Symbolic event recognition systems have been successfully applied to a variety of application domains, extracting useful information in the form of events, allowing experts or other systems to monitor and respond when significant events are recognised. In a typical event recognition application, however, these systems often have to deal with a significant amount of uncertainty. In this paper, we address the issue of uncertainty in logic-based event recognition by extending the Event Calculus with probabilistic reasoning. Markov Logic Networks are a natural candidate for our logic-based formalism. However, the temporal semantics of the Event Calculus introduce a number of challenges for the proposed model. We show how and under what assumptions we can overcome these problems. Additionally, we study how probabilistic modelling changes the behaviour of the formalism, affecting its key property, the inertia of fluents. Furthermore, we demonstrate the advantages of the probabilistic Event Calculus through examples and experiments in the domain of activity recognition, using a publicly available dataset for video surveillance.
tracking system detecting that someone is walking for a sequence of video frames. Based on such time-stamped input SDE observations, the symbolic event recognition system recognises composite events (CEs) of interest. For instance, that some people have started to move together. The recognition of a CE may be associated with the occurrence of various SDEs and other CEs involving multiple entities, e.g. people, vehicles, etc. CEs therefore, are relational structures over other sub-events, either CEs or SDEs.

Statistical approaches, e.g. probabilistic graphical models, employ Machine Learning techniques, in order to learn the situations under which CEs must be recognised from annotated examples. Such methods are data driven and they are completely dependent upon the examples of the training set. On the other hand, background knowledge (e.g. knowledge expressed by domain experts) may describe situations that do not appear in the training data or are difficult to be collected and annotated. The majority of statistical approaches employ models with limited capabilities in expressing relation among entities. As a result, the definition of CEs and the use of background knowledge is very hard. Logic-based approaches, such as the Event Calculus [Kowalski and Sergot 1986; Artikis et al. 2010a], can naturally and compactly represent relational CE structures. Based on their formal and declarative semantics, they provide solutions that allow one to easily incorporate and exploit background knowledge. In contrast to statistical methods, however, they cannot handle uncertainty which naturally exists in many real-world event recognition applications.

Event recognition systems often have to deal with data that involves a significant amount of uncertainty [Shet et al. 2007; Artikis et al. 2010a; Etzion and Niblett 2010, Section 11.2; Gal et al. 2011]: (a) Low-level detection systems often cannot detect all SDEs required for CE recognition, e.g. due to a limited number of sensing sources. Logical definitions of CEs, therefore, have to be constructed upon a limited and often insufficient dictionary of SDEs. (b) Partial and noisy observations result in incomplete and erroneous SDE streams. For example, a sensor may fail for some period of time and stop sending information, interrupting the detection of a SDE. Similarly, noise in the signal transmission may distort the observed values. (c) Inconsistencies between SDE streams and CE annotations introduce further uncertainty. When Machine Learning algorithms are used, similar patterns of SDEs may be inconsistently annotated. As a result, CE definitions and background knowledge, either learnt from data or derived by domain experts not strictly follow the annotation. Under such situations of uncertainty, the performance of an Event Recognition system may be seriously compromised.

In the presence of some of the aforementioned types of uncertainty, e.g. partial SDE streams and inconsistent annotations, the CE definitions of a logic-based Event Recognition system cannot capture perfectly the conditions under which a CE occurs. Based on such imperfect CE definitions, the aim of this work is to recognise CEs of interest under uncertainty. In particular, we propose a probabilistic version of the Event Calculus that employs Markov Logic Networks (MLNs) [Domingos and Lowd 2009]. The Event Calculus is a formalism for representing events and their effects. Beyond the advantages stemming from the fact that it is a logic-based formalism with clear semantics, one of the most interesting properties of the Event Calculus is that it handles the persistence of CEs with domain-independent axioms. On the other hand, MLNs are a generic statistical relational framework that combines the expressivity of first-order logic with the formal probabilistic properties of undirected graphical models — see de Salvo Braz et al. [2008], Raedt and Kersting [2010] and Blockeel [2011] for surveys on logic-based relational probabilistic models. By combining the Event Calculus with MLNs, we present a principled and powerful probabilistic logic-based method for event recognition.
In particular the contributions of this work are the following:

— A probabilistic version of the Event Calculus for the task of event recognition. The method inherits the domain-independent properties of the Event Calculus and supports the probabilistic recognition of CEs with imperfect definitions.

— Efficient representation of the Event Calculus axioms and CE definitions in MLNs. The method employs a discrete variant of the Event Calculus and translates the entire knowledge base into compact Markov networks, in order to avoid the combinatorial explosion caused by the expressivity of the logical formalism.

— A thorough study of the behaviour of CE persistence. Under different conditions of interest, the method can model various types of CE persistence, ranging from deterministic to purely probabilistic.

To demonstrate the benefits of the proposed approach, the method is evaluated in the real-life event recognition task of human activity recognition. The method is compared against its crisp predecessor, as well as a purely statistical model based on linear-chain Conditional Random Fields. The definitions of CEs are domain-dependent rules that are given by humans and expressed using the language of the Event Calculus. The method processes the rules in the knowledge base and produces Markov networks of manageable size and complexity. Each rule can be associated with a weight value, indicating a degree of confidence in it. Weights are automatically estimated from a training set of examples. The input to the recognition system is a sequence of SDEs expressed as a narrative of ground predicates. Probabilistic inference is used to recognise CEs.

The remainder of the paper is organised as follows. First, in Section 2, we present the target activity recognition application, in order to introduce a running example for the rest of the paper. In Section 3 we present the axiomatisation of the proposed probabilistic version of the Event Calculus for the task of Event Recognition. In Section 4 we briefly present Markov Logic Networks. Then, in Section 5 we present representational simplifications and transformations that we employ, in order to produce compact ground Markov Networks. In Section 6, we study the behaviour of the probabilistic formalism. In Section 7 we demonstrate the benefits of probabilistic modelling, through experiments in the real-life activity recognition application. Finally in Sections 8 and 9, we present related work and outline directions for further research.

2. RUNNING EXAMPLE: ACTIVITY RECOGNITION

To demonstrate our method, we apply it to video surveillance in public spaces using the publicly available benchmark dataset of the CAVIAR project\(^1\). The aim is to recognise activities that take place between multiple persons, by exploiting information about observed individual activities. The dataset comprises 28 surveillance videos, where each frame is annotated by human experts from the CAVIAR team on two levels. The first level contains simple, derived events (SDEs) that concern activities of individual persons or the state of objects. The second level contains composite event (CE) annotations, describing the activities between multiple persons and/or objects, e.g. people meeting and moving together, leaving an object, etc. In this paper, we focus on the recognition of the meeting and moving CEs, for which the dataset contains a sufficient amount of training examples.

The input to our method is a stream of SDEs, representing people walking, running, staying active, or inactive. We do not process the raw video data in order to recognise such individual activities. Instead we use the SDEs provided in the CAVIAR dataset. Thus, the input stream of SDEs is represented by a narrative of time-stamped predicates. The first and the last time that a person or an object is tracked are represented

\(^1\)http://homepages.inf.ed.ac.uk/rbf/CAVIARDATA1
by the SDEs enter and exit. Additionally, the coordinates of tracked persons or objects are preprocessed and represented by predicates that express qualitative spatial relations, e.g., two persons being relatively close to each other. Examples of these predicates are presented in the following sections.

The definitions of the meeting and moving CEs in the Event Calculus were developed in [Artikis et al. 2010b]. These definitions take the form of common-sense rules and describe the conditions under which a CE starts or ends. For example, when two persons are walking together with the same orientation, then moving starts being recognised. Similarly, when the same persons walk away from each other, then moving stops being recognised.

Based on the input stream of SDEs and the CE definitions, the aim is to recognise instances of the two CEs of interest. The CE definitions are imperfect, since under the presence of uncertainty they cannot capture perfectly all the conditions under which a CE occurs. Furthermore, the definitions are derived from experts and may not strictly follow the annotation. As a result, CE definitions do not lead to perfect recognition of the CEs.

3. THE EVENT CALCULUS

The Event Calculus, originally introduced by Kowalski and Sergot [1986], is a many-sorted first-order predicate calculus for reasoning about events and their effects. A number of different dialects have been proposed using either logic programming or classical logic [Shanahan 1999; Miller and Shanahan 2002; Mueller 2008]. Most Event Calculus dialects share the same ontology and core domain-independent axioms. The ontology consists of time-points, events and fluents. The underlying time model is often linear and may represent time-points as real or integer numbers. A fluent is a property whose value may change over time. When an event occurs it may change the value of a fluent. The core domain-independent axioms define whether a fluent holds or not at a specific time-point. Moreover, the axioms incorporate the common sense law of inertia, according to which fluents persist over time, unless they are affected by the occurrence of some event.

We base our model on an axiomisation of a discrete version of the Event Calculus in classical first-order logic. The Discrete Event Calculus (DEC) has been proved to be logically equivalent to the Event Calculus when the domain of time-points is limited to integers [Mueller 2008]. DEC is composed of twelve domain-independent axioms. However, for the task of event recognition, we focus only on the domain-independent axioms that determine the influence of events to fluents and the inertia of fluents. We do not consider the predicates and axioms stating when a fluent is not subject to inertia (releases and releasedAt), as well as its discrete change based on some domain-specific mathematical function (trajectory and antiTrajectory). Furthermore, we adopt a similar representation to that of Artikis et al. [2010a], where predicates stating the initiation and termination of fluents are only defined in terms of fluents and time-points. Table I summarises the elements of the proposed Event Calculus (MLN–EC). Variables (starting with an upper-case letter) are assumed to be universally quantified unless otherwise indicated. Predicates, functions and constants start with a lower-case letter.

\[http://decreasoner.sourceforge.net\]
Table I: The $MLN$–$EC$ predicates

<table>
<thead>
<tr>
<th>Predicate</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$happens(E, T)$</td>
<td>Event $E$ occurs at time-point $T$</td>
</tr>
<tr>
<td>$holdsAt(F, T)$</td>
<td>Fluent $F$ holds at time-point $T$</td>
</tr>
<tr>
<td>$initiatedAt(F, T)$</td>
<td>Fluent $F$ is initiated at time-point $T$</td>
</tr>
<tr>
<td>$terminatedAt(F, T)$</td>
<td>Fluent $F$ is terminated at time-point $T$</td>
</tr>
</tbody>
</table>

The $MLN$–$EC$ axioms that determine when a fluent holds are defined as follows:

\[
holdsAt(F, T+1) \iff initiatedAt(F, T) \tag{1}
\]

\[
holdsAt(F, T+1) \iff holdsAt(F, T) \land 

\neg terminatedAt(F, T) \tag{2}
\]

Axiom (1) defines that if a fluent $F$ is initiated at time $T$, then it holds at the next time-point. Axiom (2) specifies that a fluent continues to hold unless it is terminated.

The axioms that determine when a fluent does not hold are defined similarly:

\[
\neg holdsAt(F, T+1) \iff terminatedAt(F, T) \tag{3}
\]

\[
\neg holdsAt(F, T+1) \iff \neg holdsAt(F, T) \land

\neg initiatedAt(F, T) \tag{4}
\]

According to axiom (3), if a fluent $F$ is terminated at time $T$ then it does not hold at the next time-point. Axiom (4) states that a fluent continues not to hold unless it is initiated.

The predicates $happens$, $initiatedAt$ and $terminatedAt$ are defined only in a domain-dependent manner. $happens$ expresses the input evidence, determining the occurrence of a SDE at a specific time-point. A stream of observed SDEs, therefore, is represented in the $MLN$–$EC$ as a narrative of ground $happens$ predicates. As an example, consider the following fragment of a narrative:

\[
\ldots

happens(walking(id_1), 99)
happens(walking(id_2), 99)
happens(walking(id_1), 100)
happens(walking(id_2), 100)
\ldots

happens(active(id_1), 500)
happens(active(id_2), 500)
\ldots
\]

According to the above narrative, it has been observed that two persons $id_1$ and $id_2$ are walking, e.g. at time-points 99 and 100, and later at time-point 500 they are active, e.g. they are moving their arms but staying at the same position.

The predicates $initiatedAt$ and $terminatedAt$ specify under which circumstances a fluent — representing a CE — is to be initiated or terminated at a specific time-point. The domain-dependent rules of the $MLN$–$EC$, i.e. the initiation and/or termination of some fluent over some domain-specific entities $X$ and $Y$ take the following general
initiatedAt(fluent\(_1\)(X, Y), T) ⇐  
  happens(events(X), T) \land \ldots \land 
  holdsAt(fluent\(_2\)(X), T) \land \ldots \land 
  Conditions[X, Y, T]  
  \tag{5}

terminatedAt(fluent\(_1\)(X, Y), T) ⇐  
  happens(events(X), T) \land \ldots \land 
  holdsAt(fluent\(_2\)(X), T) \land \ldots \land 
  Conditions[X, Y, T]

In this work we consider finite domains of time-points, events and fluents, that are represented by the finite sets \(T, E\) and \(F\), respectively. All individual entities that appear in a particular event recognition task, e.g. persons, objects, etc., are represented by the constants of the finite set \(O\). Conditions[X, Y, T] in (5) is a set of predicates that introduce further constraints in the definition, referring to time \(T \in T\) and entities \(X, Y \in O\). The predicates \textit{happens} and \textit{holdsAt}, as well as those appearing in Conditions[X, Y, T], may also be negated. The initiation and termination of a fluent can be defined by more than one rule, each capturing a different initiation and termination case. With the use of \textit{happens} predicates, we can define a CE over SDE observations. Similarly, with the \textit{holdsAt} predicate we can define a CE over other CE, in order to create hierarchies of CE definitions. In both \textit{initiatedAt} and \textit{terminatedAt} rules, the use of \textit{happens}, \textit{holdsAt} and Conditions[X, Y, T] is optional and varies according to the requirements of the target event recognition application.

In our example application, for instance, the \textit{moving} activity of two persons is terminated when both of them are \textit{active}. This termination case can be represented using the following rule:

\[\text{terminatedAt(moving(ID}_1, \text{ID}_2), T) \Leftarrow 
\text{happens(active(ID}_1), T) \land 
\text{happens(active(ID}_2), T) \tag{6}\]

Based on a narrative of SDEs and a knowledge base composed of domain-dependent CE definitions (e.g. rule (6)) and the domain-independent Event Calculus axioms, we can infer whether a fluent holds or not at any time-point. When a fluent holds at a specific time-point, then the corresponding CE is considered to be recognised. For example, the \textit{moving} CE between persons \textit{id}_1 and \textit{id}_2 is recognised at time-point 100 by inferring that \textit{holdsAt(moving(id}_1, \text{id}_2), 100) is \textit{True}. Similarly, the \textit{moving} CE for the same persons is not recognised at time-point 501 by inferring that \textit{holdsAt(moving(id}_1, \text{id}_2), 501) is \textit{False}.

Consider the following definition of the \textit{meeting} CE between two persons in our running example.

\[\text{initiatedAt(meeting(ID}_1, \text{ID}_2), T) \Leftarrow 
\text{happens(active(ID}_1), T) \land 
\neg\text{happens(running(ID}_2), T) \land 
\text{close(ID}_1, \text{ID}_2, 25, T) \tag{7}\]

\[\text{initiatedAt(meeting(ID}_1, \text{ID}_2), T) \Leftarrow 
\text{happens(inactive(ID}_1), T) \land 
\neg\text{happens(running(ID}_2), T) \land 
\neg\text{happens(active(ID}_2), T) \land 
\text{close(ID}_1, \text{ID}_2, 25, T) \tag{8}\]
The predicate \( \text{close} \) expresses a spatial constraint stating that the distance between persons \( ID_1 \) and \( ID_2 \) at time \( T \) must be below a specified threshold in pixels, e.g. 25 pixels. According to rules (7) and (8), the \( \text{meeting} \) activity is initiated when the people involved interact with each other, i.e. at least one of them is active or inactive, the other is not running, and the measured distance between them is at most 25 pixels. The \( \text{meeting} \) CE is terminated either when people walk away from each other (rule 9), or someone is running (rule 10), or has exited the scene (rule 11).

The definition of the CE that people are moving together is represented as follows:

\[
\text{initiatedAt}(\text{moving}(ID_1, ID_2), T) \Leftarrow \begin{align*}
&\text{happens}(\text{walking}(ID_1), T) \\
&\text{happens}(\text{walking}(ID_2), T) \\
&\text{orientationMove}(ID_1, ID_2, T) \\
&\text{close}(ID_1, ID_2, 34, T)
\end{align*}
\]

\[
\text{terminatedAt}(\text{moving}(ID_1, ID_2), T) \Leftarrow \begin{align*}
&\text{happens}(\text{walking}(ID_1), T) \\
&\neg\text{close}(ID_1, ID_2, 34, T)
\end{align*}
\]

\[
\text{terminatedAt}(\text{moving}(ID_1, ID_2), T) \Leftarrow \begin{align*}
&\text{happens}(\text{active}(ID_1), T) \\
&\text{happens}(\text{active}(ID_2), T)
\end{align*}
\]

\[
\text{terminatedAt}(\text{moving}(ID_1, ID_2), T) \Leftarrow \begin{align*}
&\text{happens}(\text{inactive}(ID_1), T) \\
&\text{happens}(\text{inactive}(ID_2), T)
\end{align*}
\]

\[
\text{terminatedAt}(\text{moving}(ID_1, ID_2), T) \Leftarrow \begin{align*}
&\text{happens}(\text{running}(ID_1), T)
\end{align*}
\]

\[
\text{terminatedAt}(\text{moving}(ID_1, ID_2), T) \Leftarrow \begin{align*}
&\text{happens}(\text{exit}(ID_1), T)
\end{align*}
\]

The predicate \( \text{orientationMove} \) is a spatial constraint, stating that the orientation of two persons is almost the same (e.g. the difference is below 45 degrees). According to rule (12), the \( \text{moving} \) CE is initiated when two persons \( ID_1 \) and \( ID_2 \) are walking close to each other (their distance is at most 34 pixels) with almost the same orientation. The \( \text{moving} \) CE is terminated under several cases: (a) As specified by rule (13), when people walk away from each other, i.e. they have a distance larger than 34 pixels. (b) When none is actually moving, i.e. both are staying active, or (c) one is active while the other is inactive, represented by rules (14) and (15). (d) Finally, when one of them is running or exiting the scene, represented by rules (16) and (17), respectively.

4. MARKOV LOGIC NETWORKS

Although the Event Calculus can compactly represent complex event relations, it does not handle uncertainty adequately. A knowledge base of Event Calculus axioms and
composite event (CE) definitions is defined by a set of first-order logic formulas. Each formula imposes a (hard) constraint over the set of possible worlds, that is, Herbrand interpretations. A missed or an erroneous simple, derived event (SDE) detection can have a significant effect on the event recognition results. For example, an initiation may be based on an erroneously detected SDE, causing the recognition of a CE with absolute certainty.

We employ the framework of Markov Logic Networks\(^3\) (MLNs) [Domingos and Lowd 2009] in order to soften these constraints and perform probabilistic inference. In MLNs, each formula \(F_i\) is represented in first-order logic and is associated with a weight value \(w_i \in \mathbb{R}\). The higher the value of weight \(w_i\), the stronger the constraint represented by formula \(F_i\). In contrast to classical logic, all worlds in MLNs are possible with a certain probability. The main idea behind this is that the probability of a world increases as the number of formulas it violates decreases. A knowledge base in MLNs may contain both hard and soft-constrained formulas. Hard-constrained formulas are associated with an infinite weight value and capture the knowledge which is assumed to be certain. Therefore, an acceptable world must at least satisfy the hard constraints. Soft constraints capture imperfect knowledge in the domain, allowing for the existence of worlds in which this knowledge is violated.

Formally, a knowledge base \(L\) of weighted formulas, together with a finite domain of constants \(C\), is transformed into a ground Markov network \(M_{L,C}\). In our case, \(L\) consists of Event Calculus axioms and CE definitions, and \(C=\mathcal{T}\cup\mathcal{O}\cup\mathcal{E}\cup\mathcal{F}\). All formulas are converted into clausal form and each clause is ground according to the domain of its distinct variables. The nodes in \(M_{L,C}\) are Boolean random variables, each one corresponding to a possible grounding of a predicate that appears in \(L\). The predicates of a ground clause form a clique in \(M_{L,C}\). Each clique is associated with a corresponding weight \(w_i\) and a Boolean feature, taking the value 1 when the ground clause is true and 0 otherwise. The ground \(M_{L,C}\) defines a probability distribution over possible worlds and is represented as a log-linear model.

In event recognition we aim to recognise CEs of interest given the observed streams of SDEs. For this reason we focus on discriminative MLNs [Singla and Domingos 2005], that are akin to Conditional Random Fields [Lafferty et al. 2001; Sutton and McCallum 2007]. Specifically, the set of random variables in \(M_{L,C}\) can be partitioned into two subsets. The former is the set of evidence random variables \(X\), formed by a narrative of input ground happens predicates and spatial constraints. The latter is the set of random variables \(Y\) that correspond to groundings of query holdsAt predicates, as well as groundings of any other hidden/unobserved predicates. The joint probability distribution of a possible assignment of \(Y=y\), conditioned over a given assignment of \(X=x\), is defined as follows:

\[
P(Y=y \mid X=x) = \frac{1}{Z(x)} exp \left( \sum_{i=1}^{\lvert F_c \rvert} w_i n_i(x, y) \right)
\]

The vectors \(x \in \mathcal{X}\) and \(y \in \mathcal{Y}\) represent a possible assignment of evidence \(X\) and query/hidden variables \(Y\), respectively. \(\mathcal{X}\) and \(\mathcal{Y}\) are the sets of possible assignments that the evidence \(X\) and query/hidden variables \(Y\) can take. \(F_c\) is the set of clauses produced from the knowledge base \(L\) and the domain of constants \(C\). The scalar value \(w_i\) is the weight of the \(i\)-th clause and \(n_i(x, y)\) is the number of satisfied groundings.

\(^3\)Systems implementing MLN reasoning and learning algorithms can be found at the following addresses:
http://alchemy.cs.washington.edu
http://research.cs.wisc.edu/hazy/tuffy
http://code.google.com/p/thebeast
http://ias.cs.tum.edu/probcog-wiki
of the $i$-th clause in $x$ and $y$. $Z(x)$ is the partition function, that normalises over all possible assignments $y' \in \mathcal{Y}$ of query/hidden variables given the assignment $x$, that is, $Z(x) = \sum_{y' \in \mathcal{Y}} \exp\left(\sum_{i=1}^{F} w_i \eta_i(x, y')\right)$.

Equation (18) represents a single exponential model for the joint probability of the entire set of query variables that is globally conditioned on a set of observables. Such a conditional model can have a much simpler structure than a full joint model, e.g. a Bayesian Network. By modelling the conditional distribution directly, the model is not affected by potential dependencies between the variables in $X$ and can ignore them. The model also makes independence assumptions among the random variables $Y$, and defines by its structure the dependencies of $Y$ on $X$. Furthermore, conditioning on a specific assignment $x$, given by the observed SDEs, reduces significantly the number of possible worlds and inference becomes much more efficient [Singla and Domingos 2005; Minka 2005; Sutton and McCallum 2007].

Still, directly computing equation (18) is intractable, because the value of $Z(x)$ depends on the relationship among all clauses in the knowledge base. For this reason, a variety of efficient inference algorithms have been proposed in the literature, based on local search and sampling [Poon and Domingos 2006; Singla and Domingos 2006; Biba et al. 2011], variants of Belief Propagation [Singla and Domingos 2008; Kersting et al. 2009], Integer Linear Programming [Riedel 2008; Huynh and Mooney 2009], etc.

In this work we consider two types of inference, i.e. marginal inference and maximum a-posteriori inference (MAP). The former type of inference computes the conditional probability that CEs hold given a narrative of observed SDEs, i.e. $P(\text{holdsAt}(CE, T) = \text{True} \mid SDE)$. In other words, this probability value measures the confidence that the CE is recognised. Since it is #P-complete to compute this probability, we employ the state-of-the-art sampling algorithm MC-SAT [Poon and Domingos 2006] to approximate it. The algorithm combines Markov Chain Monte Carlo sampling with satisfiability testing and even in large state spaces with deterministic dependencies (e.g. hard-constrained formulas) it can approximate this probability efficiently. The latter type of inference identifies the most probable assignment among all $\text{holdsAt}$ instantiations that are consistent with the given narrative of observed SDEs, i.e. $\operatorname*{argmax}_{\text{holdsAt}} P(\text{holdsAt}(CE, T) \mid SDE)$. In MLNs this task reduces to finding the truth assignment of all $\text{holdsAt}$ instantiations that maximises the sum of weights of satisfied ground clauses. This is equivalent to the weighted maximum satisfiability problem.

The problem is NP-hard in general and in order to find an approximate solution efficiently we employ the LP-relaxed Integer Linear Programming method proposed by Huynh and Mooney [2009]. The weights of the soft-constrained clauses in MLNs can be estimated from training data, using supervised learning techniques. When the goal is to learn a model that recognises CEs with some confidence (i.e. probability), then the most widely adopted approach is to minimise the negative Conditional Log-Likelihood (CLL) function — derived from equation (18). This can be achieved by using either first-order or second-order optimisation methods [Singla and Domingos 2005; Lowd and Domingos 2007]. First-order methods apply standard gradient descent optimisation techniques, e.g. the voted perceptron algorithm [Collins 2002; Singla and Domingos 2005], while second-order methods pick a search direction based on the quadratic approximation of the target function. As stated by Lowd and Domingos [2007], second-order methods are more appropriate for MLN training, as they do not suffer from the problem of ill-conditioning. In a training set some clauses may have a significantly greater number of satisfied groundings than others, causing the variance of their counts to be correspondingly larger. This situation causes the standard gradient descent methods to converge very slowly, since there is no single appropriate learning rate for all soft-constrained
An alternative approach to CLL function optimisation is max-margin training, which is better suited to problems where the goal is to maximise the classification accuracy [Huynh and Mooney 2009; 2011]. Instead of optimising the CLL function, max-margin training aims to maximise the ratio between the probability of the correct truth assignment of CEs to hold and the closest competing incorrect truth assignment. In this work we assess both the second-order Diagonal Newton algorithm [Singla and Domingos 2005] and the max-margin method proposed by Huynh and Mooney [2009].

5. COMPACT MARKOV NETWORK CONSTRUCTION

The use of MLNs for inference and learning requires the grounding of the entire knowledge base used for event recognition, including the domain-independent axioms of the Event Calculus (axioms (1)–(4)). Unless optimized, this process leads to unmanageably large ground Markov Networks, where inference and learning become practically infeasible. This section presents our approach to addressing this problem.

5.1. Simplified Representation

The choice of Event Calculus dialect, as presented in Section 3, has a significant impact on the grounding process. For example, Shanahan’s Full Event Calculus [Shanahan 1999] employs axioms that contain triply quantified time-point variables. As a result, the number of their groundings has a cubic relation to the number of time-points. Furthermore, that formalism contains existentially quantified variables over events and time-points. During MLN grounding existentially quantified formulas are replaced by the disjunction of their groundings [Domingos and Lowd 2009]. This leads to a large number of disjunctions and a combinatorial explosion of the number of clauses, producing unmanageably large Markov networks.

In contrast, the proposed Event Calculus (MLN–EC) is based on the Discrete Event Calculus [Mueller 2008], where the domain-independent axioms are defined over successive time-points. For example, axiom (1) produces one clause and has two distinct variables \( F \) and \( T \). Therefore, the number of its groundings is determined by the Cartesian product of the corresponding variable-binding constraints, that is \( |F| \times |T| \).

Assuming that the domain of fluents \( F \) is relatively small compared to the domain of time-points \( T \), the number of groundings of axiom (1) grows linearly to the number of time-points. Furthermore, in MLN–EC the initiation and termination of fluents — representing CEs — are only defined in terms of fluents and time-points (see the general form (5)). This representation reduces further the number of variables and eliminates the existential quantification in the domain-independent axioms. As a result, MLN–EC produces a substantially smaller number of ground clauses, than many other dialects of Event Calculus.

5.2. Knowledge Base Transformation

In addition to choosing an Event Calculus dialect that makes the number of ground clauses linearly dependent on the number of time-points, we can achieve significant improvements in the size of the ground Markov Networks, by making the Closed World Assumption.

A knowledge base with domain-dependent rules in the form of (5) describes explicitly the conditions in which fluents are initiated or terminated. It is usually impractical to define also when a fluent is not initiated and not terminated. However, the open-world semantics of first-order logic result in an inherent uncertainty about the value of a fluent for many time-points. In other words, if at a specific time-point no event that terminates or initiates a fluent happens, we cannot rule out the possibility that the

---

4In Conjunctional Normal Form.
fluent has been initiated or terminated. As a result, we cannot determine whether a fluent holds or not, leading to the loss of inertia.

This is a variant of the well-known frame problem and one solution for the Event Calculus in first-order logic is the use of circumscription [McCarthy 1980; Lifschitz 1994; Shanahan 1997; Doherty et al. 1997; Mueller 2008]. The aim of circumscription is to automatically rule out all those conditions which are not explicitly entailed by the given formulas. Hence, circumscription introduces a closed-world assumption to first-order logic.

Technically, we perform circumscription by predicate completion — a syntactic transformation where formulas are translated into logically stronger ones. In particular, we perform a knowledge transformation procedure in which predicate completion is computed for both \textit{initiatedAt} and \textit{terminatedAt} predicates. Due to the form of CE definitions (see formalisation (5)), the result of predicate completion is applied to each CE separately, e.g. \textit{initiatedAt}(meeting($ID_1$, $ID_2$, $T$)), rather than to a generic \textit{initiatedAt}($F$, $T$) predicate. Similar to Mueller [2008], we also eliminate the \textit{initiatedAt} and \textit{terminatedAt} predicates from the knowledge base, by exploiting the equivalences resulting from predicate completion. In cases where the definitions of the initiation or termination of a specific CE are missing, the corresponding initiation or termination is considered \textit{False} for all time-points, e.g. \textit{terminatedAt}(fluent($X$, $Y$), $T$) \Leftrightarrow \text{False}.

To illustrate the form of the resulting knowledge base, consider the domain-dependent definition of \textit{meeting} — i.e. rules (7)–(11). After predicate completion, these rules will be replaced by the following formulas:

\[
\begin{align*}
\text{initiatedAt}(meeting(&ID_1, &ID_2, &T) \Leftrightarrow \\
&\happens(\text{active}(ID_1), &T) \land \\
&\neg \happens(\text{running}(ID_2), &T) \land \\
&\close(ID_1, &ID_2, 25, &T) \bigvee \\
&(\happens(\text{inactive}(ID_1), &T) \land \\
&\neg \happens(\text{running}(ID_2), &T) \land \\
&\neg \happens(\text{active}(ID_2), &T) \land \\
&\close(ID_1, &ID_2, 25, &T))
\end{align*}
\]

\[
\begin{align*}
\text{terminatedAt}(meeting(&ID_1, &ID_2, &T) \Leftrightarrow \\
&\happens(\text{walking}(ID_1), &T) \land \\
&\neg \close(ID_1, &ID_2, 25, &T) \bigvee \\
&\happens(\text{running}(ID_1), &T) \lor \\
&\happens(\text{exit}(ID_1), &T)
\end{align*}
\]

The resulting rules (19) and (20) define all conditions under which the \textit{meeting} CE is initiated or terminated. Any other event occurrence cannot affect this CE, as it cannot initiate the CE or terminate it. Based on the equivalence in formula (19), the domain-
independent axiom (1) is automatically re-written into the following specialised form\(^5\):

\[
\text{holdsAt}(\text{meeting}(ID_1, ID_2), T+1) \iff
\begin{align*}
&\text{happens}(\text{active}(ID_1), T) \land \\
&\neg\text{happens}(\text{running}(ID_2), T) \land \\
&\text{close}(ID_1, ID_2, 25, T)
\end{align*}
\]

\[
\text{holdsAt}(\text{meeting}(ID_1, ID_2), T+1) \iff
\begin{align*}
&\text{happens}(\text{inactive}(ID_1), T) \land \\
&\neg\text{happens}(\text{running}(ID_2), T) \land \\
&\neg\text{happens}(\text{active}(ID_2), T) \land \\
&\text{close}(ID_1, ID_2, 25, T)
\end{align*}
\] (21)

Similarly, the inertia axiom (2) can be re-written according to (20) as follows:

\[
\text{holdsAt}(\text{meeting}(ID_1, ID_2), T+1) \iff
\begin{align*}
&\text{holdsAt}(\text{meeting}(ID_1, ID_2), T) \land \\
&\neg\left(\text{happens}(\text{walking}(ID_1), T) \land \\
&\neg\text{close}(ID_1, ID_2, 25, T) \right) \lor \\
&\text{happens}(\text{running}(ID_1), T) \lor \\
&\text{happens}(\text{exit}(ID_1), T)
\end{align*}
\] (22)

The result of this transformation procedure replaces the original set of domain-independent axioms and domain-dependent CE definitions with a logically stronger knowledge base. The rules in the resulting knowledge base form the template that MLNs will use to produce ground Markov networks. The transformed formulas produce considerably more compact ground Markov networks than the original ones, as the clauses to be grounded are reduced. Moreover, the predicates \text{initiatedAt} and \text{terminatedAt} are eliminated and the corresponding random variables are not added to the network. This reduction decreases substantially the space of possible worlds, since the target random variables of the network (\(Y\) in equation (18)) are limited only to the corresponding \text{holdsAt} ground predicates. Specifically, the space of possible worlds is reduced from \(2^{3|F|}\times|T|\) to \(2^{|F|}\times|T|\) — where \(|T|\) and \(|F|\) denote the number of distinct time-points and fluents, respectively. These reductions improve the computational performance of the probabilistic inference. Furthermore, due to the reduced space of possible worlds, the same number of sampling iterations results in better probability estimates.

Formally, the resulting knowledge base is composed of rules having the following form:

\[
\Sigma = \left\{ \begin{array}{l}
\text{holdsAt}(\text{fluent}_1(X, Y), T+1) \iff \\
\text{happens}(\text{event}_1(X), T) \land \ldots \land \text{Conditions}[X, Y, T] \\
\vdots \\
\neg\text{holdsAt}(\text{fluent}_1(X, Y), T+1) \iff \\
\text{happens}(\text{event}_2(X), T) \land \ldots \land \text{Conditions}[X, Y, T] \\
\vdots
\end{array} \right\}
\] (23) (24)

\(^5\)This direct re-writing of (19) results to a single formula that contains the disjunction of formula (19). However, for reasons that have to do with the handling of uncertainty in MLN and will be discussed in a later section, in (21) we choose to equivalently represent it using two separate formulas.
\[ \Sigma' = \begin{cases} 
\text{holdsAt}(\text{fluent}_1(X, Y), T+1) \leftarrow \\
\text{holdsAt}(\text{fluent}_1(X, Y), T) \land \\
\neg((\text{happens}(\text{event}_j(X), T) \land \ldots \land \text{Conditions}[X, Y, T]) \lor \ldots) \\
\ldots \\
\text{holdsAt}(\text{fluent}_1(X, Y), T+1) \leftarrow \\
\neg\text{holdsAt}(\text{fluent}_1(X, Y), T) \land \\
\neg((\text{happens}(\text{event}_j(X), T) \land \ldots \land \text{Conditions}[X, Y, T]) \lor \ldots) \\
\ldots 
\end{cases} \] (25) (26)

The rules in (23)–(26) can be separated into two subsets. The former set \( \Sigma \) contains specialised definitions of axioms (1) and (3), specifying when a fluent holds (or does not hold) when its initiation (or termination) conditions are met. The latter set \( \Sigma' \) contains specialised definitions of the inertia axioms (2) and (4), specifying whether a specific fluent continues to hold or not at any instance of time.

The knowledge transformation procedure reduces the size of the produced network, based only on the rules of the knowledge base. Given a narrative of SDEs, further reduction can be achieved during the ground network construction. All ground predicates that appear in the given narrative are replaced by their truth value. Ground clauses that become tautological are safely removed, as they remain satisfied in all possible worlds [Singla and Domingos 2005; Shavlik and Natarajan 2009]. Therefore, the resulting network comprises only the remaining ground clauses, containing ground predicates with unknown truth states — i.e. groundings of \( \text{holdsAt} \).

6. THE BEHAVIOUR OF THE PROBABILISTIC EVENT CALCULUS

As mentioned in Section 4, weighted formulas in MLNs define soft constraints, allowing some worlds that do not satisfy these formulas to become likely. For example, consider a knowledge base of Event Calculus axioms and CE definitions (e.g. meeting and moving) compiled in the form of rules (23)–(26). Given a narrative of SDEs, the probability of a CE to hold at a specific time-point is determined by the probabilities of the worlds in which this CE holds. Each world, in turn, has some probability which is proportional to the sum of the weights of the ground clauses that it satisfies. Consequently, the probability of a CE to hold at a specific instance of time depends on the corresponding constraints of the ground Markov network. Thus, by treating the rules in the \( \Sigma \) and \( \Sigma' \) sets as either hard or soft constraints, we can modify the behaviour of the Event Calculus.

6.1. Soft-constrained rules in \( \Sigma \)

In order to illustrate how the probability of a CE is affected when its initiation or termination conditions are met, consider the case that the rules in \( \Sigma \) are soft-constrained while the inertia rules in \( \Sigma' \) remain hard-constrained. By soft-constraining the rules in \( \Sigma \), the worlds violating their clauses become probable. This situation reduces the certainty with which a CE is recognised when its initiation or termination conditions are met. For example, assume that the initiation rules (21) of the meeting CE are associated with weights. As a result, the meeting activity is initiated with some certainty, causing the CE to hold with some probability. Depending on the strength of the weights, the worlds that violate these rules become more or less likely. Thus, we can control the level of certainty with which a CE holds or not under the same conditions.

When the initiation conditions are met, the probability of the CE to hold increases. Equivalently, when the termination conditions are satisfied, the probability of the CE decreases. At the same time, all worlds violating hard-constrained inertia rules in \( \Sigma' \) are rejected. In the presence of SDEs leading to the partial satisfaction (i.e. satisfaction of a possibly empty strict subset) of the initiation/termination conditions, the probabil-
Fig. 1: The probability of the meeting CE given some SDE narrative. EC\textsubscript{crisp} is a crisp Event Calculus. MLN–EC\textsubscript{HI} is a probabilistic Event Calculus where rules in \( \Sigma \) are soft-constrained, while the inertia rules in \( \Sigma' \) remain hard-constrained.

Figure 1 illustrates this behaviour with the fluent meeting that initially does not hold at time 0. According to the narrative of SDEs, the meeting activity is initiated at time-points 3 and 10, e.g. satisfying the constraints imposed by rules (7) and (8) respectively. At time 20, the meeting activity is terminated by the conditions of rule (9). In crisp Event Calculus, denoted as EC\textsubscript{crisp}, after its first initiation the meeting activity holds with absolute certainty. The second initiation at time 10 does not cause any change and the CE continues to hold. The termination at time 20 causes the CE to not hold, again with absolute certainty, for the remaining time-points. In MLN–EC\textsubscript{HI} (hard-constrained inertia rules), however, the rules in \( \Sigma \) are soft-constrained. As a result, at time-point 4 the probability of meeting to hold increases to some value. Similar to EC\textsubscript{crisp}, the inertia is fully retained and the probability of meeting deterministically persists in the interval 4 to 10. In contrast to EC\textsubscript{crisp}, the second initiation at time-point 10 increases the certainty of meeting to hold. As a result, the probability of meeting is higher in the interval 11 to 20. In the same manner, the termination at 20 reduces the probability of meeting and the CE continues to hold with some reduced probability.

### 6.2. Soft-constrained inertia rules in \( \Sigma' \)

To illustrate how the behaviour of inertia is affected by soft-constraining the corresponding rules in \( \Sigma' \), consider that the rules in \( \Sigma \) are hard-constrained. Consequently, when the initiation (or termination) conditions are met, a CE holds (or does not hold) with absolute certainty. The persistence of a CE depends on its inertia rules in \( \Sigma' \). If the inertia of holdsAt is hard-constrained, the worlds in which an initiated CE does not hold are rejected. Similarly, by keeping the inertia of \( \neg \text{holdsAt} \) hard-constrained, all worlds in which a terminated CE holds are rejected. By soft-constraining these rules we control the strength of the inertia constraints. Thus, in the presence of SDEs leading to the partial satisfaction of the corresponding initiation/termination conditions, a CE may not persist with absolute certainty, as worlds that violate these constraints become likely. The persistence of holdsAt and \( \neg \text{holdsAt} \) is gradually lost over successive time-points. When allowing the constraints of holdsAt inertia to be violated, the probability of a CE gradually drops. Similarly, by allowing the constraints represent-
Fig. 2: In both figures SDEs occur leading to the partial satisfaction of the initiation/termination conditions of a CE in the interval 0 to 100. In the left figure the CE holds at time 0 with absolute certainty, while in the right figure the CE does not hold at time 0.

The inertia of \( \neg \text{holdsAt} \) to be violated, the probability of a CE gradually increases.

The lower the value of the weight on the constraint, the more probable the worlds that violate the constraints become. In other words, weight values in \( \Sigma' \) cause CE to persist for longer or shorter time periods.

Since the sum of the probabilities of \( \text{holdsAt} \) and \( \neg \text{holdsAt} \) for a specific CE is always equal to 1, the relative strength of \( \text{holdsAt} \) and \( \neg \text{holdsAt} \) rules in \( \Sigma' \) determines the type of inertia in the model. The following two general cases can be distinguished.

**Equally strong inertia constraints.** All rules in \( \Sigma' \) are equally soft-constrained, i.e. they are associated with the same weight value. Consequently, both inertia rules of \( \text{holdsAt} \) and \( \neg \text{holdsAt} \) for a particular CE impose constraints of equal importance, allowing worlds that violate them to become likely. As a result, in the absence of useful evidence, the probability of \( \text{holdsAt} \) will tend to approximate the value 0.5. For example, Figure 2(a) illustrates soft persistence for the \( \text{meeting} \) CE when it holds with absolute certainty at time-point 0, and thereafter nothing happens to initiate or terminate it. The curve \( \text{MLN–ECSI}_{\text{eq}} \) (soft-constrained inertia rules with equal weights) shows the behaviour of inertia in this case. As time evolves, the probability of \( \text{meeting} \) appears to gradually drop, converging to 0.5. If we assign weaker weights to the inertia axioms, shown by the \( \text{MLN–ECSI}_{\text{eq weak}} \) curve, the probability of \( \text{meeting} \) drops more sharply. Similarly, in Figure 2(b), the \( \text{meeting} \) CE is assumed to not hold initially. As time evolves, the probability of \( \text{meeting} \) gradually increases up to the value 0.5, as shown by the \( \text{MLN–ECSI}_{\text{eq}} \) and \( \text{MLN–ECSI}_{\text{eq weak}} \) curves respectively.

**Inertia constraints of different strength.** When the inertia rules of \( \text{holdsAt} \) and \( \neg \text{holdsAt} \) for a particular CE in \( \Sigma' \) have different weights, the probability of the CE will no longer converge to 0.5. Since the weights impose constraints with different confidence, worlds violating the stronger constraints become less likely than worlds violating the weaker ones. Depending on the relative strength of the weights, the probability of the CE may converge either to 1.0 or 0.0. The relative strength of the weights affects also the rate at which the probability of CE changes. As an extreme example, in Figure 2(a), the rules for the inertia of \( \neg \text{holdsAt} \) remain hard-constrained. By assigning weights to the rules for the inertia of \( \text{holdsAt} \), the persistence of the
CE is lost. Since the inertia constraints of holdsAt are weaker than the constraints of ¬holdsAt, worlds violating the former set of constraints will always be more likely. As a result, the probability of the CE will continue to drop, even below 0.5. The curves MLN–EC<sub>SI</sub><sub>h</sub> (soft-constrained inertia of holdsAt) and MLN–EC<sub>SI</sub><sub>weak</sub> (weaker holdsAt inertia constraints) illustrate how the probability of meeting drops sharply towards 0.0. The weaker the constraints (MLN–EC<sub>SI</sub><sub>weak</sub>) the steeper the drop. In a similar manner, when the inertia constraints of ¬holdsAt are weaker than the constraints of holdsAt, the probability of CE gradually increases and may reach values above 0.5—presented by the MLN–EC<sub>SI</sub><sub>weak</sub> cases in Figure 2(b). 

As explained in Section 5.2, the inertia rule of a specific CE may consist of a large body of conditions, e.g. rule (22). Depending on the number of conditions involved, the inertia rule of a specific CE may be decomposed into several clauses, each corresponding to a different subset of conditions. For instance, the following two clauses are added to by the inertia rule (22):

\[
\begin{align*}
\text{happens}(\text{walking}(ID_1), T) &\lor \text{happens}(\text{running}(ID_1), T) \lor \text{happens}(\text{exit}(ID_1), T) \\
&\lor \neg\text{holdsAt}(\text{meeting}(ID_1, ID_2), T) \lor \text{holdsAt}(\text{meeting}(ID_1, ID_2), T+1) \\
&\neg\text{close}(ID_1, ID_2, 25, T) \lor \text{happens}(\text{running}(ID_1), T) \lor \text{happens}(\text{exit}(ID_1), T) \\
&\lor \neg\text{holdsAt}(\text{meeting}(ID_1, ID_2), T) \lor \text{holdsAt}(\text{meeting}(ID_1, ID_2), T+1)
\end{align*}
\]  

(27) (28)

The above clauses contain literals from the termination rules of the meeting CE. Often, when SDEs that lead to the partial satisfaction of the initiation/termination conditions occur, some of these clauses become trivially satisfied. For example, at time-point 10 both persons ID<sub>1</sub> and ID<sub>2</sub> are active, while their distance is above the threshold of 25 pixels, i.e. close(ID<sub>1</sub>, ID<sub>2</sub>, 25, 10)=False. Consequently, the grounding of clause (28) at time-point 10 is trivially satisfied for all possible worlds. Although the meeting CE is not terminated at time-point 10, because clause (27) is not satisfied, the satisfaction of clause (28) reduces the probability of holdsAt for the CE. This is because the inertia at time-point 10 is now supported only by the satisfaction of the ground clause (27). In other words, the difference between the probabilities of worlds that violate the inertia of holdsAt and worlds that do not, is reduced.

To illustrate this phenomenon, consider the example cases in Figure 3(a) where only the rules about the inertia of holdsAt are soft-constrained. Both MLN–EC<sub>SI</sub><sub>y</sub> and MLN–EC<sub>SI</sub><sub>weak</sub> cases share the same knowledge base. In the MLN–EC<sub>SI</sub><sub>y</sub> case, the occurrence of SDEs causes none of the inertia clauses to become trivially satisfied. In the MLN–EC<sub>SI</sub><sub>weak</sub> case, however, the SDEs are randomly generated and cause a different subset of inertia clauses to become trivially satisfied at each time-point. In both cases the probability of the CE is reduced. In contrast to MLN–EC<sub>SI</sub><sub>y</sub>, however, the inertia in MLN–EC<sub>SI</sub><sub>weak</sub> drops more sharply, as some of the clauses in 𝜎′ are trivially satisfied by the given SDEs. Additionally, the probability of the CE to hold in MLN–EC<sub>SI</sub><sub>y</sub> persists at a different level in each time-point, since different subsets of clauses become trivially satisfied each time. Similarly, in Figure 3(b) the rules about the inertia of ¬holdsAt are soft-constrained. In contrast to MLN–EC<sub>SI</sub><sub>y</sub>, the occurrence of SDEs leads to the partial satisfaction of the initiation conditions causing the inertia in MLN–EC<sub>SI</sub><sub>weak</sub> to persist with a different confidence at each time-point, increasing the probability of the CE to hold more sharply.

Having analysed the effect of softening the inertia rules, it is worth noting that in many real cases the entire knowledge base may be soft-constrained. In this case, since
Probabilistic Event Calculus for Event Recognition

Fig. 3: In both figures no useful SDEs occur in the interval 0 to 100. In both $MLN^{–}EC_{SI^{h}}$ and $MLN^{–}EC_{SI^{h'}}$ none of the inertia clauses in $\Sigma'$ become trivially satisfied by the SDEs. However, in $MLN^{–}EC_{SI^{h}}$ and $MLN^{–}EC_{SI^{h'}}$ some inertia clauses are trivially satisfied by the SDE. In the left figure the CE holds at time 0 with absolute certainty, while in the right figure the CE does not hold at time 0.

the rules in $\Sigma$ are soft-constrained, CEs are not being initiated or terminated with absolute certainty. At the same time, CEs do not persist with certainty, as the rules in $\Sigma'$ are also soft-constrained.

Depending on the requirements of the target application, various policies regarding the soft-constraining of the knowledge base may be adopted. This flexibility is one of the advantages of combining logic with probabilities in the proposed method. Furthermore, it should be stressed that in a typical event recognition application the knowledge base will contain a large number of clauses. The strength of a constraint imposed by a clause is also affected by the weights of other clauses with which it shares the same predicates. Due to these interdependencies, the manual setting of weights is bound to be suboptimal and cumbersome. Fortunately, the weights can be estimated automatically from training sets, using standard parameter optimisation methods.

7. EVALUATION

In this section we evaluate the proposed method ($MLN^{–}EC$) in the domain of video activity recognition. As presented in Section 2, we use the publicly available benchmark dataset of the CAVIAR project. The aim of the experiments is to assess the effectiveness of $MLN^{–}EC$ in recognising CEs that occur among people, based on imperfect CE definitions and in the presence of incomplete narratives of SDEs.

$MLN^{–}EC$ combines the benefits of logic-based representation (e.g. direct expression of domain background knowledge) with probabilistic modeling (e.g. uncertainty handling). For comparison purposes, we include in the experiments two approaches that are closely related to our method. First, we include the logic-based activity recognition method of Artikis et al. [2010b], which we call here $EC_{crisp}$. Like our method, $EC_{crisp}$ employs a variant of the Event Calculus and uses the same definitions of CEs. Unlike $MLN^{–}EC$, $EC_{crisp}$ cannot perform probabilistic reasoning. Second, we include a pure probabilistic method that employs a linear-chain Conditional Random Field model [Lafferty et al. 2001], which we call here $l–CRF$. Similar to our method, $l–CRF$ is a log-linear model that performs probabilistic reasoning over an undirected proba-
bilistic network. On the other hand, l−CRF does not employ a logic-based representation.

7.1. Setup
From the 28 videos of the CAVIAR dataset, we have extracted 19 sequences that are annotated with the meeting and/or moving CEs. The rest of the sequences in the dataset are ignored, as they do not contain examples of the two target CEs. Out of 19 sequences, 8 are annotated with both moving and meeting activities, 9 are annotated only with moving and 2 only with meeting. The total length of the extracted sequences is 12869 frames. Each frame is annotated with the occurrence or not of a CE and is considered an example instance. The whole dataset contains a total of 25738 annotated example instances. There are 6272 example instances in which moving occurs and 3622 in which meeting occurs. For both CEs, consequently, the number of negative examples is significantly larger than the number of positive examples, 19466 for moving and 22116 for meeting.

The input of all three methods consists of a sequence of SDEs, i.e. active, inactive, walking, running, enter and exit. In both MLN−EC and l−CRF, the spatial constraints close and orientationMove are precomputed and their truth value is provided as input. In situations where no event occurs or the distance of the involved persons is above the highest predefined threshold, the tags none and far are given to l−CRF, respectively.

The output of the ECcrisp method consists of a sequence of ground holdsAt predicates, indicating which CEs are recognised. Since ECcrisp performs crisp reasoning, all CEs are recognised with absolute certainty. On the other hand, the output of both probabilistic methods depends on the inference type, i.e. maximum a-posteriori inference (MAP) or marginal. Given a sequence of SDEs, MAP inference outputs the most probable instantiations of CEs for all time-points. On the other hand, marginal inference outputs CEs associated with some probability for all time-points.

Table II presents the structure of the training sequences for the probabilistic methods. In particular, Table II(a) shows an example training sequence for MLN−EC. Each sequence is composed of input SDEs (ground happens), precomputed spatial constraints between pairs of people (ground close and orientationMove), as well as the corresponding CE annotations (ground holdsAt). Negated predicates in the training sequence state that the truth value of the corresponding predicate is False. Table II(b) shows the equivalent input training data for l−CRF. The random variables Person A and Person B represent the events that the two persons may perform at each time-point and variables Close and Orientation Move represent the spatial constraints between the two persons. Similar to MLN−EC, l−CRF does not contain any hidden variables between input SDEs and output CEs. As a result, training is fully supervised for both methods.

To estimate the weights in l−CRF, we use the quasi-Newton optimisation algorithm L-BFGS [Byrd et al. 1994]. For MAP and marginal inference we use the Viterbi and Forward-Backward algorithms [Culotta and McCallum 2004; Sutton and McCallum 2007]. In MLN−EC we use the second-order Diagonal Newton method of Lowd and Domingos [2007] and perform marginal inference with the MC-SAT algorithm [Poon and Domingos 2006], taking 1000 samples for each sequence. We additionally perform max-margin training and MAP inference, using the method of Huynh and Mooney [2009]. In the experiments we use the open-source software packages ALCHEMY [2005] and CRF++6.

6http://code.google.com/p/crfpp
Table II: Example training sets for CE *moving*.

<table>
<thead>
<tr>
<th>Simple Derived Events</th>
<th>Composite Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>happens(walking(id₁), 100)</td>
<td>holdsAt(moving(id₁, id₂), 100)</td>
</tr>
<tr>
<td>happens(walking(id₂), 100)</td>
<td>holdsAt(moving(id₁, id₂), 100)</td>
</tr>
<tr>
<td>orientationMove(id₁, id₂, 100)</td>
<td>orientationMove(id₁, id₂, 100)</td>
</tr>
<tr>
<td>close(id₁, id₂, 24, 100)</td>
<td>close(id₁, id₂, 24, 100)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>happens(active(id₁), 101)</td>
<td>holdsAt(moving(id₁, id₂), 101)</td>
</tr>
<tr>
<td>happens(walking(id₂), 101)</td>
<td>holdsAt(moving(id₁, id₂), 101)</td>
</tr>
<tr>
<td>orientationMove(id₁, id₂, 101)</td>
<td>orientationMove(id₁, id₂, 101)</td>
</tr>
<tr>
<td>¬close(id₁, id₂, 24, 101)</td>
<td>¬close(id₁, id₂, 24, 101)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>happens(walking(id₁), 200)</td>
<td>¬holdsAt(moving(id₁, id₂), 200)</td>
</tr>
<tr>
<td>happens(running(id₂), 200)</td>
<td>¬holdsAt(moving(id₁, id₂), 200)</td>
</tr>
<tr>
<td>¬orientationMove(id₁, id₂, 200)</td>
<td>¬orientationMove(id₁, id₂, 200)</td>
</tr>
<tr>
<td>¬close(id₁, id₂, 24, 200)</td>
<td>¬close(id₁, id₂, 24, 200)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

(a) Input narrative for *MLN–EC*.

<table>
<thead>
<tr>
<th>Person A</th>
<th>Person B</th>
<th>Close</th>
<th>Orientation Move</th>
<th>Composite Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>Moving</td>
</tr>
<tr>
<td>walking</td>
<td>walking</td>
<td>24</td>
<td>True</td>
<td>Moving</td>
</tr>
<tr>
<td>active</td>
<td>walking</td>
<td>Far</td>
<td>True</td>
<td>NotMoving</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>walking</td>
<td>running</td>
<td>Far</td>
<td>False</td>
<td>NotMoving</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

(b) Input sequence for *l–CRF*.

Table II(a) shows a training set for *MLN–EC*. The first column is composed of a narrative of SDEs and precomputed spatial constraints for *MLN–EC*, while the second column contains the CE annotation in the form of ground *holdsAt* predicates. Table II(b) shows the equivalent training set for *l–CRF*. Columns *Person A* to *Orientation Move* contain the input SDEs and spatial constraints, while the last column contains the annotation.

*MLN–EC* is tested under three different scenarios (*MLN–EC_{HI}, MLN–EC_{SI}*, and *MLN–EC_{SI}*, see Table III for a description). In all three variants of *MLN–EC*, the rules in Σ are soft-constrained while the inertia rules in Σ’ are either soft or hard.

Throughout the experimental analysis, the results for marginal inference are presented in terms of F₁ score for threshold values ranging between 0.0 and 1.0. Any CE with probability above the threshold is considered to be recognised. A snapshot of the performance using the threshold value 0.5, is presented in terms of true positives (TP), false positives (FP), false negatives (FN), precision, recall and F₁ score. Additionally, the overall performance for marginal inference is measured in terms of area under precision-recall curve (AUPRC). The number of true negatives in our experiments is significantly larger than the number of true positives. Similar to F₁ score, precision and recall, the AUPRC is insensitive to the number of true negatives. The evaluation results using MAP inference are presented in terms of true positives (TP), false positives (FP), false negatives (FN), precision, recall and F₁. All reported experiment statistics are micro-averaged over the instances of recognised CEs in the 10 folds.

### 7.2. The Methods Being Compared

Both *EC_{crisp* and *MLN–EC* employ a logic-based representation, implement a variant of the Event Calculus and contain equivalent definitions of CEs. The CE definitions of *meeting* and *moving* of *EC_{crisp* are translated into first-order logic for *MLN–EC* using the formulation proposed in Section 3. The definition of *meeting* is given by formulas
Table III: Variants of \( MLN–EC \), using hard and soft inertia rules in \( \Sigma' \).

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( MLN–EC_{HI} )</td>
<td>All inertia rules in ( \Sigma' ) are hard-constrained.</td>
</tr>
<tr>
<td>( MLN–EC_{SI}^{h} )</td>
<td>The inertia rules of ( holdsAt ) are soft-constrained, while the rest of ( \Sigma' ) remains hard-constrained.</td>
</tr>
<tr>
<td>( MLN–EC_{SI} )</td>
<td>All inertia rules in ( \Sigma' ) are soft-constrained.</td>
</tr>
</tbody>
</table>

(7)–(11), while that of \textit{moving} is given by formulas (12)–(17). In contrast to \( EC_{\text{crisp}} \), each clause in \( MLN–EC \) may be associated with a weight value, indicating a degree of confidence.

Similar to \( MLN–EC \), \( l–\text{CRF} \) is a discriminative probabilistic graphical model. The relationship among CEs at successive time-points is modelled as a Markov network, conditioned on the input evidence of SDEs. A CE at any time-point in the sequence is represented by a Boolean random variable, stating whether the CE holds or not. For example, the random variables representing the \textit{moving} CE may take either the tag value \textit{Moving} or \textit{NotMoving} at some time-point in the sequence.

However, there are also several differences between the two probabilistic methods. In \( l–\text{CRF} \), the input SDEs and the spatial constraints are represented by multivariate random variables. For instance, the input SDEs for a particular person are represented by a single random variable that can take any SDE tag value, \textit{e.g.} active, inactive, walking, etc. The relationship among random variables is defined by two types of features. The former type associates input SDEs and spatial constraints with output CEs at the same time-point, creating features for all possible instantiations. The latter type associates successive CEs, in order to form linear chains. In particular, features are constructed for each possible pair of CE instantiations at successive time-points. All features in \( l–\text{CRF} \) are associated with weights and thus all relationships are soft-constrained.

On the other hand, \( MLN–EC \) employs a logic-based representation and all features are produced from ground clauses. Domain knowledge is combined with the Event Calculus axioms, in order to form the structure of the network. For example, the relations between successive CE instantiations are formed by the inertia axioms and the corresponding initiation and termination rules. Moreover, the \( MLN–EC \) provides control over the behaviour of CE persistence, by allowing the inertia clauses to be defined either as soft or as hard constraints. The probabilistic inference in \( MLN–EC \) can deal with both deterministic (i.e. hard-constrained clauses) and probabilistic (i.e. soft-constrained clauses) dependencies, as well as arbitrary structured networks. On the other hand, the structural simplicity of \( l–\text{CRF} \) allows for specialised and significantly faster inference and learning methods.

7.3. Experimental Results of the \( EC_{\text{crisp}} \)

The CE definitions are domain dependent rules that together with the domain-independent axioms of the Event Calculus represent common sense and background knowledge. This knowledge may deviate from that implied by an annotated dataset, resulting in errors when recognising events. Therefore, regarding the annotation of the datasets, the CE definitions are imperfect. Such issues can be clearly shown by analysing the performance of \( EC_{\text{crisp}} \), which uses the CE definitions also used in \( MLN–EC \) and does not involve the representation of probabilistic knowledge.

As shown in Figure 4, \( EC_{\text{crisp}} \) achieves a similar \( F_1 \) score for both activities. However, in terms of precision and recall the situation is quite different, revealing two different cases of imperfect CE definitions. The precision for \textit{moving} is 22 percentage points higher than that of \textit{meeting}. The opposite holds for recall, with the recall for
moving being 21.6 percentage points lower than that of meeting. The lower recall values for moving indicate a larger number of unrecognised moving activities (FN). In some example sequences moving is being initiated late, producing many false negatives. Additionally, the termination rules of moving cause the CE to be prematurely terminated in some cases. For example, when the distance of two persons that are moving together becomes greater than 34 pixels for a few frames, rule (13) terminates moving. On the other hand, compared to moving, the definition of meeting results in a larger number of erroneously recognised meeting activities (FP). The initiation rule (8), for example, causes the meeting activity to be initiated earlier than it should.

Another issue caused by the definitions of meeting and moving is that the two CEs may overlap. According to rules (7)–(17), the initiation of moving does not cause the termination of meeting. Consider, for example, a situation where two people meet for a while and thereafter they move together. During the interval in which moving is detected, meeting will also remain detected, as it is not terminated and the law of inertia holds. However, according to the annotation of the CAVIAR team these activities do not happen concurrently. Furthermore, meeting appears to be annotated in cases where the relative distance of both interacting persons is greater than that used in the initiation rules (7) and (8).

On the other hand, the background knowledge may describe situations that are not included in the dataset. For example, by allowing the meeting CE to be initiated when the two persons are not very close to each other, one may achieve better results in this subset of the dataset, but erroneously recognise the meeting CE in other situations, e.g. people passing by each other. Additionally, the domain-independent property of inertia, which is included in the background knowledge, helps the method to continue to recognise the occurrence of a CE even when the narrative of SDEs is temporally incomplete, e.g. due to camera failures.

7.4. Experimental Results of the Probabilistic Methods

The experiments for the probabilistic methods are organised into two tasks. In the first task, both probabilistic methods are trained discriminatively and their performance is evaluated using 10-fold cross-validation. In the second task, we assess the value of inertia by erasing input evidence from randomly chosen successive time-points. We use the trained models from the first task, but the testing sequences are incomplete narratives of SDEs.

7.4.1. Task I. In contrast to ECcrisp, l–CRF is a probabilistic model and cannot employ background knowledge in the form of common sense rules. The recognition of a CE is affected from all input SDEs that are detected at a particular time-point, as well as from the adjacent CEs that have been recognised. The parameters of the model are estimated from the training set and thus the model is completely data driven. Compared to ECcrisp, l–CRF achieves better performance for both moving and meeting. Using marginal inference, l–CRF gives higher $F_1$ scores for most threshold values, as shown in Figures 4(a) and 4(b). For threshold 0.5, the $F_1$ score is higher than of ECcrisp by 4.6 and 13.5 percentage points for moving and meeting, respectively (see Tables IV(a) and IV(c)). The recall of moving is higher by 18.4 percentage points, while the precision is lower by 13.7 percentage points. l–CRF recognises a larger number of moving activities, increasing the number of both true and false positives. As noted in Section 7.3, this can be achieved by looser spatial constraints. Recall and precision scores for meeting are higher by 1.7 and 23.7 percentage points, respectively. In the
In the first MLN–EC scenario, indicated by MLN–EC_H1, rules in Σ are soft-constrained, i.e. they are associated with a weight value after training. Those weights control the certainty with which a CE holds when its initiation or termination conditions are satisfied. The rules in Σ′, however, remain hard-constrained and thus the behaviour of inertia for both CEs is preserved deterministically and cannot be adjusted. Compared to ECcrisp, MLN–EC_H1 achieves a higher F1 score using marginal inference for the moving CE, for most threshold values (see Figure 4(a)). For threshold 0.5, the recall of MLN–EC is higher by 17.7 percentage points than ECcrisp while precision is lower by 6 points, leading the F1 score of MLN–EC to be higher than ECcrisp by 8.2 points (Table IV(a)). This improvement in recall is caused by the weights that are learned for the termination conditions, which prevent the moving CE from terminating prematurely. Compared to l–CRF, MLN–EC_H1 achieves better F1 scores for many thresholds and higher AUPRC by 4.8 percentage points (see Tables IV(a) and IV(b)). Using MAP inference, MLN–EC_H1 achieves higher F1 score by 11 percentage points than both ECcrisp and l–CRF (see Table V(a)). Compared to ECcrisp, the recall of

Fig. 4: F1 scores using different threshold values for the moving and meeting CE.

Table IV: Results for the moving and meeting CE using marginal inference.

<table>
<thead>
<tr>
<th>Method</th>
<th>TP</th>
<th>TN</th>
<th>FP</th>
<th>FN</th>
<th>Precision</th>
<th>Recall</th>
<th>F1 score</th>
<th>AUPRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECcrisp</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MLN–EC_H1</td>
<td>5121</td>
<td>1856</td>
<td>302</td>
<td>1151</td>
<td>0.8562</td>
<td>0.8165</td>
<td>0.8330</td>
<td></td>
</tr>
<tr>
<td>MLN–EC_EH</td>
<td>5233</td>
<td>1854</td>
<td>944</td>
<td>1039</td>
<td>0.8472</td>
<td>0.8343</td>
<td>0.8407</td>
<td>0.8597</td>
</tr>
<tr>
<td>MLN–EC_Li</td>
<td>4938</td>
<td>1776</td>
<td>1326</td>
<td>1334</td>
<td>0.7410</td>
<td>0.7873</td>
<td>0.7935</td>
<td>0.8280</td>
</tr>
<tr>
<td>l–CRF</td>
<td>5160</td>
<td>1796</td>
<td>1522</td>
<td>1112</td>
<td>0.7272</td>
<td>0.8227</td>
<td>0.7967</td>
<td>0.8358</td>
</tr>
</tbody>
</table>

(a) moving, threshold 0.5.
(b) moving

(c) meeting, threshold 0.5.
(d) meeting

In general l–CRF outperforms ECcrisp in both CE. Unlike ECcrisp, l–CRF interrupts the recognition of meeting when moving starts.
Table V: Results for the moving and meeting CE using MAP inference

<table>
<thead>
<tr>
<th>Method</th>
<th>TP</th>
<th>TN</th>
<th>FP</th>
<th>FN</th>
<th>Precision</th>
<th>Recall</th>
<th>F1 score</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC_crisp</td>
<td>4099</td>
<td>19086</td>
<td>400</td>
<td>2264</td>
<td>0.8816</td>
<td>0.6381</td>
<td>0.7506</td>
</tr>
<tr>
<td>MLN–EC_HI</td>
<td>5598</td>
<td>18358</td>
<td>1128</td>
<td>674</td>
<td>0.8323</td>
<td>0.8925</td>
<td>0.8614</td>
</tr>
<tr>
<td>MLN–EC_SI</td>
<td>6902</td>
<td>18398</td>
<td>1088</td>
<td>370</td>
<td>0.8443</td>
<td>0.9410</td>
<td>0.8901</td>
</tr>
<tr>
<td>MLN–EC_SI</td>
<td>4040</td>
<td>17911</td>
<td>1575</td>
<td>2232</td>
<td>0.7195</td>
<td>0.6441</td>
<td>0.6797</td>
</tr>
<tr>
<td>l–CRF</td>
<td>4716</td>
<td>17848</td>
<td>1638</td>
<td>21825</td>
<td>0.8443</td>
<td>0.9410</td>
<td>0.8901</td>
</tr>
</tbody>
</table>

(a) moving

<table>
<thead>
<tr>
<th>Method</th>
<th>TP</th>
<th>TN</th>
<th>FP</th>
<th>FN</th>
<th>Precision</th>
<th>Recall</th>
<th>F1 score</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC_crisp</td>
<td>3099</td>
<td>20723</td>
<td>1413</td>
<td>523</td>
<td>0.6868</td>
<td>0.8556</td>
<td>0.7620</td>
</tr>
<tr>
<td>MLN–EC_HI</td>
<td>3099</td>
<td>20739</td>
<td>1397</td>
<td>523</td>
<td>0.6893</td>
<td>0.8556</td>
<td>0.7635</td>
</tr>
<tr>
<td>MLN–EC_SI</td>
<td>3067</td>
<td>21825</td>
<td>311</td>
<td>555</td>
<td>0.9079</td>
<td>0.8468</td>
<td>0.8763</td>
</tr>
<tr>
<td>MLN–EC_SI</td>
<td>1083</td>
<td>21641</td>
<td>495</td>
<td>2539</td>
<td>0.6863</td>
<td>0.2990</td>
<td>0.4185</td>
</tr>
<tr>
<td>l–CRF</td>
<td>5154</td>
<td>21906</td>
<td>230</td>
<td>468</td>
<td>0.9320</td>
<td>0.8708</td>
<td>0.9004</td>
</tr>
</tbody>
</table>

(b) meeting

MLN–EC_HI is improved by 25.3 percentage points, while its precision drops by 7 percentage points. MLN–EC_HI achieves higher recall and precision scores than l–CRF, by 14 and 9 percentage points, respectively. However, in the case of meeting, MLN–EC_HI performs worse than EC_crisp and l–CRF in marginal inference, as shown in Figure 4(b) and Table IV(c). The combination of hard-constrained inertia rules with the fact that meeting does not terminate when moving starts, push the weights of the initiation rules to very low values during training. This situation results in many unrecognised meeting instances and low precision and recall values. Max-margin training in this scenario is not affected as much as the Diagonal Newton weight learning method, leading to a model with similar behaviour and performance as EC_crisp, as shown in Table V(b).

In the MLN–EC_SI scenario, while Σ remains soft-constrained, the inertia rules of holdsAt in Σ’ are also soft-constrained. As a result, the probability of a CE tends to decrease, even when the required termination conditions are not met and nothing relevant is happening. This scenario is more suitable to our target activity recognition task and MLN–EC learns a model with a high F1 score for both CEs. In order to explain the effect of soft constraining the inertia of holdsAt, we will use again the example of meeting being recognised and thereafter moving being also recognised. Since meeting is not terminated, it continues to hold and overlaps with moving. During the overlap, all occurring SDE are irrelevant with respect to meeting and cannot cause any initiation or termination. As a result, the recognition probability of meeting cannot be reinforced, by re-initiation. As shown in Section 6, in such circumstances the recognition probability of meeting gradually decreases.

For the moving CE, the performance of MLN–EC_SI using marginal inference is similar to MLN–EC_HI, as shown in Figure 4(a) and Table IV(a). Using a threshold of 0.5, recall is higher than that of EC_crisp by 19.5 percentage points while precision is lower by 6.2 points, resulting in 9 points increase in F1 measure. Compared to l–CRF, MLN–EC_HI achieves higher F1 scores for many thresholds (see Figure 4(a)), as well as higher AUPRC by 2.4 percentage points (see Table IV(b)). In the case of MAP inference, MLN–EC_SI increases further its performance (Table V(a)). Compared to EC_crisp, recall is higher by 30 percentage points and precision drops only by 6.5 percentage points, resulting in 14 percentage points higher F1 score. MLN–EC_SI achieves higher F1 score, precision and recall than l–CRF by 14.3, 10 and 18.9 percentage points, respectively.

For the meeting CE, the performance of MLN–EC_SI using marginal inference is significantly better than of MLN–EC_HI (see Figure 4(b)). At the 0.5 threshold value, pre-
cision increases by 6.9 percentage points over $EC_{crisp}$, while recall falls by only 1 point and thus $F_1$ is higher by 3.5 points (Table IV(e)). However, the $F_1$ scores of $MLN–EC_{SI}$ remain lower than those of $l–CRF$, and its AUPRC is lower by 13.8 points (Table V(d)). Using MAP inference, the performance of $MLN–EC$ improves, but remains worse than $l–CRF$, by 2.4 percentage points in terms of $F_1$ (Table V(b)). $MLN–EC_{SI}$ performs similarly to $l–CRF$ for meeting. $MLN–EC_{SI}$ misses the recognition of meeting at time-points where the persons involved are not sufficiently close to each other according to the initiation rules (7) and (8), i.e. they have a distance greater than 25 pixels.

Finally, in the $MLN–EC_{SI}$ scenario, the entire knowledge base is soft-constrained. The weights in $\Sigma$ allow full control over the confidence that a CE holds when its initiation or termination conditions are met. Additionally, by soft-constraining the rules in $\Sigma'$, $MLN–EC_{SI}$ provides fully probabilistic inertia. However, this flexibility comes at the cost of an increase in the number of parameters to be estimated from data, as all clauses in the knowledge base are now soft-constrained. As a result, $MLN–EC_{SI}$ requires more training data. Using marginal inference $MLN–EC_{SI}$ performs almost the same as $EC_{crisp}$ in terms of $F_1$ score, but worse than $MLN–EC_{SI}$ for both CEs. In the case of MAP inference, $MLN–EC_{SI}$ performs worse than $EC_{crisp}$, as well as $l–CRF$ for both CEs.

The three variants of $MLN–EC$ used in the above experiments, illustrated the potential benefits of softening the constraints and performing probabilistic inference in event recognition. In contrast to $EC_{crisp}$, an important characteristic of $MLN–EC$ is that multiple successive initiations (or terminations) can increase (or decrease) the recognition probability of a CE. By softening the CE definitions, premature initiation or termination can be avoided. In particular, as explained above, the weights learned for the termination definitions of the moving CE reduced the number of unrecognised moving activities.

The choice of rules to be softened affects significantly the event recognition accuracy. In the presented application, for example, the $MLN–EC_{SI}$ setting is the best choice, as softening the inertia of $holdsAt$ provides advantages over crisp recognition. Depending on the target application and the availability of training data, different types of inertia rules may be softened, varying the inertia behaviour from deterministic to completely probabilistic. This is a key feature of $MLN–EC$.

7.4.2. Task II. An important difference between the proposed logic-based approach and its purely data-driven competitors, such as $l–CRF$, is that it is less dependent on the peculiarities of the training data. By incorporating background knowledge about the task and the domain, in terms of logic, it can make the recognition process more robust to variations in the data. Such variations are very common in practice, particularly in dynamic environments, such as the ones encountered in event recognition. The common assumption made in machine learning that the training and test data share the same statistical properties is often violated in these situations. In order to measure the benefits that we can gain by the combination of background knowledge and learning in MLNs, we examine the recognition of CEs under such situations. In particular, in this second task, we are comparing the performance of $MLN–EC$ and $l–CRF$ in cases where the input evidence is missing from a number of successive time-points in the test data. The models are not retrained and thus their weights remain as they were estimated in Task I.

The incomplete test sequences are generated artificially by erasing successive input SDEs and associated spatial relations at random starting points. We have generated two variants of incomplete test sequences, one containing random blank intervals of length $10$ and $20$ time-points respectively. The starting time-points of the blank intervals are chosen randomly with probability $0.01$, drawn from a uniform distribution.
Since the target CEs require the interaction of two persons, erasing events involving a single person cannot affect the performance of the recognition methods that we compare. Therefore, blank intervals are created only from time-points where both persons are involved in some SDEs. This process of artificially generating incomplete test sequences is repeated five times, generating corresponding test sets.

The recognition performance can be affected in various ways, depending on the position where evidence is erased from the test sequences. For example, when the beginning of an activity is erased, its recognition will be delayed, increasing the number of false negatives. Similarly, erasing information at the end of an activity will delay the termination of a CE, resulting in a higher number of false positives. In cases where missing information appears during an activity, the recognition of the CE may be interrupted, increasing the number of false negatives. The recognition performance may even be improved in some cases, due to the removal of SDEs that would cause false positives or false negatives.

Out of all the methods examined in Task I the performance of EC_{crisp} and MLN–EC_{HI} is bound to be affected less by the missing data, due to the use of deterministic inertia. This is because the erased evidence will often be in the interval that a CE is (not) being recognised. In these cases, the erased evidence will not affect the inertia of the CE and the CE will remain (not) recognised. EC_{crisp} and MLN–EC_{HI} are only affected when the evidence is erased at the beginning or at the end of an activity, which is less frequent. For this reason, we chose to exclude these two methods from Task II.

Furthermore we exclude the MLN–EC_{SI}, which performed significantly worse than MLN–EC_{SI} in Task I. Therefore, in this task we compare only MLN–EC_{SI} against l–CRF.

In the rest of this section we will denote medium and large incomplete sequences the test sequences that contain random blank intervals of 10 and 20 time-point duration, respectively. The evaluation measures are the same as in Task I. Figure 5 presents the results in terms of $F_1$ score for the two methods, using marginal inference. The bar charts of Figure 6, on the other hand, present the average AUPRC of the two methods compared also to the AUPRC when no data is removed (from Tables IV(b) and IV(d) of Task I). The threshold values range between 0.0 and 1.0. Using similar illustration, Figures 7, 8 and 9 present the results of the two methods using MAP inference. All results are averaged over five runs and error bars display the standard deviation.

Unlike MLN–EC_{SI}, l–CRF appears to be affected significantly from incomplete evidence. Using marginal inference on the moving CE in the original test sequences (see Table IV(a)) l–CRF achieved an $F_1$ score of 0.7967. At the same threshold value, the average $F_1$ score of l–CRF drops to 0.566 and 0.53 for medium and large incomplete sequences, respectively. MLN–EC_{SI} is affected much less, achieving $F_1$ scores of 0.836 and 0.832, for medium and large incomplete sequences, respectively. In terms of AUPRC (Figure 6(a)), the performance of l–CRF also drops by 13 points, while MLN–EC_{SI} is almost unaffected. When MAP inference is used, the effect of the removal of data seems to be larger. The recall of l–CRF falls by more than 45 percentage points, causing the $F_1$ score to drop by more than 30 percentage points (Figures 8(a) and 7(a)). The number of recognised moving activities is reduced, resulting in an increase in precision, but with high variance (Figure 8(a)). The precision of MLN–EC_{SI} remains close to the original test set, with a small variance. However, its recall drops and causes the reduction of $F_1$ score by 8 and 9 percentage points for medium and large incomplete sequences, respectively.

In the case of the meeting CE, MLN–EC_{SI} also seems to resist more than l–CRF to the distortion of the data. The $F_1$ score is higher than that of l–CRF for many threshold values, using marginal inference (see Figures 5(b) and 5(d)). For threshold 0.5 in the original test sequences l–CRF achieved an $F_1$ score that was higher by 10 percentage points.
points than that of $MLN-EC_{SI}$. However, when data are removed its $F_1$ score for the same threshold drops much lower than that of $MLN-EC_{SI}$ (a difference of more than 15 percentage points). The AUPRC of $l-CRF$ also drops much more (more than 10 points) than that of $MLN-EC_{SI}$ (see Figure 6(b)). The effect is even higher when MAP inference is used for $meeting$ CE. In particular, the recall of $l-CRF$ drops more than 60 percentage points (see Figure 9(b)), while that of $MLN-EC_{SI}$ drops much less. Thus, the $F_1$ score of $MLN-EC_{SI}$ reduces by less than 10 percentage points, while that of $l-CRF$ is 50 percentage points lower than in the original data (see Figure 7(b)).

In summary, this task showed that the logic-based $MLN-EC$ method is more robust than its purely statistical $l-CRF$ counterpart, when data are removed from the test set, rendering it less similar to the training set. This is due to the fact that $l-CRF$ is completely data driven and does not employ any background knowledge. On the other hand, $MLN-EC$ employs background knowledge, including the domain independent axioms of inertia. Consequently, the persistence of a CE is modeled differently by $l-CRF$ and $MLN-EC$. $l-CRF$ learns to maintain the state of CEs under some circumstances that appear in the training data. However it does not model the inertia
Probabilistic Event Calculus for Event Recognition

MLN–EC_{st}  l–CRF
MLN–EC_{st}  l–CRF

AUPRC

MLN–EC_{st}  l–CRF
MLN–EC_{st}  l–CRF

F_1 score

Precision

(a) moving (b) meeting
Fig. 6: Average AUPRC scores over five runs when data are removed. Marginal inference is used.

(a) moving (b) meeting
Fig. 7: Average F_1 scores over five runs when data are removed. MAP inference is used.

(a) moving (b) meeting
Fig. 8: Average precision over five runs when data are removed. MAP inference is used.

explicitly. Therefore, when the circumstances change its performance is hurt significantly. MLN–EC on the other hand enjoys the explicit modelling of inertia, provided as background knowledge. Even when this inertia is softened, it remains a strong bias in the model. As a result, MLN–EC avoids overfitting the training data and behaves robustly when the data changes.
8. RELATED WORK

Event Calculus is related to other formalisms in the literature of commonsense reasoning, such as the Situation Calculus [McCarthy and Hayes 1968; Reiter 2001], the Fluent Calculus [Thielscher 1999; 2001], the action language C+ [Giunchiglia et al. 2004; Akman et al. 2004] and Temporal Action Logics [Doherty et al. 1998; Kvarnström 2005]. Action formalisms provide domain-independent axioms in order to represent and reason about the effects of events and support the property of inertia. Comparisons and proofs of equivalence between formalisms for commonsense reasoning can be found in Kowalski and Sadri [1997], Van Belleghem et al. [1997], Chittaro and Montanari [2000], Miller and Shanahan [2002], Schiffl and Thielscher [2006], Mueller [2006, Chapter 15], Craven [2006] and Paschke and Kozlenkov [2009].

Probabilistic extensions of the Situation Calculus have been proposed in the literature, in order to support noisy input from sensors, stochastic events and model Markov Decision Processes, e.g. see Bacchus et al. [1995], Pinto et al. [2000], Mateus et al. [2001], Hajishirzi and Amir [2008] and Reiter [2001, Chapter 12]. Furthermore, Hölldobler et al. [2006] proposed a probabilistic extension of the Fluent Calculus. Both Situation Calculus and Fluent Calculus, as well as in their probabilistic variants use a tree model of time, in which each event may give rise to a different possible future. A point in time is represented by a situation, which is a possible sequence of events. As a result, events are represented to occur sequentially and atemporally.

In the Event Calculus, as well as in C+ and Temporal Action Logics, there is a single time line on which events occur. This is a more suitable model for event recognition, where the task is to recognise CEs of interest in a time-stamped sequence of observed SDEs.

PC+ is a probabilistic generalisation of C+ that incorporates probabilistic knowledge about the effects of events [Eiter and Lukasiewicz 2003]. PC+ supports nondeterministic and probabilistic effects of events, as well as probabilistic uncertainty about the initial state of the application. Similar to the aforementioned probabilistic variants of commonsense reasoning languages, the method focuses on planning under uncertainty while inertia remains deterministic.

In MLN–EC we can have customisable inertia behaviour by adjusting the weights of the inertia axioms, as shown in Sections 6 and 7. To deal with uncertainty we employ the framework of Markov Logic Networks for probabilistic modelling and automatically estimate the weights from training data.

A related approach that we have developed in parallel is that of Skarlatidis et al. [2013]. The method employs an Event Calculus formalism that is based on probabilistic logic programming and handles noise in the input data. Input SDEs are assumed...
to be independent and are associated with detection probabilities. The Event Calculus axioms and CE definitions in the knowledge base remain hard-constrained. Given a narrative of SDEs, a CE may be recognised with some probability. Any initiation or termination caused by the given SDEs increases or decreases the probability of a CE to hold. Inertia is modelled by the closed-world semantics of logic programming and is restricted to be deterministic. It is worth-noting that the MLN approach presented in this paper does not make any independence assumption about the input SDEs.

Shet et al. [2007] proposed an activity recognition method that is based on logic programming and handles uncertainty using the Bilattice framework [Ginsberg 1988]. The knowledge base consists of domain-specific rules, expressing CEs in terms of SDEs. Each CE or SDE is associated with two uncertainty values, indicating a degree of information and confidence respectively. The underlying idea of the method is that the more confident information is provided, the stronger the belief about the corresponding CE becomes. Another logic-based method that recognises user activities over noisy or incomplete data is proposed by Filippaki et al. [2011]. The method recognises CEs from SDEs using rules that impose temporal and spatial constraints between SDEs. Some of the constraints in CE definitions are optional. As a result, a CE can be recognised from incomplete information, but with lower confidence. The confidence of a CE increases when more of the optional SDEs are recognised. Due to noisy or incomplete information, the recognised CEs may be logically inconsistent with each other. The method resolves those inconsistencies using the confidence, duration and number of involved SDEs. In contrast to these methods, our work employs MLNs that have formal probabilistic semantics, as well as an Event Calculus formalism to represent complex CEs.

Probabilistic graphical models have been successfully applied to a variety of event recognition tasks where a significant amount of uncertainty exists. Since event recognition requires the processing of streams of time-stamped SDE, numerous event recognition methods are based on sequential variants of probabilistic graphical models, such as Hidden Markov Models (HMM) [Rabiner and Juang 1986], Dynamic Bayesian Networks (DBN) [Murphy 2002] and linear-chain Conditional Random Fields (CRF) [Lafferty et al. 2001]. Such models can naturally handle uncertainty but their propositional structure provides limited representation capabilities. To overcome this limitation, graphical models have been extended to model interactions between multiple entities [Brand et al. 1997; Gong and Xiang 2003; Wu et al. 2007; Vail et al. 2007], to capture long-term dependencies between states [Hongeng and Nevatia 2003] and to model the hierarchical composition of events [Natarajan and Nevatia 2007; Liao et al. 2005]. However, the lack of a formal representation language makes the definition of structured CEs complicated and the use of background knowledge very hard.

Recently, statistical relational learning (SRL) methods have been applied to event recognition. These methods combine logic with probabilistic models, in order to represent complex relational structures and perform reasoning under uncertainty. Using a declarative language as a template, SRL methods specify probabilistic models at an abstract level. Given an input stream of SDE observations, the template is partially or completely instantiated, creating lifted or propositional graphical models on which probabilistic inference is performed [de Salvo Braz et al. 2008; Raedt and Kersting 2010].

Among others, HMMs have been extended in order to represent states and transitions using logical expressions [Kersting et al. 2006; Natarajan et al. 2008]. In contrast to standard HMM, the logical representation allows the model to represent compactly probability distributions over sequences of logical atoms, rather than propositional symbols. Similarly, DBNs have been extended using first-order logic [Manfredotti 2009; Manfredotti et al. 2010]. A tree structure is used, where each node corre-
sponds to a first-order logic expression, e.g. a predicate representing a CE, and can be related to nodes of the same or previous time instances. Compared to their propositional counterparts, the extended HMM and DBN methods can compactly represent CE that involve various entities.

Our method is based on Markov Logic Networks (MLNs), which is a more general and expressive model. The knowledge base of weighted first-order logic formulas in MLNs defines an arbitrarily structured undirected graphical model. Therefore, MLNs provide a generic SRL framework, which subsumes various graphical models, e.g. HMM, CRF, etc., and can be used with expressive logic-based formalisms, such as the Event Calculus. The inertia axioms of our method allow the model to capture long-term dependencies between events. Additionally, adopting a discriminative model, the method avoids common independence assumptions over the input SDEs.

Markov Logic Networks have been used for event recognition in the literature. Biswas et al. [2007] combine the information provided by different low-level classifiers with the use of MLNs, in order to recognise CEs. Tran and Davis [2008]; Kembhavi et al. [2010] take into account the confidence value of the input SDEs, which may be due to noisy sensors. A more expressive approach that can represent persistent and concurrent CEs, as well as their starting and ending points, is proposed by Helaoui et al. [2011]. However, that method has a quadratic complexity to the number of time-points.

Morariu and Davis [2011] proposed an MLN-based method that uses interval relations. The method determines the most consistent sequence of CEs, based on the observations of low-level classifiers. Similar to Tran and Davis [2008]; Kembhavi et al. [2010] the method expresses CEs in first-order logic, but it employs temporal relations from the Interval Algebra [Allen 1983]. In order to avoid the combinatorial explosion of possible intervals, as well as to eliminate the existential quantifiers in CE definitions, a bottom-up process eliminates the unlikely CE hypotheses. The elimination process can only be applied to domain-dependent axioms, as it is guided by the observations and the Interval Algebra relations. A different approach to interval-based activity recognition, is the Probabilistic Event Logic (PEL) [Brendel et al. 2011; Selman et al. 2011]. Similar to MLNs, the method defines a log-linear model from a set of weighted formulas, but the formulas are represented in Event Logic [Siskind 2001]. Each formula defines a soft constraint over some events, using interval relations that are represented by the spanning intervals data structure. The method performs inference via a local-search algorithm (based on MaxWalkSAT of Kautz et al. [1997]), but using the spanning intervals it avoids grounding all possible time intervals. In our work, we address the combinatorial explosion problem in a more generic manner, through the efficient representation of the domain-independent axioms. Additionally, we use a transformation procedure to further simplify the structure of the Markov network. The transformation is performed at the level of the knowledge base and is independent of the input SDEs.

Sadilek and Kautz [2012] employ hybrid-MLNs [Wang and Domingos 2008] in order to recognise successful and failed interactions between humans, using noisy location data from GPS devices. The method uses hybrid formulas that denoise the location data. Hybrid formulas are defined as normal soft-constrained formulas, but their weights are also associated with a real-valued function, e.g. the distance of two persons. As a result, the strength of the constraint that a hybrid rule imposes is defined by both its weight and function — e.g. the closer the distance, the stronger the constraint. The weights are estimated from training data. However, the method does not employ any generic formalism for representing the events and their effects and thus it uses only domain-dependent CE definitions. On the other hand, the use of a hybrid
approach for numeric constraints is an interesting alternative to the discretisation adopted by our method.

9. CONCLUSIONS
We addressed the issue of imperfect CE definitions that stems from the uncertainty that naturally exists in event recognition. We proposed a probabilistic version of the Event Calculus based on Markov Logic Networks (MLN–EC). The method has declarative and formal (probabilistic) semantics, inheriting the properties of the Event Calculus. We placed particular emphasis on the efficiency and effectiveness of our approach. By simplifying the axioms of the Event Calculus, as well as following a knowledge transformation procedure, the method produces compact Markov networks with reduced complexity. Consequently, the performance of probabilistic inference is improved, as it places on a simpler model. MLN–EC supports flexible CE persistence, ranging from deterministic to probabilistic, in order to meet the requirements of different applications. Due to the use of MLNs, the method lends itself naturally to learning the weights of event definitions from data, as the manual setting of weights is sub-optimal and cumbersome. MLN–EC is trained discriminatively, using a supervised learning technique. In the experimental evaluation, MLN–EC outperforms its crisp equivalent on a benchmark data. MLN–EC matches the performance of a linear-chain Conditional Random Fields method. Furthermore, due to the use of the Event Calculus, MLN–EC is affected less by missing data in the test sequences than its probabilistic akin.

There are several directions in which we would like to extend our work. In many applications the input SDE observations are accompanied by a degree of confidence, usually in the form of probability. Therefore, we consider extending our method in order to exploit data that involves such confidence values, either in the form of additional clauses (e.g. Tran and Davis [2008], Morariu and Davis [2011]), or by employing different inference algorithms (e.g. Jain and Beetz [2010]). Furthermore, we would like to address the problems that involve numerical constraints by adopting a hybrid-MLN (e.g. Sadilek and Kautz [2012]) or a similar approach. We also consider extending our formalism in order to support temporal interval relations, using preprocessing techniques (e.g. Morariu and Davis [2011]), or by employing different representation and inference methods (e.g. Brendel et al. [2011], Selman et al. [2011]). As shown in Section 7, the MLN–EC with soft-constrained inertia performs well. We would like to extend our method to automatically soften the right subset of inertia axioms. Finally, we would like to examine structure learning/refinement methods for the CE definitions, since they are often hard to acquire from experts.

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