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Space-domain non-iterative approach for SPECT/CT systems considering attenuation and space-variant detector response

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Abstract

A quantitative analysis of emission planar image reconstruction from projections by an object dependent, exact, direct approach in the space-domain considering both object attenuation and space-variant impulse response of SPECT/CT systems is proposed. That approach is compared with iterative methods and non-object-dependent exact methods in both the space domain and the frequency one. Since the mean-projection precorrection method is the concern of some actual 3D methods of compensation for distance-dependent spatial resolution and is thought right for competing with different methods able to quantify the tracer density in the object of interest, it is also examined in the course of the analysis.

The direct approach may also augment the simulation power of the Matlab Image Processing Toolbox concerning the direct and inverse Radon transform from parallel projection data, the Toolbox being actually restricted to the ideal transform in the frequency domain.

Keywords: Image reconstruction from projections; Iterative methods; Mean projection; Attenuated Radon transform blurred by space-variant point spread function

1. Introduction

An ideal method of image reconstruction by real gamma camera (GC) systems for single photon emission computed tomography (SPECT) should handle a shift-variant point spread function (psf), a correlated non-stationary noise model, attenuation and scattering. Several methods have been proposed in literature to gain that purpose more or less completely.

Direct reconstruction methods are always attractive, and projection correction before reconstruction is theoretically superior since it removes or reduces the inconsistencies in the reconstruction algorithm. Mean imaging, a long accepted method for quantitating data from planar images [1–3], would also be the concern of some recent 3D methods of compensation for distance-dependent spatial resolution [4].

The alternative use of approximate iterative reconstruction methods allows for the inclusion of a model for the projection process into the reconstruction algorithm, with the hope the inclusion of that model would provide more accurate reconstruction image. Unfortunately, those methods are not exempt from drawbacks and limitations essentially due to computation time, difficulty in realizing a reliable control of their structural instabilities manifested in high variance in the estimated image pixel intensities, and more or less accurate approximation of the elements of the system matrix.

As for the noise analysis of the image reconstructed by iterative methods, though on the basis of assumptions unlikely occurring at each iteration by the actual system, it has been demonstrated the ensemble iteration noise in both the maximum-likelihood expectation-maximization (MLEM) [5,6] method and methods accelerating the MLEM one like the ordered-subsets of the projection data (OSEM) [7] and the rescaled block-iterative expectation-maximization of the EM (RBIEM) algorithm [8–10] can be modelled by a log-normal distribution characterized by a width increasing with increasing iterative number [11–14]. That asymmetric distribution will then lead to significant correlation between image pixels.

The more or less involved methods used to control those instabilities lead to degraded image resolution [15,16], biases [17], time consuming adaptive filtering strategies [18] based on local statistics [19–22], and also artefacts in reconstructed image when applying inter-smoothed reprojections into OSEM.
iterations by the recent log-normal distribution inspired robust wavelet de-noising method [23].

When using maximum a posteriori (MAP) solutions to penalize noisy images, the thorough difficulty consists in selecting both the penalty term and the scalar weight of the prior. The regularization process is always realizable, but can the weight term be assigned univocally and simply [24,25]? Does the adopted a priori model supporting the selected penalty term come up to the actual physical reality? Though recent approaches use both quadratic stabilizing functional and boundaries from anatomically based imaging modalities to provide prior information about sites where no smoothing is desired [26,27], an effective and reliable knowledge of the random field specifying the regularizing image would be needed when applying the a priori model.

MAP-EM [28,29] and MAP-RBI [30,31] methods are available as well as ensemble noise analysis of some MAP-EM algorithms [32]. Alternative MAP reconstruction approximated methods able to quickly compute statistical properties in plugin form rather than ensemble statistical ones, aside from the particular optimizing algorithm, are in rapid progress [33–35]. Unfortunately, the assumptions at the basis of those methods are not well suited to the case of lesions with varying size and activity.

At last, a recent study [36] searching for the interaction between the system matrix accuracy and the quality of the image reconstructed by both penalized weighted least square (PWLS) [56] and OSEM algorithms, confirms the general behaviour of any iterative algorithm: when the data are inconsistent with the measured ones, the algorithm will produce artifacts by having a try to resolve that inconsistency.

The goal of the present work is twofold. First, the simulation power of the Matlab Image Processing Toolbox concerning the Radon Transform from parallel projection, actually restricted to the ideal transform, will increase once adapted to real situations providing it for both non-homogeneous attenuation in the examining section and space-variant resolution of the instrument acquiring the parallel projection data. Second, owing to the synthesized drawbacks due to emission image reconstruction by iterative methods, taking advantage of the actual multimodality imaging systems, the present work aims to develop a direct approach in the space-domain able to determine the ideal projection when considering attenuation and space-variant point spread function. The proposed object-dependent exact approach will result more tractable than the non-object-dependent exact method in the frequency domain described in [37–40] and get over the limitations due to different exact methods when dealing with non-homogeneous attenuation object [41].

According to Lipinski et al. [27], common assumptions for the direct approach will be: (1) the borders between anatomical compartments coincide with borders between areas with homogeneous uptake of radioactivity; (2) the anatomical compartments can be separated and identified by appropriate segmentation of the CT images.

Taking advantage of the analysis concerning the 2D emission image reconstruction from projections, the limitations resulting from the use of mean projection to solve quantitatively the same problem in closed form shall also be pointed out. The scatter phenomenon, detailed noise analysis and the projection denoising problem, the cross contamination between 2D image slices as well as the strategy to adopt in case the previous assumptions for the direct approach are not fulfilled and the point spread function is not known exactly shall be neglected; owing to the complexity inherent in those subjects, they shall be treated in future work.

2. Methods

2.1. Statement of the 2D image reconstruction problem from projections

Onwards, \( \rho(x,y), p_{\mu a}, p_{\mu c}, h_d, h_c, \bar{h}_d \) and \( \bar{h}_c \) will signify one unknown activity density distribution in a section of the body, the direct exponential projection (or premodified projection) independent of the attenuator boundary, the conjugate exponential projection, the GC direct impulse response, the GC conjugate impulse response, and the arithmetic mean and the geometric mean of \( h_d \) and \( h_c \), respectively.

According to the two-dimensional approximation, if the \((x,y)\) orthogonal coordinate system is rotated by an angle \( \theta \) and the new axes are labelled by \( \xi \) and \( \eta \) (see Fig. 1; for each view, the axis \( \eta \) is normal to the collimator plane), by neglecting the scatter phenomenon and assuming both the attenuation coefficient distribution \( \mu(x,y) \) is a constant equal to \( \mu \) in the emitting domain and the GC system is a linear one, the superposition integral for each view may be written as follows [37]:

\[
\begin{align*}
   p_{\mu a}(\xi; \theta) &= \int \int \rho(\xi', \eta) h_d(\xi - \xi', \eta) e^{\mu \eta} \, d\xi' \, d\eta \\
   p_{\mu c}(\xi; \theta) &= \int \int \rho(\xi', \eta) h_c(\xi - \xi', \eta) e^{-\mu \eta} \, d\xi' \, d\eta
\end{align*}
\]

Fig. 1. The 2D parallel-beam geometry.
By (1) and (2), the arithmetic mean and the geometric mean of the direct and conjugate projections may, respectively, be written as
\[ p_{\mu_d}(\xi;\theta) = \frac{p_{\mu_d} + p_{\mu_c}}{2} = \int_{\xi} \int_{\eta} \rho(\xi',\eta) h_d e^{\mu \eta} + h_c e^{-\mu \eta} \frac{1}{2} d\xi' d\eta \] (3)
\[ p_{\mu_c}(\xi;\theta) = \int_{\xi} \int_{\eta} \rho(\xi',\eta) \sqrt{h_d h_c} d\xi' d\eta \] (4)

Assuming \( h_d = (h_d + h_c)/2 \) is space-invariant, Ying-Lie [1] writes:
\[ p_{\mu_d} = \int_{\xi} \int_{\eta} \rho(\xi',\eta) h_d(\xi - \xi') \cosh(\mu \eta) d\xi' d\eta \]
\[ = \int_{\pi} \left( \int_{\eta} \rho(\xi',\eta) \cosh(\mu \eta) d\eta \right) \bar{h}_d(\xi - \xi') d\xi' \]
\[ = \int_{\xi} q(\xi') \bar{h}_d(\xi - \xi') d\xi' = q(\xi') * \bar{h}_d(\xi - \xi') \] (5)

The solution of the image reconstruction problem comes then down to the determination of the function \( q(\xi') \) by deconvolution of \( p_{\mu_d} \) with \( \bar{h}_d \), and the next solution of the integral equation \( q(\xi') = \int \rho(\xi',\eta) \cosh(\mu \eta) d\eta \) which was obtained in [1] with reference to the special case of a circular emitting domain.

Let us assume the GC depth-variant geometric impulse response may be modelled in each slice by the Gaussian function:
\[ h = \frac{1}{\sqrt{2\pi} \sigma(\eta_{sc})} e^{-\xi^2/(2\sigma^2(\eta_{sc}))} \] (6)

\( \eta_{sc} \) being the source-to-collimator distance in the \( (\xi, \eta) \) orthogonal coordinate system.

A generally adopted first degree linear model approximation [37,42–44] relating \( \eta_{sc} \) and the depth-variant collimator resolution, \( R_c \), is
\[ R_c = \text{FWHM}_{mc} = a_m + b_m \eta_{sc} \] (7)

If \( R_c \) is the crystal intrinsic resolution, the resolution of the GC system and the corresponding \( \sigma(\eta_{sc}) \) shall then be
\[ \text{FWHM}_{gc} = \sqrt{R_c^2 + R_c^2} \]
\[ \sigma(\eta_{sc}) = \frac{\text{FWHM}_{gc}}{8 \ln 2} = \sqrt{\frac{R_c^2 + (a_m + b_m \eta_{sc})^2}{8 \ln 2}} \] (8)

For a generic view, the direct projection and the conjugate one of a punctiform radioactive source at depth \( \eta \) in the object section are characterized by values of \( \eta_{sc} \) equal to \( \eta_{sc}^d = r_{gc} - \eta \) and \( \eta_{sc}^c = r_{gc} + \eta \), respectively, being the GC radius of rotation. So, owing to (8), the variances corresponding to the two projections are
\[ \sigma_d^2 = \sigma^2(\eta_{sc}^d) = c_2 \eta^2 + c_1 \eta + c_0 = P(\eta), \]
\[ \sigma_c^2 = \sigma^2(\eta_{sc}^c) = c_2 \eta^2 - c_1 \eta + c_0 = Q(\eta) \] (9)

with
\[ k = \frac{1}{8 \ln 2}, \quad c_0 = k[R_c^2 + (a_m + b_m r_{gc})^2], \quad c_1 = -2b_m k(a_m + b_m r_{gc}), \quad c_2 = k b_m^2 \]

By (6) and (9), the impulse responses \( \bar{h}_d, \bar{h}_c, \bar{h}_s, \) and \( h_g = \sqrt{h_d h_c} \) can be determined.

As shown in Fig. 2, both \( \bar{h}_d \) and the two kernels \( y_1 = \bar{h}_d \cosh(\mu \eta), \ y_2 = (h_d e^{\mu \eta} + h_c e^{-\mu \eta})/2 \) in (5) and (3), respectively, are not space-invariant; for an assigned position of the source in the working slice, the corresponding \( \eta \) value varies according to the acquiring view. Moreover, by the adopted model in (6) and the system parameters specified in the Fig. 2 caption, the function \( y_1 \) coincides with the \( y_2 \) one for \( \eta \leq 2 \text{ cm} \). It follows the inadequacy of Eq. (5) for large \( \eta \) values.

Well as far as \( h_g \) is concerned, by (6) and (9), \( h_g \) can be rendered explicit as follows:
\[ h_g = \frac{1}{\sqrt{2\pi} \sqrt{c_2^2 \eta^4 + (2c_0 c_2 - c_1^2) \eta^2 + c_0^2}} \times \exp \left\{ -\frac{c_2 \eta^2 + c_0}{2} \left[ \frac{c_2^2 \eta^4 + (2c_0 c_2 - c_1^2) \eta^2 + c_0^2}{(\xi - \xi')^2} \right] \right\} \] (10)

So, the kernel associated with \( p_{\mu_g} \) in the integral Eq. (4) besides being space-variant (Fig. 2) is also so complex to forbid a closed form solution of the same equation.

### 2.2. Space-domain direct approach

In this section we will refer to direct projections only, so the ‘d’ subscript will no more be inserted in both the projection and psf symbols.

For each view, let us say \( \eta_{\text{min}}, \eta_{\text{max}} \) the minimum \( \eta \) value and the maximum one corresponding to the interesting emitting domain in the actual view, and \( \xi_1(\eta), \xi_2(\eta) \) the smaller \( \xi \) value and the greater one corresponding to the intersection of the same domain with the straight line \( \eta = \text{constant}, \ \eta \) varying in the interval \( I_\eta \in [\eta_{\text{min}}, \eta_{\text{max}}] \) (Fig. 1).

By (1), the projection \( p_{\mu_0} \) corresponding to the depth-variant impulse response \( h_0 \) will be
\[ p_{\mu_0}(\xi;\theta) = \int_{\eta} \int_{\eta} \rho(\xi',\eta) h_0(\xi - \xi') e^{\mu \eta} d\xi' d\eta \]
\[ = \int_{\xi} \left[ \int_{\eta} \rho(\xi',\eta) e^{\mu \eta} d\eta \right] h_0(\xi - \xi') d\xi' \]
\[ = \int p_{\mu_d}(\xi') h_0(\xi - \xi') d\xi' \]
\[ = p_{\mu_d}(\xi') * h_0(\xi - \xi') \] (11)
the attenuated Radon transform convolved with the function \( p_{0,\mu}(\eta) \), it results:

\[
\int_{\xi_{\min}}^{\xi_{\max}} \frac{e^{\mu \eta}}{\sqrt{2\pi\sigma_0}} \left[ \exp \left( -\frac{\xi^2 - 2\eta\xi + \eta^2}{2\sigma^2} \right) \right] d\eta
\]

implies a deconvolution operation.

2.3. Single emitting domain

Let us say \( \rho \) the constant activity density distribution in the interesting domain; owing to the model (6) and the first relation in (9), Eqs. (1) and (11) can respectively be written as [46, p. 303, no. 7.4.32]:

\[
p_{\mu_0}(\xi;\theta) = \frac{\rho}{\sqrt{2\pi\sigma_0}} \int_{\eta_{\min}}^{\eta_{\max}} e^{\mu \eta} \left[ \exp \left( -\frac{\xi^2 - 2\eta\xi + \eta^2}{2\sigma^2} \right) \right] d\eta
\]

Eqs. (12) and (13) by themselves are obviously suitable in simulation problems. In problems of image reconstruction, they provide us with the wanted relation between \( p_{\mu_0} \) and \( p_{\mu} \); it results:

\[
p_{\mu_0}(\xi;\theta) = \frac{\rho}{\sqrt{2\pi\sigma_0}} \int_{\eta_{\min}}^{\eta_{\max}} e^{\mu \eta} \left[ \exp \left( -\frac{\xi^2 - 2\eta\xi + \eta^2}{2\sigma^2} \right) \right] d\eta
\]

As shown in Fig. 1, the functions \( \xi_1(\eta) \) and \( \xi_2(\eta) \) take on values included in the interval \( I_\xi = [\xi_{\min}, \xi_{\max}] \), the minimum \( \xi \) value, \( \xi_{\min} \), and the maximum one, \( \xi_{\max} \) being determined by the emitting domain in the actual view. Both \( I_\xi \) and \( I_\eta \) vary according to the view, so are the two functions.

The inner integral in Eqs. (12) and (13) can be considered as the weight assigned to the integrand function in the outer integral for each \( \eta \) value. When dealing with \( p_{\mu_0} \), that weight is the part of a Gaussian function with the space-invariant standard

\[
\sigma^2 = \frac{\rho^2}{\sqrt{2\pi\sigma_0}^2} \int_{\eta_{\min}}^{\eta_{\max}} e^{\mu \eta} \left[ \exp \left( -\frac{\xi^2 - 2\eta\xi + \eta^2}{2\sigma^2} \right) \right] d\eta
\]
deviation, $\sigma_0$, determined by the interesting emitting domain at the actual $\eta$ value. When dealing with $p_\mu$, the weight is the part of a Gaussian function with the space-variant variance $P(\eta)$ determined by both the interesting emitting domain at the actual $\eta$ value and the same $\eta$ value via the stated psf behaviour (Fig. 1).

The way of depending of the two functions $\xi_1(\eta)$ and $\xi_2(\eta)$ on $\eta, \eta \in I_p$, may be determined once known the emitting domain by segmentation.

Once known $p_{\mu 0}$ by (14), the attenuated Radon transform can consequently be computed by (11). On the other hand, for an emitting domain with constant density, the attenuated Radon transform and the ideal one can, respectively, be written as

$$p_{\mu d}(\xi; \theta) = \rho \int_{\eta_{\text{max}}}^{\eta_{\text{min}}} e^{\mu \eta} d\eta$$

In the case of quite general domain, by directly solving the integrals in (15) and (16), it results:

$$p_{\mu d}(\xi; \theta) = \frac{\mu (\eta_{\text{max}}(\xi) - \eta_{\text{min}}(\xi))}{e^{\mu \eta_{\text{max}}(\xi)} - e^{\mu \eta_{\text{min}}(\xi)}} p_{\mu 0}(\xi; \theta)$$

Let us detail the above results in the case of an elliptic emitting domain, that domain being versatile enough when used in both mathematical phantoms and real situations (for example, it may be the ellipse that has the same second moments as the interesting region).

Referring to [47] for details, an elliptic domain in the $(\xi, \eta)$ plane, can be expressed as

$$a_{11} \xi^2 + a_{22} \eta^2 + 2a_{12} \xi \eta + 2a_{13} \xi + 2a_{23} \eta + a_{33} = 0$$

It is

$$\xi_{1/2}(\eta) = f_1 + f_2 \eta + \sqrt{f_3 \eta^2 + f_4 \eta + f_5}$$

with

$$f_1 = -\frac{a_{13}}{a_{11}}, \quad f_2 = -\frac{a_{12}}{a_{11}}, \quad f_3 = \frac{a_{12}^2 - a_{11} a_{22}}{a_{11}^2}, \quad f_4 = \frac{2(a_{12} a_{13} - a_{11} a_{23})}{a_{11}^2}, \quad f_5 = \frac{a_{23}^2 - a_{11} a_{33}}{a_{11}^2}.$$ 

So, for an elliptic domain, the arguments of the erf functions in (17) are

$$\xi_{1/2}(\eta) - \xi = \frac{\sqrt{2}}{\sqrt{2} \sqrt{\nu(P(\eta))}} \sqrt{-f_1 - f_2 \eta + \sqrt{f_3 \eta^2 + 2f_4 \eta + f_5}},$$

$$\xi_{1/2}(\eta) - \xi = \frac{\sqrt{2}}{\sqrt{2} \sigma_0} \sqrt{f_1 - f_2 \eta + \sqrt{f_3 \eta^2 + 2f_4 \eta + f_5}}/\sigma_0.$$  

(18)

The generic ray parallel to the $\eta$ axis intersects the ellipse in the two points given by

$$\eta_{1/2} = \frac{-(a_{23} + a_{12} \xi) \pm \sqrt{\Delta}}{a_{22}}$$

The attenuated Radon transform and the ideal one can then be written in closed form as

$$p_{\mu d}(\xi; \theta) = \rho \int_{\eta_{\text{max}}}^{\eta_{\text{min}}} e^{\mu \eta} d\eta = \frac{2\rho}{\mu} \exp \left[-\frac{\mu(a_{23} + a_{12} \xi)}{a_{22}}\right] \sinh \left(\frac{\mu \Delta}{a_{22}}\right)$$

From (19) and (20), it results:

$$p_{\mu d}(\xi; \theta) = \frac{\mu \sqrt{\Delta}}{a_{22}} \exp \left[-\frac{(\mu(a_{23} + a_{12} \xi))/a_{22}}{\mu \Delta/a_{22}}\right] p_{\mu 0}(\xi; \theta)$$

(21)

2.4. Multiple emitting domains

According to [3,4], once known the shape, $\Omega$, of each emitting domain from the high resolution CT image, the mathematical exponential projection, $p_{\mu \text{real}}(\xi; \theta)$, corresponding to that domain, can be determined for an unitary $\rho$ value by (12), (13), and either (15) in the general case or (19) in the case of elliptic emitting domain.

Owing to the assumed GC linearity, the denoised and scatter corrected measured exponential projection, $p_{\mu \text{meas}}(\xi; \theta)$, will be connected with the $p_{\mu \text{real}}(\xi; \theta)$ corresponding to the different emitting domain by

$$p_{\mu \text{real}}(\xi; \theta) \cong \rho_1 [p_{\mu \text{meas}}(\xi; \theta)]_{\Omega_1} + \cdots + \rho_{N_d} [p_{\mu \text{meas}}(\xi; \theta)]_{\Omega_{N_d}}$$

(22)

$N_d$ being the number of emitting domains.

The unknown constant density in each domain will then be determined, for example, by weighted least square procedure.

3. Results and discussion

The integrals in (14) cannot be solved in closed form, so the Matlab Symbolic Mathematics Toolbox (MSMT) is not able to compute them.

An approximation of the integrand functions by just one polynomial would have been useful to solve simply those integrals. Unluckily, the result of that approximation is hardly ever good even though reasonable high polynomial order is employed. In fact, though a least squares polynomial approximation for the error function resulting faster than the infinite series approximation reported in [46,p. 297, no. 7.1.5] is feasible, \footnote{The function erf(\(x\)) can be considered saturated at the level one for \(x \geq 5\). When least squares polynomials of order \(n = 20\) and \(n = 25\) are used to approx-} both the
Fig. 3. Left: direct, conjugate, geometric mean, and ideal projections as functions of $\xi$ corresponding to an UHRES collimator, angular sampling of $5^\circ$, and two emitting domains. The first domain is an elliptic one with centre at $P_0=(x_0, y_0)=(2.5, 0)$ cm, principal axes inclined at an angle $\varphi=45^\circ$ with respect to the positive $x$ axis, lengths of the major and minor axes equal to $(2a_e, 2b_e)=(10, 6)$ cm, respectively, and $\theta=0^\circ$. The second domain is a circular one with centre at $P_1=(x_1, y_1)=(4, 0)$ cm and unitary radius. Both domains have unitary activity and $\mu=0.15$. (--) pd: attenuated and space-variant psf blurred direct projection corresponding to $\theta=0^\circ$. (- - -) pcm: conjugate projection; (⃝⃝⃝) pc = pcm($\xi$); (+++) pg: the geometric mean projection ($pd \cdot pc$)$^{1/2}$; ( ⋯ ) pid: the reconstructed ideal projection corresponding to $\theta=0^\circ$. Right: the corresponding error function $|pg-pid|$ as function of $\xi$.

The kind and the variability of the functions characterizing the erf argument as well as the less or more wide saturating effect caused by the same erf function, get the integrand to become less polynomial-like in its behaviour and, as a consequence, the approximation error increases.

Future work will examine different approximation methods; to obtain immediate results, methods of numerical integration may be an alternative.

Matlab provides the three functions $\text{trapz}$, $\text{quad}$, and $\text{quadl}$ for numerically computing integrals, the last two functions requiring the integrand function must be furnished in closed form. The expressions (18) meet that requirement in case the emitting domain can be shaped as an elliptic or a circular one. If one will not succeed in determining an analytic expression for the domain shape, the $\text{trapz}$ function will be employed.

Fig. 3 shows the direct projection (pd), the conjugate projection (pc), the geometric mean projection (pg), the ideal projection reconstructed by the direct approach (pid), and the absolute difference (err) between the geometric mean projection and the ideal one corresponding to one view, an elliptic emitting domain, and a circular one the only circular domain being critical. At the latest, the pg projection succeeds in following the low frequencies in the ideal projection, but it is not good at following the high frequencies in the same projection.

Fig. 4 shows the variability of the err function corresponding to some views in the case of both emitting domains being critical as specified in the figure caption.

Fig. 5 details both the err function in Fig. 4 at the view $\theta=90^\circ$ and the corresponding pd, pc, pg, and pid projections.

Figs. 6 and 7 show the images corresponding to the two critical domains reconstructed by the pd, pg, and pid projections respectively. The simple $\text{iradon}$ function in Matlab has been used to that end; no artefact is present in the Fig. 7 reconstruction and the extent of the noise does not impair contrast. For comparison, the results achieved by the application of Eq. (22) shall follow since the determination of the interesting covariance matrix is closely connected with the determination of the noisy scatter free projections and the real knowledge of the point spread function.

The use of the actual geometric mean projection resulting less noisy than the arithmetic one at low counting, does...
then not simplify computing and is itself unable to quantify correctly the activity distribution specially in the case of multiple critical emitting domains at different depths in the object section.

The previous analysis points out the need of considering simultaneously the two physical phenomena, attenuation and space-variant psf blurring, when aiming at quantitation of radioactivity in small domain. In fact, even with patient specific attenuation correction, the SPECT relatively poor spatial resolution will however limit the accuracy of quantitative study, so signal loss for structures with dimensions smaller than about two to three times the FWHM of the image system is inevitable (partial volume error [48]). The direct approach is able to comply simply and efficiently with that need. On the contrary, the precision of object-dependent methods using physical models of the image process including the effects of attenuation

Fig. 5. Details of projections and error function corresponding to the view $\theta = 90^\circ$ in Fig. 4. For the rest, see Fig. 3.

Fig. 6. The images of the two critical domains reconstructed by the direct projections (Id) and the geometric mean projections (Igm), respectively.

Fig. 7. The image (Iid) of the two critical domains reconstructed by the ideal projections.
and non-ideal collimation [2] but employing an approximate iterative approach, is still object size and activity dependent, the reprojection and backprojection operations needed to their implementation being unfortunately prone to error and computing demanding, though recent hardware solution to speed up those operations has been proposed [49]. Moreover, even though the partial volume error would either be considered negligible with regard to some specific problem or corrected independently of attenuation (see, for example [29] in [48]) and algorithms like the Generalized Chang method or the Iterative Precorrection one [50] based on the simple filtered backprojection method are employed to correct for attenuation, an error image must be computed. That image is reconstructed from error projections obtained by subtracting the reprojected data from the original data. In addition to the just said drawbacks due to the iterative application of reprojection and backprojection operations, the subtraction of low counts is an inopportune matter [51] and the effects on signals resulting from both interpolation and filtering by fixed window needed on the two iterative algorithms are variable and not at all negligible [47] depending on the frequency and amplitude of the same signals.

Unlike analytical approaches for compensation of uniform attenuation like the Bellini method [52] or the equivalent [53] method of Inouye et al. [54], the direct approach does not impose the two limitations that the emitting domain must be convex and the centre of the rotating gamma camera must lie within that domain. Consequently, in the case of multiple emitting domain, once known the contours of the domains, the direct approach is able to determine the activity in each domain without the limitation occurring when one tries to compensate for non uniform attenuation by the Bellini method just like in [41]; in that case, the attenuation correction may properly be applied to the only domain satisfying the two above specified limitations provided that its activity is the greatest of all the activities in the remaining domains. Moreover, computing difficulties (double interpolation, selection of the number of terms in the series expansion, inevitable smoothing of the statistical noise components involved in the projection data) must be added to the just said structural limitation inherent in both the Bellini and the Inouye analytical approaches.

A non-object-dependent exact method in the frequency domain which, like the direct approach, aims at the determination of an explicit relationship between the ideal sinogram and the sinogram measured with parallel projections in 2D SPECT systems but degraded by uniform attenuation and distance-dependent spatial resolution of the system has been proposed in [37–40]. That method requires a linear approximation of the variance characterizing the point spread function Gaussian model in (9), imposes a structural low-limit in frequency, and greatly amplifies the data noise and the finite sampling effect due to the finite sampling employed in both the projection space and the image one. Since the quantitative accuracy as well as the noise characteristics of the reconstructed image have an unclear connection to those parameters, the design of an optimal filter to suppress the amplification of both the noise and the finite sampling effect will result a simply matter not at all; moreover, the same limitation and drawback pointed out with regard to the Bellini method go on. Compared to the previous method, the object-dependent direct approach is simpler, the deconvolution operation in (11) able to determine the attenuate Radon transform, $p_{\text{att}}$, involves a Gaussian function rather than a destabilizing increasing one, and a clear relation among errors are stated by (14) and related interesting relations.

The actual resolution of CT representation matrix [55] together with advanced methods of shape representation and the availability of opportunely numerous sample of $\eta$ values in the interval $I_\eta$ independently of the gamma camera collimator sampling, may well help to reduce the error of the $p_\mu$ factor in (14).

4. Summary

The a priori information about the anatomic contour of the interesting emitting domain with constant density and constant attenuation coefficient allows us to go simply back to the ideal projection starting from the measured space-variant blurred attenuated Radon transform. Specially, the detection of small masses will get better by working with ideal projection. The direct approach gets rid of the constraint concerning both the convex domain requirement and the psf oneness for all views.

The use of ideal projection in a linear algorithm is always an attractive matter for different reasons: (1) the ideal projection obtained by the direct method will practically be noisy; but, its error analysis as well as the determination of the covariance matrix of the reconstructed image, whatever its use may be, can be obtained in plug-in form without excessive difficulty and the restrictive assumptions pointed out in Section 1 with regard to the iterative reconstruction methods. Let us remember the covariance matrix, besides being needed to quantify the reliability of the reconstructed image and select a priori models, is also critical to the PWLS image reconstruction method, channelized Hotelling and human observer studies; (2) the linear algorithm allows us to have subpixel information which implies the correct knowledge of the resulting random field; if necessary, we shall then be able to postfilter or denoise the reconstructed image without using models borrowed from systems whose behaviour may be quite different from the actual one (see the Introduction with regard to the MAP penalizing solution); (3) at last, each projection may be processed independently of the different projections being acquired in the course of the measurement, so a parallel implementation is feasible; moreover, the speed of the whole system will benefit by resorting to hardware solutions whenever they will be realizable. The direct approach will be of assistance in implementing such a system.

Obviously, the region of interest (ROI) can be selected by the physician; but, automatic maps of the section are desirable. System identification and/or classical or fuzzy pattern recognition methods shall then be employed; the resulting system configuration will be the object of future work.

References


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