flexibility [32]), informational (i.e., fluctuating information availability [8], financial (i.e., cost and profit sharing). This multi-dimensional dynamic space along with the coordinated and distributed decision-making lead us to the understanding of modern SCs as multi-structural active systems with structure dynamics [44], [47].

Besides, the SC execution is accomplished by permanent changes in the internal network properties and the environment. The changes set limits on the SC performance. The limits on the performance require the stability and robustness analysis [62] and establishing SC adaptation to a real execution environment. The dynamics, feedbacks and not determined considerations of future make SC process non-stationary and non-linear [64].

In addition, in recent years, the works on covering SC dynamics have been extended by developments in information technologies such as RFID (Radio Frequency Identification), SCEM (Supply Chain Event Management) and mobile business provide a constructive basis to incorporate the stages of SC planning and execution [58], [81]. However, along with considerable advancements in discrete SC optimization, the domain of SC dynamics analysis and adaptive (re)planning and (re)scheduling still remains ill-investigated. Increase in complexity and multi-dimensionality of SCM problems necessitates that methodology of operations research closes with systems and CT, artificial intelligence, and informatics [46].

Summarizing, problems of SC planning and scheduling are challenged by high complexity, combination of continuous and discrete processes, integrated production and transportation operations as well as dynamics and resulting requirements for adaptability and stability analysis. Therefore, SCs are to be considered as adaptive systems. A possibility to address the above-named issues opens CT and OPC in particular.

CT is becoming of a greater interest to researchers and practitioners [8], [10], [21], [25], [40], [44], [45], [47], [72], [73], [87]. CT is favorable in the cases of many dynamically changing control parameters, obtaining analytical solutions or properties, and in investigating different mutual impacts of SC planning and scheduling parameters (e.g., demands, resource and channel capacities, lead-time, lot-sizes, and inventories) on the SC tactical and operative performance (i.e., service level and costs). In some cases (e.g., if many changes, many stages, and many periods), it is convenient to transit from a discrete problem statement to continuous solution procedure, and then represent the result again in discrete terms due to particular accuracy of continuous time models.

Attraction of CT can be seen as the next crucial step in the development of SCM theory to reflect the real-time dynamics and dynamic optimization of SC structures and processes as well as explore robustness, stability, and adaptability in the real-time mode taking into account non-linearity, non-stationarity and uncertainty in SCs.
3. PRO- AND CONTRA OF CONTROL THEORETIC APPROACHES TO SCM

Dynamics in SCs can be referred to both the dynamics of a process under optimization (dynamics of the transition from an input to an output state) and the real-time dynamics regarding the feedback-loop consideration and adaptation in accordance to an actual execution environment. Attraction of CT can be favorable for both the SC synthesis and analysis stages.

Sarimveis [72] underline the resemblance of SCs to engineering dynamic systems. Previously, CT has been extensively applied for management and economics applications [74], [75]. In the SCM domain, application of CT has been increasingly developed since 1980s beginning with the works on inventory control (e.g., by Axsaeter [4]) and SC dynamics (e.g., by Wikner et al. [89]). The applied tools vary from classical transfer function analysis to model predictive control.

In reviewing the literature, the following problem domains can be indicated:

- Dynamic inventory control policies,
- Optimal multi-stage, multi-period production planning,
- Analysis of disturbances and fluctuations (e.g., bullwhip-effect, and stability),
- Integrated marketing-production decisions,
- Adaptation and real-time control, and

Let us consider the existing literature on these five domains.

**Linear dynamic production-inventory control policies**

Beginning with the study by Holt et al. [41] that seems to be the first study on using calculus of variations to solve production-inventory problems, integrated production-inventory problems with lot-size and capacity optimization have been extensively considered in this domain [4], [5], [24], [29], [33], [40], [65], [73]. For example, Disney et al. [24] and Hoberg et al. [40] investigated recently the effects of inventory control policies on order and inventory variability with linear classical CT. Bensoussan et al. [8] considered possible information delay and incompleteness in the ordering policies for inventory decisions. General models and additional literature overview can be found in the work by Sethi and Thompson [75].

However, the authors frequently point out certain limitations in linear CT analytics. However, the authors frequently point out certain limitations in linear CT analytics. For example, linearity requires strong assumptions regarding demand backordering and negative orders (i.e. physical returns) and imposes heavy burdens on possible relationships between parameters as you pointed out. These limitations can be eliminated by advantages of system dynamics theory as shown in the study by Villegas and Smith [85] on analysis of inventory and order oscillations trade-off. One of the advantages of SD approaches is to deal with the non-linear issues of SC dynamics. For further reading on application of classical CT to logistics and SCM, we refer the readers to the studies by Ortega and Lin [65] and Sarimveis et al. [72].

**Optimal planning of multi-stage, multi-period, non-stationary processes**

Optimal design, planning and scheduling is another application of CT to SCM domain [53], [47], [72]. Due to their dynamic and uncertain nature, production-inventory problems can be naturally formulated as OPC problems. Dynamic programming [7] and Pontryagin’s maximum principle [70] are the standard procedures to obtain an optimal state feedback control law for deterministic and stochastic optimal control problems.
OPC is a method for solving dynamic optimization problems, when those problems are expressed in continuous time and the value of a goal criterion (or a number of criteria) are accumulated over time. OPC is a deterministic control method as opposite to the stochastic optimal control [30]. One of the basic milestones in modern OPC, along with dynamic programming, is the maximum principle that was developed in 1950s by Russian mathematicians among whom the central character was Lev Semenovich Pontryagin.

Maximum principle is an original method for computing OPC when optimizing system behavior over many periods of time under constrained control subject to several decision variables where other techniques can become analytically and computationally difficult to apply. The initial formulation of maximum principle was concerned with the problem of transfer a space vehicle from one orbit to another with minimum time and minimum fuel consumption.

According to the maximum principle, the optimal solution of the instantaneous problems can be shown to give the optimal solution to the overall problem [9], [38], [71], [75]. Maximum principle basically generalizes the calculus of variations and builds the basis of the OPC theory. Even the development of the maximum principle has stimulated the application of OPC to many industrial and engineering applications.

The work by Fan and Hwang [28] and Hwang et al. [42] were among first studies on application of discrete maximum principle to multi-level and multi-period master production scheduling and inventory control. Hwang et al. [42] determined production planning as optimal control action and corresponding trajectory of state variables (the inventory) by means of the maximum principle subject to minimization of costs.

The stream of production scheduling has been continued by Kinemia and Gershwin [50], who applied hierarchical method in designing the solution procedure to the overall model, and Kogan and Khmelnitsky [49], [54], who applied the maximum principle in discrete form to planning continuous-time flows in flexible manufacturing systems and transited from the hierarchical approach to heuristic rules for OPC calculation.

Although the OPC application has been widely understood at the tactical planning level, the research on OPC for detailed dynamic production and transportation scheduling in the integrated SC context is fairly recent, although there is a wealth of works in these direction from the enterprise perspective and treatment those problems in isolation (production scheduling and routing). This rapidly emerging field of integrated, customer-oriented SC scheduling [16] can become a new application area of OPC for SCM.

**Analysis of disturbances and fluctuations**

Another fundamental domain of CT is related to analysis of disturbances, fluctuations, robustness and stability of SCs. There is considerable variation in the definitions of terms related to SC uncertainty, robustness, and performance [52]. Basically, there are three main properties of an SC which can be analyzed regarding uncertainty. These are: (1) the ability to cope with volatility and continue plan execution after being perturbed, (2) the ability to remain stable and achieve the planned performance in the presence of disturbances, and (3) the ability to maintain, execute and recover (adapt) the planned execution along with the achievement of the planned (or adapted, but yet still acceptable) performance. In the systems and control theories, these properties are analyzed as stability (property 1), robustness (property 2), and disaster-tolerance (property 3).
The understanding of stability depends much on the system considered as well as on methods and goals of systems analysis. The study by Daganzo [21] applied classical CT and Lyapunov’s stability metrics to the SCM domain. Disney and Towill [24] applied a discrete linear CT model to determine the dynamic stability of vendor managed inventory SCs. Warburton et al. [88] provided a stability boundary for the continuous time SC ordering decision with regard to BIBO (bounded-in-bounded-out) stability. In some studies, bullwhip-effect has been addressed from control theoretic perspective [22], [66], [67].

Although stability analysis is an useful tool, it is subject to many restrictions if applied in the classical form of Lyapunov’s stability or BIBO stability. First, the classical models imply natural movement of objects. Second, they typically consider very small deviations of control and output variables. Third, stability analysis can help in estimating SC volatility in any concrete state. But it is not enough to stabilize the SC – the SC should also bring profits; hence, the inclusion of performance considerations (i.e., the robustness analysis) is required as the next step. Finally, classical stability analysis is concerned with funding equilibrium states for mechanical and automatic systems.

**Integrated production-marketing decisions**

The work in this domain has started with the study by Pekelman [68] and has been continued by Sethi’s advertising model [75], optimal pricing and production model by Feichtinger and Hartl [29], and many other applications up to now (e.g., [1], [31], [39], [53]). In recent years, continuous-time maximum principle has been applied to revenue management and dynamic pricing as a stochastic optimal control problem [59].

**Adaptation and real-time control**

The very extensive area of CT applications to SCM domain is related to the adaptation and real-time control. Actually, it is the area for many SC scholars and professionals to which they refer first while talking about the CT. Adaptive control (AC) is a control strategy with some form of recursive system identification (Astrom and Wittenmark 2008). However, classical AC approach has not found a wide application to the SCM domain so far except of some studies, e.g., by Kim et al. (2005).

A popular technique of SC adaptation in the modern CT is the model predictive control (MPC) [10], [69], [71], [73]. In MPC, a system model and current and historical measurements of the process are used to predict the system behaviour at future time instants. A control-relevant objective function is then optimized to calculate a control sequence that must satisfy the system constraints. The MPC approach is not simply to run the planning frequently, but rather to develop decision policies [87]. Applications of MPC to multi-echelon production–inventory problems and SCs have been examined previously in the literature. Perea et al. [69] modeled multi-plant, multi-product polymer process through difference equations and schedule optimisation in MPC framework. Braun et al. [10] developed a decentralized MPC implementation for a six-node, two-product, and three-echelon demand network problem. In the study by Puigjaner et al. [71], a multi-stage stochastic model has been employed. However, the stabilizing controllers still remain a critical bottleneck in MPC [60].

A critical issue in applying MPC to SCM is the centralized controller and its functions. In technical systems, the controller is a technical device (e.g., a sensor) that adapts the system behavior based on error identification within milliseconds. The controller in the SC is a manager, or more precisely, a number of managers with possible interest conflicts. Even if a deviation in the SC execution has been identified (e.g., track delay identification with the help
of RFID), the MPC controller will not be able to change anything in this situation. The role of this model will be to identify the deviations, to notify the SC managers, to estimate the impact of this disturbance on SC performance, and to develop any recommendations on the adaptation. That is why additional research is needed for analyzing the applicability of MPC to human-driven SC adaptation.

Based on the state-of-the-art analysis, we now derive the basic problem domains for application of different control theoretic approaches to SCM (see Table 1).

Table 1. Methodical contributions to the supply chain dynamics domain

<table>
<thead>
<tr>
<th>Problem domain</th>
<th>Control theoretic approaches</th>
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<tbody>
<tr>
<td></td>
<td>Automatic regulation CT</td>
</tr>
<tr>
<td>Design, Planning and Scheduling</td>
<td>Limited application</td>
</tr>
<tr>
<td>Inventory control</td>
<td>Linear inventory control policies</td>
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<tr>
<td>Analysis of disturbances and fluctua-</td>
<td>Lyapunov’s / BIBO stability; Bullwhip effect</td>
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<tr>
<td>tions</td>
<td></td>
</tr>
<tr>
<td>Adaptation and real-time control</td>
<td>Limited application, because of SC discreteness</td>
</tr>
<tr>
<td>Production-marketing decisions</td>
<td>Limited application</td>
</tr>
<tr>
<td>Advantages</td>
<td>Dynamics in decisions, especially in the inventory domain, non-stationary performance indicators</td>
</tr>
<tr>
<td>Possible limitations</td>
<td>Problem dimensionality; delay time; numerical instability,</td>
</tr>
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</table>

Table 1 provides a systematic overview of the correspondence between practically important SCM problems and CT. In analyzing the Table 1, it can be concluded that OPC approaches can be efficiently used for optimal design, planning and scheduling of complex multi-stage, multi-period and multi-commodity SCs. However, the important precondition is the centralized information over the whole network and relevant parameters. In practice, it is rather rare that enterprises really share data on inventories, demands, and capacities. Therefore, OPC can be efficiently applied for centralized information strategies. Such cases are very common in SCs, e.g., with strong original equipment manufacturers, by applying the CPFR (collabora-
tive planning, forecasting and replenishment) coordination strategy or in SCs which are managed by logistics service providers. However, even if the decentralization and antagonistic goals of enterprises in an SC do not allow the implementation of the OPC, the optimal solutions under the assumptions of full information sharing may be considered as an orientation for quality estimation of the decentralized plans and schedules that can be found, e.g., with the help of agents.

The linear CT can be efficiently applied to multi-echelon inventory control problems with full information sharing, e.g. vendor-managed inventory. In addition, information delays can be efficiently investigated with linear CT [8]. Besides, bullwhip-effect and stability of SCs can be approached in the frequency domain of the linear CT, however, under the assumption of relatively small dimensionality. For the stage of execution control (again, under the assumption of immediate and full information sharing), MPC methods can be applied.

In further observing Table 1, the following research issues can be indicated:

- **Issue 1 – Synthesis models - complexity and content of the planning models and their interconnection with control models.** SCs are becoming more and more complex. It is becoming more and more difficult to represent the ever more complex SCM problems within only one model taking into account dynamic SC processes.

- **Issue 2 – Analysis models - executeability, robustness, stability and adaptation.** The achievement of the planned performance can be inhibited by perturbation impacts. This forces the research on replanning and control to make SCs reliable and flexible enough to be able to adapt in the case of perturbations and to remain stable and robust by recovering disruptions once disturbed.

With regard to the above-mentioned research gaps, attracting CT methods is becoming timely and crucial. *SCs resemble control systems as a multi-stage dynamic flow of materials with information feedbacks.* CT contains a rigor quantitative basis for planning multi-stage dynamic systems. CT tools can be applied to a wide range of systems, from discrete linear to stochastic non-linear systems with both stable and dynamically changing structures. In Table 2, we summarize possible applications of modern CT results to SCM domain.

### Table 2 - Applications of modern CT to SCM

<table>
<thead>
<tr>
<th>The main results of modern CT</th>
<th>Implementations for SC control</th>
</tr>
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<tbody>
<tr>
<td>Criteria for existence of a solution</td>
<td>Model verification for SC control</td>
</tr>
<tr>
<td>Criteria for controllability and attainability</td>
<td>Control processes verification for a given time interval / Determination of the constraints on SC goals</td>
</tr>
<tr>
<td>Criteria for uniqueness of OPC</td>
<td>Analysis of possibility to obtain an optimal plan</td>
</tr>
<tr>
<td>Necessary and sufficient conditions of optimality</td>
<td>Preliminary analysis of optimal program controls; generation of basic SC planning algorithms</td>
</tr>
<tr>
<td>The program control and feedback control</td>
<td>SC planning, scheduling and execution control models on united methodological basis</td>
</tr>
<tr>
<td>Criteria for stability and sensitivity</td>
<td>Evaluation of SC robustness, stability and sensitivity for environmental impacts and alteration of input data</td>
</tr>
</tbody>
</table>
CT takes into account dynamics, real dimensions, non-linearity and non-stationary of SC processes. However, although SCs resemble control systems, they have some peculiarities which do not allow direct application of CT methods. Classical CT leads us to the fields of automatic control. In SCs, the controllers are human beings. SC tuning occurs not within milliseconds but with a time delay. SC managers consciously tend to take risks. In the light of multi-criteria problems, the decisions in SCs are typically taken based on individual psychological risk perceptions and preferences. Hence, interactive tools for multi-criteria decision-making are needed. Decision-making in SCs is of a discrete nature. In technical control systems, it is assumed that control is selected continuously. Besides, in OCT, differential equations are expressing the process dynamics of the systems’ behavior in the input-output context. This non-linear mathematics is very complicated and is more suitable for automatic control.

4. OPTIMAL PROGRAM CONTROL FOR ADAPTIVE SUPPLY CHAIN PLANNING AND SCHEDULING

4.1 Practical needs for application of optimal control and maximum principle to SC optimization

Recall that according to the maximum principle, the optimal solution of the instantaneous problems can be shown to give the optimal solution to the overall problem. If so, it is a very convenient approach to naturally decompose a problem dynamics horizontally into some subproblems to which optimal solutions can be found, e.g., with the help of mathematical programming, and then link these solutions with the help of OPC.

This property is of a great practical importance for SC optimization. Indeed, it is frequently difficult or impossible to accumulate all the necessary information on SC dynamics at the initial planning point of time. In this setting, adaptive planning and scheduling concepts are frequently applied when a plan is modified periodically by change in the SC parameters or the characteristics of control influences on the basis of information feedback about a current system state, the past and the updated forecasts of the future [44].

Another practical challenge is flexible resource, capacity, and flow allocation to dynamically changing environmental and internal conditions (e.g., demand, SC structure, collaboration and coordination rules). Integrated logistics planning by 4PL (fourth party logistics) service providers face along with a detailed treatment of dynamic parameters such as varying capacities in problems with multiple plants and distribution centres at different locations. An increasing number of companies now adopt make-to-order and assemble-to-order concepts. In many industries (e.g., perishable or seasonal products or process industry), finished orders are frequently delivered to customers immediately or shortly after production without intermediate inventory. In general, specific SC collaboration and coordination evidence to extend the models of SC optimization to the dynamics domain.

Last but not least – a crucial topic is the impact of uncertainty and disruptions. Big centralized models for planning the whole time horizon are very sensitive to changes in data availability. The existence of a great diversity of different dynamic characteristics in those problems SCs can significantly impact SC performance. SCM is based on information sharing and coordination, and many SC optimization model assume full information availability. However, due to dynamic changes and coordination problems in the SC it is frequently impossi-
ble. If such a disturbance takes place two issues occur: Is the SC able to continue its operation? Can mathematical models work with incomplete or delayed information? In the light of the above-mentioned practical challenges, the application of OPC and maximum principle to SC optimization can be very favorable.

4.2. Methodical challenges for application of OPC to SC optimization

The application of the OPC to SC optimization is not a trivial problem. Discrete time and discrete quantities of SC operations in both production and logistics SC parts can make the SC planning and scheduling problems intractable. Despite OC models make it possible to reflect dynamics, the consideration of sequencing and resource allocation in these models is significantly complicated by specific mathematical features.

For example, the derived function from the arising sectionally continuous functions is infinity [63]. In addition, such problems as numerical instability, nonexistence of gradients, and non-convexity of state space should be named. In addition, the problem of continuous time and state variables in canonical OPC statements and discrete times and quantities in SCs exist. SC scheduling could not be performed in applying conventional form of OPC formulation.

However, the maximum principle permits the decoupling of the dynamic problem over time using what are known as adjoint variables or shadow prices into a series of problems each of which holds at a single instant of time. This property of optimal control is very helpful when interconnecting mathematical programming (MP) and OPC elements.

4.3. Synthesis (planning) domain

In the studies by Ivanov et al. [47] and Ivanov and Sokolov [44], an original SC representation as a dynamic system with changing structural characteristics has been developed. This idea is based on the observation that during the planning horizon, different structural elements (decision-makers, processes, products, control variables, constraints, goals, perturbations, etc.) are involved in decision-making on SC planning, and not all of them at the same time. In moving on through the planning period, these elements appear and disappear from the decision-making. If so, there is no need to consider all the structural elements at the same time in a large planning problem in steady-state environments. Moreover, the solution procedure becomes undependable from the continuous optimization and can be of discrete nature, e.g., a linear programming, transportation problem, or integer allocation problem [45].

This idea is to some extent similar to those in combining MP and meta-heuristics that uses an exact method over restricted portions of the solution space subject to a given problem of a very large feasible space. By taking optimal decisions within these certain intervals, we can address the problems of significantly smaller dimensionality. This means, that the set of feasible solutions is presented dynamically, but the solution at each point of time are calculated at the local section and for deterministic problems very small dimensionality. This is very important as the computational time decrease considerably even if a large number of nodes or arcs area considered and additional constraints are introduced. Besides, the a priori knowledge of the SC structure, and moreover, structure dynamics, is no more necessary.

Let us present a possible scheme for applying maximum principle to an SC scheduling problem that is similar to job shop scheduling [45]. The scheduling model is formulated as a linear non-stationary finite-dimensional controlled differential system with the convex area of admissible control. The non-linearity is transferred to the model constraints. This allows us to
ensure convexity and to use interval constraints. Besides this, the required consistency between OPC and MP models is ensured – although the solver works in the space of piecewise continuous functions, the control actions can be presented in the discrete form as in MP model.

The developed model formulation satisfies the conditions of the existence theorem by Lee and Markus which allows us to assert the existence of the optimal solution in the appropriate class of admissible controls and to calculate the OC with the help of maximum principle. This is the essential structural property of the proposed approach in order to apply discrete optimization for OPC calculation. In maximizing Hamiltonian in OC computing, this makes it possible to solve the assignment problem and the flow distribution problem both in discrete and continuous manner. In this aspect, the proposed approach differs from the scheduling with the help of maximum principle with only continuous control variables or discrete maximum principle. The model can work with both continuous and discrete processes. The discretization is possible since the optimization problem is in fact a MP problem. This is mainly due to the fact that the governing dynamics in the supply network are linear in the state (but not in the control) variables.

On the basis of the maximum principle, the original problem of OPC is transformed to the boundary problem. Then a relaxed problem is solved to receive OPC vector (i.e., the SC schedule). For OC computation, the main and conjunctive systems are integrated. The control vector at time \( t = t_0 \) returns a maximal value to the Hamiltonian. Then we make the first integration step with the value of control vector at time \( t_0 \) and again implement the maximum principle to receive the next value for time \( t=t_1 \). The process of integration is continued until the end conditions are satisfied and convergence accuracy is adequate. For obtaining the vector of conjunctive equation system, the Krylov-Chernousko method is used that is based on joint use of modified successive approximations method and branch-and-bound method.

Let us turn to the adaptation level. At the scheduling stage, different levers (material inventory, financial reserves, and IT) to mitigate uncertainty and to ensure SC execution control under the presence of disturbances are built. As the SC execution is inevitably followed by changes of both environment and SC itself, the adjustment of SCs is needed. A convenient way to approach this issue is the concept of adaptation.

The main purpose of the adaptation framework is to ensure SC tuning with regard to changes in the execution environment and planned values of performance indicators. In Figure 2, the adaptive framework is presented (based on Skurikhin et al. [80]).

![Figure 2. SC adaptive scheduling and execution control](image_url)
A hierarchy of adjustment actions is brought into correspondence with different deviations in the SC execution [43], [47]. At the constructive level, the above-mentioned integration is realized by the state variables and conjunctive variables that memorize the schedule execution and use this information for schedule update using the maximum principle.

4.4. Analysis domain

A crucial application area of CT to SCM is SC dynamics analysis. Even in this area, the potential of CT can be applied to SCM to great extent. For example, it becomes possible to investigate the SC robustness as an ability to continue schedule execution and to achieve the planned output performance in the presence of disturbances.

With an original representation of the SC schedule as OPC, robustness objective can be integrated as a non-stationary performance indicator in SC scheduling decisions. We propose to apply the method of attainable sets (AS) to calculate the robustness index for SC schedules and to obtain the attainable sets for interval data with no a priori information about perturbation impacts, i.e., for the most severe case of non-stationary perturbations.

The AS approach is to determine a range of operating policies (the union of which is called as an AS) during the scheduling stage over which the system current performance can be guaranteed to meet certain targets, i.e., the output performance [44]. The AS characterizes all possible states of the SC schedule subject to different variations of SC parameters in nodes and channels (e.g., resource capacity availability).

The AS is calculated from the main OPC vector. Here, the perturbation impacts play the role of control variables. In varying these perturbations at each instant of time over the schedule within the time interval and setting these variations into the initial differential system, a set of points where the SC schedule can be steered to is generated. In other words, a set of alternative OPC vectors is generated through admissible variations of perturbation impacts and forms herewith the AS of the SC schedule under disturbances.

However, if the dimensionality of control and state vectors is high, the construction of an AS is a rather complicated problem. That is why an AS is usually approximated in different forms. E.g., the construction of AS can be restricted to a simple (rectangular) form through the four-point orthogonal projection of the convex attainable sets in the state and performance spaces on each other.

With the help of AS, it becomes possible to create the dynamic projection of the SC schedule execution on different uncertainty scenarios. Therefore, AS can be used to calculate the corresponding robustness metric for SC schedules. This metric can be used for ranging alternative SC schedules and corresponding SC plans subject to individual risk perceptions of decision-makers.

With the help of AS, scheduling decisions can be brought into correspondence to the higher level decisions and used as estimates for input data needed for making planning decisions. The schedule can be analysed with regard to both output performance indicators and robustness. If none of the generated schedules provides a satisfactory level of performance and robustness, parameters of the SC plans (e.g., resource capacities, inventories, lot-sizes, delivery data, etc.) can be tuned. Besides, such an analysis can reveal that a very cost-intensive SC plan attains the same schedule robustness as a more cost-efficient SC plan. An SC planner can analyse the perturbation impact on SC schedules and how these changes influence the planning output performance indicators.
5. FUTURE RESEARCH DIRECTIONS

CT can be applied to some problems in SC planning and control, however, not always. In many aspects, SCM domain-specific modifications of CT methods are required. The objective of future research program should be to develop fundamental theory and optimization based methodologies and computational tools that enable SC managers and engineers to analyze, design and evaluate SCs as dynamic systems which are economically attractive, while at the same time exhibit good non-stationary performance characteristics like stability, flexibility, controllability, robustness, and resilience. This can be achieved if the variety of CT advantages (whole multi-stage SC dynamics, feedbacks, time-independent dimensionality, etc.) could be embedded into the models of SC design, planning, and control. The future research can involve the following main strands:

SC planning and control: Here the research program is concerned with the development of dynamics control policies for the design of adaptable SCs with many dynamic characteristics. Novel SC synthesis concepts should be explored together with life-cycle aspects, leading to new designs with dynamic changes in economic efficiency and process non-stationary.

Integration of operability objectives in SC planning decisions: Here work has to be centered on the development and implementation of novel analytical tools to simultaneously assess process stability, flexibility, controllability, robustness, resilience of SC systems and the systematic incorporation of these tools at the planning and execution control levels.

SC optimization under uncertainty - theory, algorithms and applications: Here the development of the interdisciplinary mathematical theory, numerical algorithms and efficient computational tools for the solution of SC planning and control problems with dynamic characteristics, which arise in the context of the work described in the other two research strands, is needed.

In the light of the above-mentioned research strands, the following avenues for future research can be identified.

Multi-dimensional planning, control and interdisciplinary models

It can be observed from the results of Table 1 that different methods for tackling SC dynamics have different application areas, advantages and disadvantages. These possible limitations reduce the application areas and lead to may unrealistic assumptions in the models which rarely reflect multi-stage, multi-period and multi-product nature of real decision-making problems.

For example, lead times are not fixed and are not known with high accuracy, inventory levels should be bounded below by zero and above due to warehouse capacities so as the production rates which are limited by the machinery capacities. Another limitation is that single stage or at best dyadic systems are usually studied, assuming production of a single product or aggregated production. In real life systems, various products are produced and moved with different production rates, order sizes and different lead times, which, however, share common machinery and storage facilities. Horizontal integration is often represented by considering the supply chain stages in a raw, while interconnections between different level and same level stages are ignored [72]. Finally, dynamics of transport tariffs, raw material costs, labor costs and inventory costs are rarely taken explicitly into account.

At the same time, the possible limitations of certain methods are frequently exactly compensated in the advantages of other methods. Although those methods appear to differ in targets,
presumptions, application areas, enabling technologies, and research methodologies, each compliments the others and endeavors to improve decision-making [46]. In the light of this, the interdisciplinary contributions can be stated as a future research avenue to reflect multidimensional dynamic characteristics of SC processes.

Let us provide an example [44]. In integrated SC production and transportation scheduling models, different goals, variables and constraints can be expressed either in static or in dynamic form. In applying only one solution method, e.g., mathematical programming or OPC, significant difficulties in representing both static and dynamic aspects are frequently encountered. Therefore, it can be sensible to distribute static and dynamic elements between different models where the corresponding static or dynamic elements can be expressed in the best way.

Besides, during the planning horizon, different structural elements (decision-makers, processes, products, control variables, constraints, goals, perturbations, etc.) are involved in decision-making on SC planning, and not all of them at the same time. In moving on through the planning period, these elements appear and disappear from the decision-making. If so, there is no need to consider all the structural elements at the same time and to solve planning problems of enormous dimensionality in steady-state environments. By taking decisions within certain intervals (so called intervals of structural constancy [47], we can solve problems of significantly smaller dimensionality.

As these intervals can differ in their duration and these durations are hardly predictable because of perturbations, the splitting of the planning period into those intervals should occur not at the planning stage, but according to the natural logic of time and events, that is in SC dynamics. The transitions between the multi-structural states (both planned and perturbation-driven, e.g., demand fluctuations) is modelled with the help of OPC. Within the intervals of structural constancy, the planning problems will be solved with the help of MP with attracting OPC for representation of time-dependent elements, e.g., transportation frequency.

**Non-stationary performance analysis**

Recent studies indicated different approaches to analyzing impacts of uncertainty and dynamics on SC performance and emphasized that SCs need to be considered with regard to non-stationary dynamic aspects, a real performance and perturbed execution environment. The consideration of stability, volatility, robustness and resilience is either explicit or implicit in these approaches, and is certainly receiving more attention.

Regarding analysis of SCs in terms of dynamics and uncertainty, further investigation in robustness and stability are needed. With the robustness and stability analysis, CT opens constructive ways for integration of non-stationary operability performance objectives in SC synthesis (planning) decisions. In addition, a crucial future research direction should become the investigations in the property to maintain, execute, and recover (adapt) their behavior, i.e. in resilience and disaster-tolerance of SCs. These categories incorporate the analysis of states (stability) and structures (robustness).

Finally, linking SC complexity measurement to a trade-off between maximization of performance and ensuring robustness/stability is of great potential to unveil insights in SCM. Highly complex SCs potentially imply that more resources and effort are required to synchronize and coordinate activities within the network [2]. Understanding and finding SC designs and plans with effective and efficient constellations of complexity, robustness, flexibility, adaptability and resilience is also a promising research area. In this respect, advanced graph theory can
help in analyzing such network properties as density, diversity, linkage, homogeneity, and criticality which are argued in recent literature to be highly related to severity of SC disruptions [20]. This research can provide professionals with tools to specify performance and resilience of the systems and to improve both design and execution control policies according to customers’ values.

**Intellectualization of control**

The incorporation of intelligent IT (e.g., agents or RFID) into CT, also known as intellectualization of control, can provide a variety of methods and tools for dynamics in the SC domain. This can become the area where the knowledge of SC managers and control specialists can be effectively integrated taking advantages of modern IT, e.g., for investigating SC dynamics in different non-stationary environments (not only in discrete or stochastic) or applying RFID to SC monitoring and adaptation [58].

Special focus should be directed to the subjective human decisions on adaptation and multi-criteria decision-making. Control schemes for the real-time adaptation of autonomy level are needed. The SC system and its operators should be modelled symmetrically as dynamic system components according to behavioural levels with different disturbance severity and reaction times. Comparing the human behaviour with the SC execution (model in MPC), a human reference model can be identified for interrelating basic classes of disturbances and corresponding adaptation actions. Schemes should be developed for the adaptation of the autonomy level in real-time based on the individual characteristics. As a result, guidelines can be provided for the dependable design of interfaces between the human operators and the SC.

6. CONCLUSIONS

One of the fascinating features of CT is the extraordinary wide range of its possible applications. The first strong contribution of CT to operations and SCM regarding the dynamics is the interpretation of planning and execution processes not as isolated domains but as an adaptive process. Second, an advantage of CT is the possibility of solving problems with non-stationary and non-linear processes due to the independency of time variable. Continuous dynamic models let us establish and optimize SC performance indicators in dynamics that are difficult to express within a static and discrete time models. Third, the possibility of covering the SC dynamics at the process level and the changes in SC and environment is also a strong contribution of CT. Fourth, CT allows the consideration of goal-oriented formation of SC structures and the solution of problems in this system as a whole. Fifth, as it has been proved in our research, OPC can applicable for SC planning and scheduling with both discrete and continuous processes. In using CT, important categories of SC analysis such as stability, robustness and adaptability can be taken into consideration. In addition, the dimensionality of OR-based problems can be relieved with the help of distributing model elements between an OR-based (static aspects) and a CT-based (dynamic aspects) model.

Although in certain case its application can lead to computational problems, it is possible to formulate the OPC model in such a way to allow efficient OPC computation. For this purpose, a combined application of CT, OR and system dynamics can be favourable. In addition, ipso facto that an SC plan and schedule can be formulated as OPC is a great advantage subject to further dynamics analysis that is a crucial application area of CT to SCM. Even in this area, the CT can be applied to SCM to great extent and enlarge the scope of SC analysis that is currently rather limited.
Another task of future research is to compare centralized and decentralized control strategies. Centralized OPC strategies may potentially generate better solutions rather than decentralized control. At the same time, centralized SC control can be only conditionally performed due to the decentralized decision-making nature in SCs. Here, differential game OPC-based models can be investigated.

The incorporation of intelligent IT into CT, also known as intellectualization of control, can provide a variety of methods and tools for dynamics in the SC domain. This can become the area where the knowledge of SC managers and control specialists can be effectively integrated taking advantages of modern IT, e.g., for investigating SC dynamics or applying RFID to SC monitoring and adaptation.

We can conclude that the success of SC planning and control depend a great deal on tackling uncertainty and dynamics in SCs that evolve over time. CT provides a variety of methods and tools for the SCM domain and allows to take into account dynamics, real dimensions, non-linearity and non-stationary of SC processes. In addition, with the help of CT, non-stationary performance indicators such as robustness and stability can be investigated in their fullness and consistency with operations planning and execution control within a conceptually and mathematically integrated framework.

In future, an interdisciplinary collaboration between SCM and control, and system dynamics researchers is needed. Often, researchers in logistics and SCM apply interesting CT techniques to solve existing problems, but do not have close contact to the progress of research in this field. Similarly, CT researchers are often not aware of the possibilities to connect their work to state-of-the-art logistics and SCM. In limiting the decision-making support on only methods of CT, the domain of SC dynamics would still remain ill-investigated. The combined application of CT with OR and artificial intelligence can potentially enrich the possibilities to develop useful solutions to many practical problems.

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