Stationarity of Mobility Models Constructed with LEMMA

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Abstract—The Layered Mobility Model Architecture (LEMMA) stipulates that a mobility model can be divided into five distinct layers, which communicate unidirectionally via simple interlayer interfaces. It has been shown that this general framework can be used for the creation of empirical and theoretical models, and that any mobility model can be represented by the form specified by LEMMA. The fundamental result of this paper is the definition and the proof of several theorems related to the stationarity of probabilistic mobility models specified under the form defined by LEMMA. We support our findings with the help of simulation, by performing simulations with 300 different mobility models over a range of nine network setups.

I. INTRODUCTION

Wireless network studies are often using simulations as means to evaluating the different propositions. A crucial part of these simulations is the selected mobility model - the laws that govern node movements - as they can seriously influence the results, e.g. as shown in [1] the ranking of routing algorithms may depend on the used mobility model. Because it is difficult to define which qualities make a mobility model acceptable for all (e.g. universally good) or even for some specific scenarios there have emerged two opposite approaches to mobility modeling - empirical and theoretical. The former creates models based on some empirical evidences (such as WLAN association logs [2], surveys [3], etc.) and tries to reproduce the captured behavior. The latter constructs the movement trajectories based on existing theoretic works, such as studies of the human behavior [4], or the mathematically tractable models, e.g. Random Waypoint [5]. Even though the theoretical models are usually considered "non-realistic" they are the most used ones [6].

The Layered Mobility Model Architecture - LEMMA we introduced earlier [7], [8] defines a framework which allows the creation of mixed models, e.g. containing theoretical and empirical layers, thus bridging the gap. This is made possible because of the strict semantic separation of the layers and the minimal interlayer communication interfaces. In [9] we proved that any stochastic mobility model may be represented under the form specified by LEMMA, thus making it an ideal candidate for model analysis.

In this paper we provide sufficient conditions for the stationarity of a stochastic mobility model defined with LEMMA, and we support our findings with simulations. First, in Section II is introduced the layered architecture, along with the formal definition of mobility model and the specification of other important results related to LEMMA. Section III is the core of our work, where we state and prove the theorems necessary for proving model stationarity based solely on the properties of their layers. Then in Section IV we give the results of the simulations we have performed as a support to the theoretic work from the previous section. Finally, we present the related work in Section V before concluding this work in Section VI.

II. PRELIMINARIES

In this section, we are going to briefly introduce the layered architecture and the basic notions we will be using. For an in-depth discussion on the constructions and the proofs of the theorems, please refer to [7], [9].

A. Mobility Model

**Definition II.1.** Informally, we can say that a mobility model is a mathematical or computer model which can be used to analyze or simulate some (or all) of the aspects of entities movements. Formally, this can be translated as:

\[ m : \Omega \rightarrow E^T \]  

where \( T \) is an index set which may be considered as the time (discrete or continuous), \( E \) is the destination space, \( \Omega \) is the parameter space of the model, and \( E^T \) designates a function \( T \rightarrow E \).

The above definition is very natural, simply stating that a mobility model, given some parameters, produces a trajectory in the required space \( T \rightarrow E \) (i.e. associates a point \( e \in E \) for each time instant \( t \in T \)). Additionally, there are some restrictions that make the above definition mathematically tractable, such as the requirement that \( (\Omega, \mathcal{F}), (E, \mathcal{E}), \) and \( (T, \mathcal{T}) \) are measurable spaces, and that \( m \) is \( \mathcal{F}/\mathcal{E}^T \)-measurable, where \( \mathcal{E}^T \) is the \( \sigma \)-algebra generated by the cylinder sets over the product space \( \otimes_{t \in T} \mathcal{E} \) [10], [11].

B. Layered Architecture Description

The environment in LEMMA is standardized and contains the simulation area, the zones in it, the constraints and the movement influencing factors. Based on these entities, each mobility model is divided into five distinct layers, each having a dedicated task. The communication between the layer is...
always top-to-down and happens via simple interfaces (see Fig. 1).

Here, we are only going to describe the different layers constituting a mobility model:

- **Strategy** - the high-level movement decision-making process. Takes the list of zones from which it selects the next zone to be visited. Furthermore, it specifies how long should the node stay in that zone after reaching it.

- **Zone-to-coordinates mapper** - translates the high-level zone location given by the strategy to “low-level” coordinates suitable for the tactic.

- **Tactic** - the trajectory-generating process. It generates a route from point $a$ (e.g. the current node position) to point $b$ (the destination point - supplied by the mapper), satisfying the set of constraints set by the user.

- **Movement dynamics** - layer specifies the speed and acceleration, and possibly some small deviations from the generated trajectory.

- **Stay** - the layer governing the node once it reaches the destination zone.

### C. Stochastic Processes and LEMMA

A model can be defined by following the LEMMA Model Representation (LMR). This means, that it has been divided into layers, each layer having the functions defined earlier. Here, we are going to give a formal definition of what is meant by LMR when the mobility model is a stochastic process.

First, we start by defining the set of all zones taking part of a scenario $\mathcal{Z} = \{Z_i|i = 0...k < \infty, Z_i \subseteq E, Z_i \neq \phi, Z_i$- closed}. This set depends entirely on the scenario creator, and should be non-empty. Selecting the zones to be closed sets is a convenience which allows us to select points from their borders. Finally, we will denote the union of all zones as $\mathcal{Z} = \cup \mathcal{Z}$.

We will denote with $\mathfrak{B}$ the Borel $\sigma$-algebra. Then, let us define for convenience $\mathcal{G}$ as the set of all $\mathfrak{B}([0;1])/\mathcal{E}$-measurable functions $g: [0;1] \rightarrow E$, and $\mathcal{D}$ as the set of all strictly monotone increasing $\mathfrak{B}([0;1])/\mathcal{T}$-measurable functions $d: [0;1] \rightarrow T$.

### Definition II.2. If the mobility model $m$ is a stochastic process, then we will call the processes corresponding to the different layers (see II-B) - layer processes. Each is defined in its proper, probabilistic space and is a measurable function in the corresponding spaces. These processes all depend on the specific choice of the zones $\mathcal{Z}$, which we will omit for brevity.

(i) A strategy layer process is: $L_{str} : \Omega_{str} \rightarrow (\mathcal{Z} \times T)^N$

(ii) A mapper layer process is: $L_{map} : \Omega_{map} \times \mathcal{Z} \rightarrow \mathcal{Z}^N$

(iii) A tactic layer process is: $L_{tac} : \Omega_{tac} \times \mathcal{Z} \rightarrow \mathcal{G}^N$

(iv) A dynamic layer process is: $L_{dyn} : \Omega_{dyn} \times \mathcal{G} \rightarrow (\mathcal{G} \times \mathcal{D})^N$

(v) A stay layer process is: $L_{sta} : \Omega_{sta} \times T \times \mathcal{Z} \rightarrow (\mathcal{G} \times \mathcal{D})^N$

Note that throughout this work we are going to use stochastic process and random function notations, which we will only refer them as a stochastic process. This equivalence and the necessary transformations, are given in [10].

In a previous work [9] we have proved that any set of layer processes defines a valid mobility model, and more importantly, that any mobility model may be represented with LEMMA. In the next section we show that it is possible to obtain global model knowledge by only having some of the individual layer properties, e.g. our architecture can simplify mobility model analysis.

### III. STATIONARY MOBILITY MODELS

#### A. Stationarity

It is important to know if a model is terminating or stationary, as its type affects the simulation results and if handled improperly can lead to biased conclusions. A terminating simulation is executed until a particular event occurs, upon which the simulation is stopped. A non-terminating simulation can have an arbitrary duration, during which the behavior of the entities should follow some kind of regular behavior. In all cases, node mobility should not affect the performance of the simulated network in unexpected ways. An important characteristic of the mobility models is thus whether if they possess a stationary regime. Here, we are going to provide a way to construct stationary mobility models out of independent layers, where stationary process is a stochastic process with probability laws that do not change over time.

#### Theorem III.1. A probabilistic mobility model $m$ constructed in the form specified by LEMMA (excluding the stay layer) is stationary if its layers are all stationary, pairwise independent processes. Additionally, all curves in $\mathcal{G}$ and $\mathcal{D}$ should be with a finite length.

Note that this is a sufficient, but not a necessary condition, e.g. it may be possible to construct a stationary mobility model even though some of its layers may be non-stationary processes.

**Proof:**

We are going to use the following variants of the layer processes (modifications equivalent to the original ones). Furthermore, without loss of generality, we will assume that all layer processes are defined on the same space $\Omega$. 

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Fig. 1. Node environment and movement process layers.
Modification of the strategy layer process:
\( L'_{\text{str}, z} : \Omega \rightarrow Z^N \) and \( L'_{\text{str}, t} : \Omega \rightarrow T^N \)

Modification of the mapper layer process:
\( L'_{\text{map}} : \Omega \rightarrow (\mathbb{Z})^N \)

Modification of the tactic layer process:
\( L'_{\text{tac}} : \Omega \rightarrow (\mathbb{Z}^3)^N \)

Modification of the dynamic layer process:
\( L'_{\text{dyn}} : \Omega \rightarrow ((\mathbb{G} \times \mathbb{D})^2)^N \)

Using these modifications we can express the compositions of the layers as follows:
\[
L'_{\text{ms}} := L'_\text{map} \circ L'_{\text{str}, z} \\
L'_{\text{tms}} := L'_{\text{tac}} \circ L'_{\text{ms}} \\
L'_{\text{dms}} := L'_{\text{dyn}} \circ L'_{\text{tms}}
\]

The above compositions may be expressed with the help of the operators \( f_2(f, a) := f(a) \) and \( f_3(f, a, \theta) := f(a, \theta a) \):
\[
L'_{\text{ms}}(\omega, n) := f_2(L'_{\text{map}}(\omega, n), L'_{\text{str}, z}(\omega, n)) \\
L'_{\text{tms}}(\omega, n) := f_3(L'_{\text{tac}}(\omega, n), L'_{\text{ms}}(\omega, n), \theta) \\
L'_{\text{dms}}(\omega, n) := f_2(L'_{\text{dyn}}(\omega, n), L'_{\text{tms}}(\omega, n))
\]

where \( \theta \) is a measure-preserving shift operator which transforms \( X = (X_n) \) into \( \theta X = (X_{n+1}) \). We will use the following lemmas in order to prove the theorem:

**Lemma III.2.** If \( X : \Omega \rightarrow F^N \) and \( Y : \Omega \rightarrow B^N \) are pairwise independent, stationary processes (defined in appropriate spaces), and \( F \subseteq A^B \) are measurable functions, then \( V := f_2(X, Y), V : \Omega \rightarrow A^N \) is a stationary process.

**Proof:** By definition \( V(\omega, n) = f_2(X(\omega, n), Y(\omega, n)) \), where \( X(\omega, n) \) is a measurable function such that \( X(\omega, n) : B \rightarrow A \). The random process \( (X, Y) \) of two pairwise independent, stochastic random variables is also stationary. Finally, \( f_2 \) is measurable (\( F \) are measurable functions, and \( X \) and \( Y \) are measurable random functions) which by Lemma 9.1 from [11] provides the stationarity of \( V \).

**Lemma III.3.** If \( X : \Omega \rightarrow F^N \) and \( Y : \Omega \rightarrow B^N \) are pairwise independent, stationary processes (defined in appropriate spaces), \( F \subseteq A^B \times B^A \) are measurable functions, and \( \theta \) a measure-preserving shift transformation, then \( W := f_3(X, Y, \theta), W : \Omega \rightarrow A^N \) is a stationary process.

**Proof:** As \( \theta \) is a measure preserving transformation, \( \theta Y \) is a stationary process, which analogically to III.2 provides the measurability of \( f_3 \), and thus, the stationarity of \( W \).

From lemma III.2 follows that \( L'_{\text{ms}} \) is stationary, because \( L'_{\text{map}} \) and \( L'_{\text{str}, z} \) are such. From lemma III.3 follows that \( L'_{\text{tms}} \) is stationary, because \( L'_{\text{tac}} \) and \( L'_{\text{ms}} \) are stationary, and \( \theta \) is a measure-preserving shift operator. \( L'_{\text{dms}} \) is stationary because \( L'_{\text{dyn}} \) and \( L'_{\text{tms}} \) are stationary, and because of lemma III.2. This provides us with a process choosing the trajectory segments and their duration, which possesses a stationary regime. The finiteness of the trajectories and the movement durations provides the stationarity of \( m \).

**Corollary III.4.** A probabilistic mobility model constructed by the form specified by LEMMA (excluding the stay layer) is stationary if one of the two conditions is fulfilled:
1. Both \( L'_{\text{tms}} \) and \( L'_{\text{dyn}} \) are stationary, pairwise independent processes.
2. \( L'_{\text{ms}}, L'_{\text{tac}} \) and \( L'_{\text{dyn}} \) are stationary, pairwise independent processes.
3. All curves in \( G \) and \( D \) should be with a finite length.

**Proof:** The result follows directly from Theorem III.1 and just gives the precision that an appropriate combination of non-stationary layer processes may be used in conjunction to construct a stationary process.

**Theorem III.5.** A process specified as described in Theorem III.1 with a pause stay layer is stationary.

**Proof:** Adding a pause stay layer (e.g. there is no movement specified during the move duration) to a model \( m \), specified with the layers processes \( L_{\text{str}}, L_{\text{map}}, L_{\text{tac}} \) and \( L_{\text{dyn}} \) makes the process equivalent to a process \( m^1 \) specified with the layers \( L_{\text{str}}, L_{\text{map}}, L_{\text{tac}} \) and \( L'_{\text{dyn}} \), where \( L'_{\text{dyn}} \) is a modification of \( L_{\text{dyn}} \) which the stay time \( t_s \) generated by the stationary and independent process \( L_{\text{tms}} \) transforms the pair \((g,d) \in G \times D \) to \((g^1,d^1) \in G \times D \) as follows:
\[
g^1(x) := \begin{cases} 
  g(1) & x \in (0, \frac{1}{2}] \\
  g(2) & x \in (\frac{1}{2}, 1]
\end{cases}
\]
\[
d^1(x) := \begin{cases} 
  d(1) & x \in [0, \frac{1}{2}] \\
  d(2) & x \in (\frac{1}{2}, 1]
\end{cases}
\]

Indeed, the modification applied to the dynamic layer prolongs each movement cycle with the pause duration, and the trajectory is modified, so that there is no movement during the pause time. This dynamic layer is indeed stationary, as both the original dynamic layer, and the strategy layer (specifying the pause duration) are pairwise independent stationary processes, and the transformation \((g,d) \rightarrow (g^1,d^1) \) is measurable. Thus, from Theorem III.1 \( m^1 \) is stationary, which as we noted is the equivalent of \( m \) with pauses.

Note that this construction is also applicable to a more general case, where the stay layer process is a limited movement around the endpoint of the node, the extent of these limits being dependent on the simulation environment.

**IV. Simulations**

**A. Methodology**

In order to show experimentally the results from the previous section, we have performed a series of simulations. We have selected stationary layer processes for 5 strategies, 5 mappers, 4 tactics and 3 dynamics, summarized in Table I, and have evaluated the stationarity measures of all possible resulting mobility models (300 models). They were evaluated in nine different setups in a 1D environment shown in Fig. 2 - four patterns for the 1000 m simulation area, five patterns for the 2000 m area. The average node density was fixed to 1/10 nodes/meter, which gives 100 nodes for the 1000 m simulations and 200 nodes for the 2000 m simulations.
Fig. 2. The different environments used for the simulations. All four types of zone dispositions have been used with the 1000m and 2000m simulation areas. For the 2000m area we have also performed simulations with zones with length 200m following pattern D.

The choice of the simulation environment was motivated by the existence of well-established, readily available statistical tests for stationarity of 1D time series. Additionally, the illustrative power of such time series is greater when compared to their 2D and 3D counterparts, as it is much easier to demonstrate the evolution over time of a 1D process. Moreover, this is not an impediment to the application or the validity of the findings described in the previous section, as the simulations discussed hereafter are only a demonstration of the theoretic results.

Each of the simulations was 22000 s long and was ran 10 times, each time with a different random generator seed (each experiment had a different seed). The nodes were initially placed uniformly on the simulation area, and first 2000 s of the simulation were discarded to remove any initialization bias.

The analyzed data include the position and the speed of each node, as well as the distribution of the nodes over the simulation area, and the distribution of their speeds. All data were sampled at 1 second intervals, and were then submitted to the augmented Dickey-Fuller test (ADF) [12], [13]. The ADF tests the data against the null hypothesis of a unit disk root, with alternative hypothesis of stationarity of the data. Thus, the null hypothesis has to be rejected for a sample to be generated from a stationary process.

B. Simulation Results

All movement traces without exception were recognized by the ADF test as stationary time series (e.g. the null hypothesis was rejected). Moreover, the majority of the results obtained from the test are statistically significant at the 1% level (as summarized in Table II), with the rest being statistically significant at the 5% level. Sample realizations of a mobility model and its node sojourn density is shown in Fig. 3.

V. RELATED WORK

Several frameworks have been proposed, aimed either at mobility model creation, or studying models’ mathematical properties. The former are more “practical” in the sense that they are based purely on empiric observations and engineering approaches (e.g. activity modeling) and eventual theoretical

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>NAME AND DESCRIPTION OF THE DIFFERENT LAYERS USED IN THE SIMULATIONS.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategies</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>Always select a fixed zone.</td>
</tr>
<tr>
<td>Enumerated</td>
<td>Select the first zone, then the second, etc. until reaching the last zone, then repeat.</td>
</tr>
<tr>
<td>Uniform</td>
<td>Select the zone with probability of a uniform distribution.</td>
</tr>
<tr>
<td>Binomial</td>
<td>Select the zone with probability of a binomial distribution.</td>
</tr>
<tr>
<td>Geometric</td>
<td>Select the zone with probability of a geometric distribution.</td>
</tr>
<tr>
<td>Mappers</td>
<td></td>
</tr>
<tr>
<td>Midpoint</td>
<td>Select the center of the zone.</td>
</tr>
<tr>
<td>Upper point</td>
<td>Select the maximal point of the zone.</td>
</tr>
<tr>
<td>Lower point</td>
<td>Select the minimal point of the zone.</td>
</tr>
<tr>
<td>Uniform</td>
<td>Uniformly select the point of the zone.</td>
</tr>
<tr>
<td>Normal</td>
<td>Select the point with normal distribution limited to the area of the zone.</td>
</tr>
<tr>
<td>Tactics</td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>Move linearly towards the destination.</td>
</tr>
<tr>
<td>Tango</td>
<td>Move 2/3 of the way to the destination, then step back 1/3 of the way, then go to the destination point.</td>
</tr>
<tr>
<td>Reversed Tango</td>
<td>Move 1/3 away from the destination, then move 1 2/3 in the direction of the destination (by surpassing it), and then step back with 1/3 of the distance, until reaching the endpoint.</td>
</tr>
<tr>
<td>Random Tango</td>
<td>With a probability p move with Tango, and probability 1 – p with Reversed Tango.</td>
</tr>
<tr>
<td>Dynamics</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>Move at a constant speed.</td>
</tr>
<tr>
<td>Uniform</td>
<td>Draw the speed from a uniform distribution and move with it on the entire trajectory.</td>
</tr>
<tr>
<td>Exponentially spaced</td>
<td>Draw the speed from a uniform distribution and move for a duration drawn from an exponential distribution.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>PERCENT OF TRACES PER TRACE TYPE FOR WHICH THE ADF TEST RESULT WAS STATISTICALLY SIGNIFICANT AT THE 1% LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node position</td>
<td>77.5%</td>
</tr>
<tr>
<td>Node speed</td>
<td>100%</td>
</tr>
<tr>
<td>Node sojourn distribution</td>
<td>100%</td>
</tr>
<tr>
<td>Node speed distribution</td>
<td>97.9%</td>
</tr>
</tbody>
</table>
theoretical works in these cases do not benefit from the studies are an exception to their typical use cases. Furthermore, the models, making the specification of non-trivial ones a laborious task. These studies include the works of Yoon et al. [17], where the authors introduce a framework which allows the study of several cases of random mobility models which generate node movement based on random selection of the values of (speed; time) or (speed; distance). The different dependencies are studied (e.g. both speed and time are independent random variables) and the steady state average speeds are derived. In [18] Le Boudec and Vojnović defined the Random Trip Mobility Model, which provides a theoretical basis founded on the Palm calculus for creating stationary mobility models. The framework provides specific conditions that have to be fulfilled by a mobility model in order to be declared Random Trip. Even though some of the conditions are straightforward, others are more elaborated and may require considerable investigation.

VI. CONCLUSION

In this paper we have outlined LEMMA as introduced earlier in [7], [8], which is a consistent and universal framework for mobility model definition [9]. We have stated and proved important results related to the stationarity of a mobility model specified under the form of LEMMA by only putting restrictions on the participating layers. This not only gives the possibility to easily define new stationary mobility models, a task which until now was a laborious and non-trivial process, but also serves as an example of the mathematical tractability of the models created with LEMMA. Thus, along with the concrete results, it gives the more general conclusion that LEMMA can be used as a framework for the analysis of mobility models.

Finally, we have supported our findings with simulations - we have simulated 300 different mobility models in 9 environmental setups and have received strong illustration of our theoretical findings.

REFERENCES