ADAPTIVE SECOND-ORDER ASYNCHRONOUS CCI CANCELLATION: MAXIMUM LIKELIHOOD BENCHMARK FOR REGULARIZED SEMI-BLIND TECHNIQUE

Alexandr M. Kuzminskiy* and Yuri I. Abramovich**

*Bell Laboratories, Lucent Technologies
The Quadrant, Stonehill Green, Westlea, Swindon SN5 7DJ, UK, ak9@lucent.com
**Intelligence, Surveillance & Reconnaissance Division, DSTO
PO Box 1500, Edinburg SA 5111, Australia, yuri@cssip.edu.au

Abstract

An asynchronous interference cancellation problem is addressed when training and working intervals are available containing the desired signal and arbitrary overlapping interference. A likelihood ratio (LR) maximization approach is developed for estimation of the structured correlation matrices over both training and working intervals for the Gaussian data model and exploited to obtain a performance benchmark for ad-hoc estimators. A regularized non-iterative estimation of the antenna array coefficients is proposed, which employs the autocorrelation matrix estimation as a weighted sum of the autocorrelation matrices estimated over the training and working intervals. It is shown by means of simulation in TDMA and OFDM environments that the regularized semi-blind solution significantly outperforms the conventional estimators and demonstrates performance close to the LR based benchmark.

1. INTRODUCTION

Conventional space-time equalization and interference cancellation techniques in wireless communications exploit known training symbols to estimate the weight vector of an antenna array. The underlying assumption for these techniques is that the training data is reliable since the co-channel interference (CCI) overlaps with the training symbols of the desired signal. Normally, this is the case for the synchronous CCI, which has the same time-frequency structure as the desired user. Asynchronous cells, packed transmission and other modern techniques lead to more complicated asynchronous or intermittent CCI scenarios [1,2 and others], where the interference may partially overlap or not overlap with the training data of the desired signal.

It is pointed out in [1] that stationary filtering can be exploited to enhance the desired signal and reject the synchronous CCI if information data for the signal of interest is involved in the estimation of the weight coefficients together with the training data. Iterative semi-blind algorithms are used for cancellation of the asynchronous CCI in [1,2 and others]. Although iterative receivers may be an effective solution to the considered problem, they are computationally expensive. Availability of training and working intervals with partially overlapping interference allows us to design an analytical semi-blind solution, which involves data from both training and working intervals into estimation of antenna array coefficients. This simple solution can be applied itself or as initialization for different iterative schemes.

We consider the basic narrowband scenario with training and working time intervals containing the desired signal and arbitrary overlapping CCI. We propose an estimation technique based on “stochastic” likelihood ratio (LR) optimization. The basic non-asymptotic LR optimization technique is developed and applied to different problems in [3 and others]. The main idea is that the local solutions (outliers) found by some optimization procedure, which are far away from the global LR maximum, can be found by means of comparison with some pre-calculated threshold. This threshold depends on the LR distribution for the actual parameters, which does not depend on the scenario. Here we use the LR optimization approach for estimation of the structured correlation matrices over both training and working intervals in the asynchronous CCI scenario and propose to exploit the developed procedure to obtain a performance benchmark for ad-hoc estimators.

We develop a regularized non-iterative estimation of the antenna array coefficients, which uses the autocorrelation matrix estimation as a weighted sum of the autocorrelation matrices averaged over the training and working intervals. We demonstrate that this regularized semi-blind solution significantly outperforms the conventional estimators and demonstrates the performance close to the LR based benchmark according to the developed LR optimization technique.

In Section 2 we describe the data model and formulate the problem. In Section 3 an LR based optimization technique is presented. A regularized analytical semi-blind solution is presented in Section 4. The simulation results are given in Section 5. Section 6 concludes the paper.
2. DATA MODEL AND PROBLEM FORMULATION

We consider the following narrowband data model of the signal received by an antenna array of $K$ elements:

$$
\mathbf{x}(n) = \mathbf{h}s(n) + \sum_{m=1}^{M} \mathbf{g}_m \mathbf{u}_m(n) + \mathbf{z}(n),
$$

where $n = 1 \ldots N$ is the time index; $\mathbf{x}(n) \in \mathcal{C}^{K \times 1}$ is the vector of observed outputs of an antenna array; $s(n)$ is the desired signal, $\mathbb{E}[|s|^2] = p_s$, $\mathbb{E}[s(n_1)s^*(n_2)] = 0$, $n_1 \neq n_2$, where $\mathbb{E}[-]$ denotes expectation; $\mathbf{u}_m(n), m = 1 \ldots M$ are the $M < K - 1$ components of CCI:

$$
\mathbb{E}\{\mathbf{u}_m(n_1)\mathbf{u}_m^*(n_2)\} = \begin{cases}
  p_m, & n_1 = n_2 \in \mathcal{N}_m \\
  0, & n_1 = n_2 \in \mathcal{N}_m, n_1 \neq n_2
\end{cases},
$$

$\mathcal{N}_m$ is the appearance interval for the $m$-th interference component; $\mathbf{z}(n) \in \mathcal{C}^{K \times 1}$ is the vector of noise, $\mathbb{E}[\mathbf{z}(n)\mathbf{z}^*(n)] = p_0 \mathbf{I}_K$, $\mathbb{E}[\mathbf{z}(n_1)\mathbf{z}^*(n_2)] = 0$, $n_1 \neq n_2$ and $\mathbf{h} \in \mathcal{C}^{K \times 1}$ and $\mathbf{g}_m \in \mathcal{C}^{K \times 1}$ are the vectors modeling linear propagation channels for the desired signal and interference. All propagation channels are assumed to be stationary over the whole data slot and independent for different antenna elements and slots. The desired signal, noise, and all interference components are assumed to be independent circular Gaussian processes. The training interval of $N_t$ samples, where $K + 1 < N_t < N$, and its position inside data slot are known at the receiver: $s(n), n \in \mathcal{N}_t$. The working data interval $\mathcal{N}_d$ of $N_d = N - N_t$ samples is defined as the rest of the slot. The interference appearance intervals are not known at the receiver, nor are all the propagation channels, interference or noise powers. For simplification of Gaussian modelling we assume that the training samples $s(n), n \in \mathcal{N}_t$ are generated as i.i.d. complex Gaussian random values. While the actual power of the useful signal $p_s$ is unknown, the power of the training signal $p_t$ is set to 1. Note, that this modelling means that strictly speaking we have to simulate different random-like training sequences to accurately fit into the Gaussian assumption. In what follows for analytical derivations we accept Gaussian random-like training signal model, while for actual simulations we model the desired signal and CCI as independent streams of random $(\pm 1 \pm j 1)/\sqrt{2}$ symbols. Despite this distinction, performance of the second-order based adaptive solution is shown to closely follow theoretical bound derived via Gaussian assumptions.

Under the introduced model, the covariance matrices at the training and working data intervals are:

$$
\mathbf{R}_t = \mathbf{v}\mathbf{v}^* + \sum_{m=1}^{M} \hat{p}_m^d \mathbf{g}_m \mathbf{g}_m^* + p_0 \mathbf{I}_K,
$$

$$
\mathbf{R}_d = \mathbf{v}\mathbf{v}^* + \sum_{m=1}^{M} \hat{p}_m^d \mathbf{g}_m \mathbf{g}_m^* + p_0 \mathbf{I}_K,
$$

where $\mathbf{v} = \sqrt{p_s} \mathbf{h}$, $\hat{p}_m^d \geq 0$ and $\hat{p}_m^d > 0$ are the power coefficients depending on the appearance interval $\mathcal{N}_m$ and actual power $p_m$ of each interference source. Here we assume a midamble position of the training sequence, where the $M$-component CCI always appears in the working interval, but some of components may not appear in the training interval. Note, that the covariance matrices (3) and (4) are the “averaged” over corresponding intervals actual time-dependent (nonstationary) covariance matrices.

For such a “stationarized” Gaussian model the optimal linear spatial filter for recovering the desired signal $s(n)$, $n \in \mathcal{N}_d$ can be applied to give

$$
\hat{s}(n) = \mathbf{w}_{opt}^* \mathbf{x}(n),
$$

$$
\mathbf{w}_{opt} = \mathbf{R}_d^{-1} \mathbf{v}.
$$

The problem is to estimate the vector $\mathbf{w}_{opt}$ using an a priori information and all the data available at the receiver in the training and working data intervals.

3. LR MAXIMIZATION TECHNIQUE OVER THE TRAINING AND WORKING INTERVALS

During the training interval we observe $(K + 1)$-variate independent Gaussian training vectors $\mathbf{x}(n) = [s(n), \mathbf{x}^T(n)]^T$, $n \in \mathcal{N}_t$. Taking into account that $N_t > K + 1$, the sufficient statistic at the training interval is

$$
\hat{\mathbf{R}}_t = N_t^{-1} \sum_{n \in \mathcal{N}_t} \mathbf{x}(n)\mathbf{x}^*(n) = \begin{bmatrix} \hat{p}_t & \hat{\mathbf{r}}_t^* \\ \hat{\mathbf{r}}_t & \hat{\mathbf{R}}_t \end{bmatrix},
$$

where $\hat{p}_t = N_t^{-1} \sum s(n)s^*(n), \hat{\mathbf{r}}_t = N_t^{-1} \sum s(n)^* \mathbf{x}(n)$, and

$$
\hat{\mathbf{R}}_t = N_t^{-1} \sum \mathbf{x}(n)\mathbf{x}^*(n) \text{ and }
$$

$$
\mathbb{E}[-\hat{\mathbf{R}}_t] = \begin{bmatrix} 1 & \mathbf{v} \\ \mathbf{v}^* & \mathbf{R}_t \end{bmatrix}.
$$

During the working interval we observe $K$-variate independent Gaussian vectors $\mathbf{x}(n), n \in \mathcal{N}_d$ and again because of $N_d > K$, the sufficient statistic at the working interval is

$$
\hat{\mathbf{R}}_d = N_d^{-1} \sum \mathbf{x}(n)\mathbf{x}^*(n), \text{ where } \mathbb{E}[-\hat{\mathbf{R}}_d] = \mathbf{R}_d.
$$

Let us express the “stationarized” (over the working interval) interference-only covariance matrix as

$$
\mathbf{R}_{CCI} = \sum_{m=1}^{M} \hat{p}_m^d \mathbf{g}_m \mathbf{g}_m^* + \mathbf{U}_M \mathbf{A}_M \mathbf{U}_M^* = \mathbf{U}_M \mathbf{A}_M \mathbf{U}_M^*,
$$

where $\mathbf{U}_M, \mathbf{A}_M$ is the eigen decomposition of the matrix $\mathbf{R}_{CCI}$, i.e. $\mathbf{U}_M^* \mathbf{U}_M = \mathbf{I}_M, \mathbf{A}_M > 0$. Therefore, $\mathbf{R}_d = \mathbf{v}\mathbf{v}^* + \mathbf{U}_M \mathbf{A}_M \mathbf{U}_M^* + p_0 \mathbf{I}_K$

and since $\hat{p}_m^d \neq \hat{p}_m^d$, we get

$$
\mathbf{R}_t = \mathbf{v}\mathbf{v}^* + \mathbf{U}_M \mathbf{A}_M \mathbf{U}_M^* + p_0 \mathbf{I}_K,
$$

$$
\mathbf{B}_{M-m} = \mathbf{V}_{M-m} \mathbf{E}_{M-m}^{-1} \mathbf{V}_{M-m}^* \geq 0, \text{ where } \mathbf{V}_{M-m} \mathbf{V}_{M-m}^* = \mathbf{I}_{M-m}, \mathbf{E}_{M-m} > 0 \text{ and } m (0 \leq m < M) \text{ is the number of sources "missing" at the training interval. The case } m = M, \text{ i.e. none of the interference sources
}$$
overlap with the training interval, corresponds to \( B_0 = 0 \), i.e. \( R_t = v v^* + p_t I_K \). In this paper we assume that the total number of interference sources \( M \) and the number of "missing" sources \( m \) are known or have been estimated. Then the admissible set of the optimization parameters can be introduced as follows:

\[
\hat{A}_t = \begin{bmatrix} 1 & b^* \\ b & A_t \end{bmatrix} > 0,
\]

(8)

\[
A_d = b b^* + D_M L_M D_M^* + d I_K > 0,
\]

(9)

\[
A_t = b b^* + D_M F_{M-m} D_M^* + d I_K > 0,
\]

(10)

where \( D_M \) is a \( K \times M \) matrix, \( F_{M-m} \) is a \( M \times M \) matrix of rank \( M - m \) (again, \( m = M \) means that \( F_0 = 0 \)), \( b \) is a \( K \times 1 \) vector, \( L_M \) is a \( M \times M \) diagonal matrix and \( d \) is a positive scalar.

Since sufficient statistics \( \hat{R}_t \) and \( \hat{R}_d \) are non-degenerate matrices, the ML estimates could be obtained via maximization of a monotonic function of the product of the two likelihood ratios:

\[
\text{Find} \max_{\hat{A}_t, A_d} \gamma(\hat{A}_t, A_d),
\]

(11)

\[
\gamma = (\gamma_t)^{N_t/N_d} \gamma_d,
\]

(12)

\[
\gamma_t(\hat{A}_t) = \frac{\text{det}(\hat{A}_t^{-1}\hat{R}_t)\exp(K + 1)}{\exp[\text{tr}(\hat{A}_t^{-1}\hat{R}_t)]},
\]

(13)

\[
\gamma_d(A_d) = \frac{\text{det}(A_d^{-1}\hat{R}_d)\exp(K)}{\exp[\text{tr}(A_d^{-1}\hat{R}_d)]},
\]

(14)

For Monte-Carlo simulations in Section 5 the following initialization procedure has been adopted: \( d_0 = (K - M - 1)^{-1} \sum_{i=1}^{K-M-1} \lambda_i(\hat{R}_d); b_0 = \hat{r}_t; D_0 = [d_0; \ldots; d_0]; L_0 = \text{diag}[d_0; \ldots; d_0]; F_0 = D_0^* (\hat{R}_t - \hat{r}_t \hat{r}_t^* - d_0^2 I_K) D_0^*; F_{M-m} = \text{RT}(F_{M-m}, m) \), where \( \lambda_i(\hat{R}_d) \) are the eigenvalues of matrix \( \hat{R}_d \) in ascending order, \( d_0 \) are the eigenvectors of matrix \( \hat{R}_d - \hat{r}_t \hat{r}_t^* \) that correspond to the \( M \) largest non-negative eigenvalues \( t_0^2 \) and \( \text{RT}(F, m) \) is the rank truncation operation. Therefore, all the parameters in (11) - (14) are initialized and an optimization procedure over parameters \( b, d, D_M \) and \( L_M \) can be applied with

\[
F_{M-m} = \text{RT}\left[ F_{M-m} \left( \hat{R}_t - b^* b + d I_K \right) P_{M}^{d*} \right],
\]

where \( P_{M}^{d*} = \left( D_M^* D_M^{d*} \right)^{-1} D_M^{d*} \) and \( j \) is the current iteration number. When a numerical solution to (11) - (14) is found, then \( w_{LR} = A_d^{-1} b \text{LR} \).

Automatically, any initialization and locally convergent optimization algorithm cannot guarantee finding the global maximum of the non-convex function in (11). In this situation

\[
\text{it is critically important to have a technique, which allows us to decide with some probability if the local solution is far away from the global maximum, i.e. it can be classified as an outlier. If the outlier is found, another initialization can be applied or it can be just disregarded as in our case, when we are looking for the ML benchmark for ad-hoc techniques.}

The LR based outliers selection technique is developed in [3] for the "stochastic" ML problem. Here we apply it in the considered scenario. The basic idea is that the global maximum of the LR function is always exceeds the LR function for the actual parameters, i.e. for the global solutions \( \hat{R}_t \) and \( \hat{R}_d \) we have

\[
\gamma_{\text{max}} = \gamma_t(\hat{R}_t)^{N_t/N_d} \gamma_d(\hat{R}_d) > \gamma_t(\hat{R}_t)^{N_t/N_d} \gamma_d(\hat{R}_d).
\]

Furthermore, \( \gamma_t(\hat{R}_t) \) and \( \gamma_d(\hat{R}_d) \) do not depend on \( \hat{R}_t \) and \( \hat{R}_d \). In our case:

\[
\gamma_t = \gamma_t(\hat{R}_t) = \frac{\exp(K + 1)\text{det}(\hat{C}_t)}{\exp[(N_t^{-1} \hat{C}_t)^{1/2}]}.
\]

(15)

\[
\gamma_d = \gamma_d(\hat{R}_d) = \frac{\exp(K)\text{det}(\hat{C}_d)}{\exp[(N_d^{-1} \hat{C}_d)^{1/2}]}.
\]

(16)

where \( \hat{C}_t \sim \text{C}\text{W}(N_t, K + 1, I_{K+1}) \) and \( \hat{C}_d \sim \text{C}\text{W}(N_d, K, I_K) \) are the matrices derived from a complex-valued Wishart distribution.

The outliers selection procedure is as follows:

- For the given values of \( K, N_t \) and \( N_d \) and "missing target" probability \( p_t \) find the threshold \( \gamma_0 \) from equation

\[
\text{Prob} \{ \gamma_t(N_t/N_d, \gamma_0 < \gamma_0) = p_t \}.
\]

The required distribution of \( N_t/N_d, \gamma_0 \) can be obtained analytically [4] or by means of Monte-Carlo simulations as in Section 5.

- Find a local solution to the nonlinear constrained optimization problem (11) using the presented initialization and calculate the LR value for this solution \( \gamma \).

- If \( \gamma > \gamma_0 \), accept this solution, otherwise classify it as an outlier.

4. SEMI-BLIND REGULARIZED SOLUTION

One known way to modify the conventional LS estimator to take into account an additional information on a particular problem is to consider the modified (regularized) LS criterion:

\[
\hat{w} = \arg \min_w \sum_{n=1}^{N_t} |s(n) - w^* x(n)|^2 + \rho F(w),
\]

(17)

where \( \rho > 0 \) is a regularization parameter and \( F(w) \) is some regularization function.
In the considered scenario the working interval may be affected by the interference components not presented at the training interval. Thus, selection of the regularization function containing data from the working interval allows us to introduce ability to cancel the asynchronous CCI.

One possibility is to use the quadratic function \( F(w) = w^* R_d w - \hat{r}_t^* w - w^* \hat{r}_t \) leading to the semi-blind solution \[ \hat{w}_{SB} = [(1-\delta)\hat{R}_t + \delta \hat{R}_d]^{-1} \hat{r}_t, \]

where \( 0 \leq \delta = \rho/(1+\rho) \leq 1 \) is the regularization coefficient. One can see that (18) contains the conventional LS \( \hat{w}_L = \hat{R}_t^{-1} \hat{r}_t \) and the burst-based LS \( \hat{w}_{LSB} = \hat{R}_d^{-1} \hat{r}_t \) as the particular cases when \( \delta = 0 \) and \( \delta = 1 \) accordingly.

Thus, one can expect that for some intermediate value of \( \delta \) estimator (18) can demonstrate ability to cancel the asynchronous interference. Our goal is to use a technique developed in Section 3 and compare the ad-hoc estimator (18) with the ML solution in the scenario presented in Section 2.

5. SIMULATION RESULTS

We simulate a four element antenna array and 2-conponent interference according to the asynchronous CCI scenario. The desired signal and interference are simulated as independent streams of random symbols \( \pm \hat{r}_t \). All propagation channels are generated as independent complex Gaussian vectors with unit variance and zero mean. The data slot parameters are: \( N_t = 20, N_d = 80 \). Total signal-to-interference ratio is fixed \( SIR = 0 \) dB and variable signal-to-noise ratio (SNR) is considered. The LR function and MSE performance are estimated in 1000 trials for each SNR value. A priori LR distribution for the known parameters is estimated by means of 40000 Monte-Carlo trials according to equations (15), (16). The threshold \( \gamma_0 \) is calculated for \( \rho_0 = 10^{-7} \). The standard MATLAB optimization routine “fmincon” is used for LR maximization. The MSE performance is estimated for the LS, LSB, and SB estimators over all trials and for the LR based solution over the selected and disregarded trials separately. The regularization coefficient in SB is roughly selected to get the best results.

An a priori LR distribution together with the LR functions before and after optimization are presented in Fig. 1 for variable SNR. Comparison of the initialization and maximization curves shows that effective optimization is applied here because in the most cases the LR values after optimization become higher than the LR values for the actual parameters (a priori distribution). One can see the efficiency of the proposed procedure for outliers finding: the selected trials have significantly lower MSE compared to the outliers, e.g. more than 10 times lower for SNR = 20 dB.

The MSE performance is presented in Fig. 2, in the same scenario. The main conclusion is that the regularized SB algorithm significantly outperforms the conventional LS and LSB estimators and demonstrates the results close to the ML solution.

6. CONCLUSION

An asynchronous interference cancellation problem has been considered. The potential efficiency was studied for the Gaussian data model. An LR maximization approach for estimation of the structured correlation matrices over both training and working intervals was developed. A regularized second-order estimation of the antenna array coefficients has been proposed. It was shown by means of simulation in a TDMA environment that the regularized semi-blind solution significantly outperforms the conventional estimators and demonstrates the performance close to the LR-based benchmark.

7. REFERENCES