Abstract—Electronics has greatly contributed to the development of internal combustion engine. This progress has resulted in reducing environmental degradation, and yet continuing to support improvements in performance. Regarding gasoline engine, a considerable step forward has been achieved by Common Rail (CR) technology able to exactly regulate the injection pressure during whole engine speed range. As a consequence, the injection of a fixed amount of fuel is more precise and it is possible to perform multiple injections for combustion cycle.

In this paper, the authors present a mean value model aimed at the control of a CR system for a Gasoline Direct Injection (GDI) engine. The model is based on the descriptions of electro-valve, including the actuator circuit, and the fuel pressure in the rail. The performances of the proposed model are finally depicted through comparisons with experimental data collected by a CR system mounted on a 2.0 liters spark ignition engine, showing a good accuracy and reliability.

I. INTRODUCTION

Today, an huge effort is devoted by companies to preserve the earth’s environment. In particular, automobile industry has to comply both the reduction of pollutant emissions enforced by international regulations and the improvement of performance required by the customers.

In this scenario, the GDI engines with High Pressure (HP) fuel injection systems, based on the CR architecture, can be considered a good solution to this aim. In fact, electronically controlled HP fuel injection system holds an important role concerning both the emission control strategy and the improvement of internal combustion engine performance [1], [2]. High pressure injection allows to finely atomize the fuel spray and to promote fuel and air mixing, resulting in significant combustion improvements [3], [4]. Following the previous consideration about the pollutant emissions and consequent role of HP system to reduce it, the prediction of CR equipment behavior is of practical relevance in design of automotive engine.

In literature, extensive research is conducted for diesel engines to reduce both NOx and particulate (soot) emissions [5], [6], [7], or to improve the mass transfer characteristics to the catalytic surface through the analysis of a diesel oxidation catalyst [8]. In spark ignition engine, the CR system can be useful to reduce the exhaust emission and fuel consumption and to improve the driving dynamics [9].

For example, for diesel engines, the use of CR injection systems improves performance and customers perception. In this case the modern CR systems allow multiple injections with a limited dwell time between injections, but the use of multiple injections generates a series of pressure waves, due to the closure of the injectors, which make constant injection pressure a pure theoretical assumption.

In this perspective, the CR system represents a quenched oscillating system, in which the pressure waves generated by the first injection pulse produce a variation in the injection pressure of the subsequent injections [10]. Many CR injection models have been previously proposed, based on the equations of the physics underlying the process or alternatively developed through simulation packages. The form of these models is suitable for mechanical design but is often too complex to be appropriate for control purposes.

In [11] and [12] innovative approaches in automotive control application have been considered, respectively based on mean value engine models and the discrete-continuous interactions in the fuel injection system, due to the slow time-varying frequency of the HP pump cycles and the fast sampling frequency of sensing and actuation. Whereas, a rather complete mathematical model for a common-rail injection-system dynamics numerical simulation was developed in [13] to support experimentation, layout, and control design, as well as performance optimization.

In [14] and [15] an accurate model considers the fuel injection system of diesel engines with a distributor-type pump and properly represents the unsteady flow in the pipeline connecting the jerk pump and the injector. Then the resulting hyperbolic equations are solved by using an explicit scheme of the predictor corrector type.

The authors in [16] consider the same complex fluid-dynamic phenomena and study the instability in a CR injection system for high speed diesel engines. The good accuracy of all these models is counterpoised by the complexity of the resulting equations.

In this context, the paper presents a regulation oriented model of a common-rail system for GDI engines. The proposed model is obtained exploiting the descriptions of electro-valve dynamics and of steady fuel pressure in rail. Moreover, in this work, the analysis of CR system is completed by the study of pressure alternating component, present in the HP circuit and generated by the pump in absence of injections. The robustness of the model is tested comparing some measurable variables with experimental...
The paper is outlined as follows. The CR system is briefly described in section II. The electro-valve dynamic model is detailed in section III, taking into account the electric circuit devoted to the actuation of the valve. Then, in section IV, the model capturing the mean pressure in the CR is illustrated and a brief experimental analysis of the pressure alternating component discussed in section V. Finally, the experimental equipment is presented and some validation results are commented in section VI. Conclusions and future activities end the paper.

II. COMMON RAIL SYSTEM

The main objective of a CR system is to supply the electro-injectors with high pressure fuel independently by the quantity of fuel. The main benefit is, therefore, to decouple the regulation of the pump by the operation of the injectors, differently from traditional injection systems where the mechanic pump generates a pressure varying with the quantity of fuel to be injected.

The CR plant for spark ignition engine, shown in Fig. 1, is composed mainly by two separated sections: a low-pressure circuit, consisting of a fuel tank (1), a fuel feed pump (2) with a preliminary filter (3) and a low-pressure pipe; an high-pressure circuit formed by a mechanical pump (4), an high-pressure pipe, a common manifold (5) equipped with a pressure sensor (7), a flow stopper, a pressure regulator valve (8) and four injectors (6). The low pressure electro-pump (4 − 5 (bar)) forces the fuel from the tank toward the HP mechanical pump, crossing the filter aimed at cleaning the fuel. The mechanical pump compresses the fuel (100-120 (bar)) and sends it into the common manifold (named common-rail) equipped with the electro-injectors. The manifold is designed so as to filter the pressure oscillations in the fuel due to the pump and the intermittent working of the injectors. Finally, the fuel pressure in the manifold is regulated through the sensor and the electro-valve that flows the excess of fuel back into the tank [17].

![Common rail injection system description](image)

Fig. 1. Common rail injection system description; 1) fuel tank; 2) low pressure pump; 3) fuel filter; 4) high pressure pump; 5) common manifold; 6) electro-injector; 7) pressure sensor; 8) pressure regulation electro-valve.

The HP pump, shown in Fig. 2, is formed by three small pistons arranged in radial position (radial-jet) at an angular distance of 120 degrees. The pump is dragged by the engine through the camshaft and it does not require the phasing since the instant and time of injection are set by the control unit, which manages the opening of the injectors.

![High pressure pump scheme](image)

Fig. 2. High pressure pump scheme: 1) piston; 2) triangular cam; 3) shaft; 4) exit hole.

Therefore, in order to regulate the injection pressure, a mathematical model able to capture the behavior of the electro-valve and of the fuel pressure inside the CR in a wide range of engine working conditions is proposed and in the following presented.

III. PRESSURE REGULATION ELECTRO-VALVE

A. Electro-Valve Model

The valve devoted to the regulation of the pressure in the common manifold can be considered as a variable orifice without inertial effects, if the mechanical dynamics are neglected. Under this assumption, the pressure regulation electro-valve has been modeled through an electrical circuit governed by the following equation

\[
\frac{di}{dt} = -Ri + v_d,
\]

where also inductance variations and the counter-electromotive force have been neglected. The parameters \( L (\text{H}) \) and \( R (\Omega) \) indicate, respectively, inductance and the electric resistance of coil, and \( v_d(t) \) is the instantaneous voltage applied to its poles.

The values of \( R \) and \( L \) have been experimentally estimated (see Table I), forcing the valve with the battery voltage with 10 (Hz) of modulation. The comparison between dynamic model (1) and the experimental data is shown in Fig. 3, and it is possible to note how the model reproduces the electric current showing a good level of reliability and accuracy.

B. Actuation Circuit Model

Fig. 4 shows the simplified electronic circuit used to drive the electro-valve. Neglecting the voltage drop on the power transistor despite the battery \( V_b(t) \), the voltage to the coil terminals can be rewritten as follows

\[
v_d(t) = V_b(t)v_c(t),
\]

where

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>( R )</td>
<td>( \Omega )</td>
<td>5.41</td>
</tr>
<tr>
<td>( L )</td>
<td>mH</td>
<td>18.74</td>
</tr>
</tbody>
</table>

The alternating motion of the three small pistons is assured by a triangular cam connected to the pump’s shaft and each pumping group has an intake and exhaust valve. The combined action of the three pumping groups allows to reach a pressure between 50 and 100 (bar), maintaining low level of residual pressure into the external manifolds.
Fig. 3. The first plot depicts the comparison between the simulated (solid line) and experimental (points) electro-valve current response to a voltage input traced in the second plot.

where \( v_c(t) \in \{0, 1\} \) is the signal modulating in PWM the control voltage.

Fig. 4. A simple electric circuit aimed at actuating the electro-valve.

To this aim, the PWM frequency has been experimentally selected, \( f_m = 1500 \) (Hz), considered as the best compromise between reducing current ripple and a limited switching action from the components. This modulation frequency is approximately thirty times the electro-valve cutoff frequency (the electrical time constant is \( \tau = R/L = 3.46 \) (ms), for which \( f_3 = 1/(2\pi\tau) = 46 \) (Hz) and \( f_m/f_3 \approx 32 \), therefore ensuring a good attenuation of the harmonics introduced by the same modulation.

Coherently to what described, the voltage \( v_d \) can be approximated with its average value, \( V_m \); that is, integrating (2) on the modulation period \( T_m = 1/f_m \), it is obtained

\[
V_m = \left( \frac{a\delta + b}{100} \right) V_b,
\]

where \( \delta \) indicates the duty-cycle percentage of \( v_c(t) \).

The coefficients \( a \) and \( b \) take into account the not-ideal modulation (\( a = 1 \) and \( b = 0 \) in the ideal case); their values, experimentally estimated, are listed in Table II.

Note that, since \( \delta \) varies in the interval [0; 100], the voltage 3 could be negative or bigger than the battery voltage, that is clearly unfeasible. So, to avoid this, the relation 3 must be saturated in the interval [0; \( V_b \)].

Finally, replacing (3) into equation (1), the complete electro-valve model equipped with the electric circuit is obtained

\[
\frac{di}{dt} = -\frac{R}{L}[i - f_i(\delta(t), V_b(t))],
\]

where

\[
f_i(\delta, V_b) := \frac{V_b}{R} \left( \frac{a\delta + b}{100} \right)
\]

describes the current in stationary conditions as function of \( \delta \) (control variable) and \( V_b \) (not manipulable input). The battery voltage is usually measured in an ECU (Electronic Central Unit) and taking into account to adapt the maps of the main engine actuators (injectors, spark-plugs, throttle, valves, etc.).

It is important to note that equation (4) describes now the average current behavior in the modulation period \( T_m \). The instantaneous current, including the ripple due to the PWM, will be however governed by equation (1).

Fig. 5 shows the comparison between simulated and experimental current response of the electro-valve and actuation circuit in steady conditions. The good prediction of the steady model 5 is confirmed.

Fig. 5. Comparison of modeled \( f_i(\delta, V_b) \) (predicted) and measured (observed) steady current of the electro-valve and the actuator circuit.

IV. COMMON-RAIL PRESSURE MODEL

A detailed model of the fluid-dynamics phenomena and fuel propagation in the common-rail system is not the target of this work, aimed at designing a simple phenomenological model suited for control purposes. Therefore, a mean value pressure model has been obtained through an experimental approach.

To this aim, observing pressure data in Fig. 6, obtained varying the current and the shaft speed \( N \) of the HP pump \( (N := N_c/2 \) where \( N_c \) is engine speed), the following model for the average pressure \( \bar{p} \) in the manifold has been obtained

\[
\bar{p} := f_p(i, N) = c(N)i + d(N),
\]

whose parameters \( c \) (bar/A) and \( d \) (bar) have been firstly estimated for each \( N \) and then fitted with the following

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>-</td>
<td>1.0573</td>
</tr>
<tr>
<td>( b )</td>
<td>%</td>
<td>-5.6084</td>
</tr>
</tbody>
</table>
polynomial models (plotted in Fig. 7):

\[ c(N) = c_3 \left( \frac{N}{10^3} \right)^3 + c_2 \left( \frac{N}{10^3} \right)^2 + c_1 \left( \frac{N}{10^3} \right) + c_0 \] (7)

\[ d(N) = d_3 \left( \frac{N}{10^3} \right)^3 + d_2 \left( \frac{N}{10^3} \right)^2 + d_1 \left( \frac{N}{10^3} \right) + d_0 \] (8)

whose coefficients, estimated by means of a least squares algorithm, are reported in Table III. The degree of polynomials has been chosen maximizing the well known adjusted \(R^2\) statistic, which is generally the best measure of fit quality when additional coefficients are added to a model.

We point out that the accuracy of the pressure model (6) could be further improved introducing a quadratic dependency on the current, but we preferred not to increase the complexity of a model that is mainly developed for control purposes. A feedback regulator will have to compensate model errors mainly at higher pressure.

V. ANALYSIS OF PRESSURE ALTERNATING COMPONENT

The model (6) describes the mean value, over a cycle of a HP pump, of the total rail pressure \(p(t)\). During the control system design, it could be useful to have further quantitative information, even if approximated, on the alternating pressure \(p_ac(t) := p(t) - \bar{p}\) generated by the HP pump in absence of injections.

To this aim, we analyzed the amplitude of pressure oscillations at different mean pressure values \(\bar{p}\) and pump speeds \(N\). For each steady working condition we calculated the maximum and minimum percentage of overshoot and undershoot of pressure with respect to mean value, respectively \(\sigma_+ = 100(p_{\text{max}} - \bar{p})/\bar{p}\) and \(\sigma_- = 100(\bar{p} - p_{\text{min}})/\bar{p}\), where \(p_{\text{max}}\) and \(p_{\text{min}}\) are the maximum and minimum values of \(p(t)\) observed on a enough long time interval. The results of these analysis are shown in Fig. 10. In particular, the first and second plot depict respectively the experimental values of \(\sigma_+\) and \(\sigma_-\) got for all steady conditions. The trending curves of such distributions, averaged over \(N\), have been

<table>
<thead>
<tr>
<th>(c(N))</th>
<th>Value</th>
<th>(d(N))</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_0)</td>
<td>-97.9354</td>
<td>(d_0)</td>
<td>113.4366</td>
</tr>
<tr>
<td>(c_1)</td>
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<td>(c_3)</td>
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<td>(d_4)</td>
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</tbody>
</table>
calculated in terms of $\bar{p}$, $\sigma_+ = 85.275 \ e^{-0.0296 \ \bar{p}}$ and $\sigma_- = 54.272 \ e^{-0.0229 \ \bar{p}}$, and their traces are superimposed in the first two plots and compared in the third.

![Fig. 10. Results of pressure alternating component: (first plot) maximum percentage of overshoot $\sigma_+$; (second plot) minimum percentage of undershoot $\sigma_-$; (third plot) comparison between the trending curves $\sigma_+(\bar{p})$ and $\sigma_-(\bar{p})$.](image1)

Analyzing the trends of $\sigma_+(\bar{p})$ and $\sigma_-(\bar{p})$, one can state that the percentage ripple decreases for higher mean pressure, until reaching values lower than 10% for pressure close to 100 (bar). For values higher than 50 (bar), the two curves can be considered superimposed, that is the negative and positive peaks of the alternating pressure are the same modulus values. On the contrary, for lower pressure than 50 (bar), the overshoot values are always grater than undershoot ones, until reaching the maximum difference, $\approx 20\%$, at 10 (bar).

For sake of brevity, we report for a single case the harmonic analysis of the alternating pressure measured at 60 (bar) and a pump speed of 500 (rpm). In Fig. 11, we observe a spectrum where the first four harmonics can be considered enough to describe pressure ripple, with a main harmonic of 3 (bar) of amplitude (i.e. 5% of 60 (bar)). Note that, the motion of radial pistons in the HP pump is evidently reflected in the first three harmonics of the spectrum, whose fundamental frequency is in agreement with the theoretical one, i.e. $f_p \approx 8.3$ (Hz); in fact, the HP pump completes a whole round in a time period $T_P = 1/f_P = 60/N$ (s).

We point out that in presence of injections further oscillating components would appear in $p_{ac}(t)$ at higher frequency ($f_l = n f_P$, where $n$ is the number of injectors), as well as a continuous one (disturbance). This last determine a decreasing of the mean pressure $\bar{p}(t)$ that a feedback control action will have to compensate rapidly in order to reestablish the desired injection pressure.

### VI. EXPERIMENTAL RESULTS

The performances of the proposed model are highlighted comparing the model outputs, obtained through equations (4)–(8), with experimental data collected from a CR system (see Fig. 12) mounted on a 2.0L GDI engine.

The selected duty-cycle input signal, commanded during the experiment, is reported in the second plot of Fig. 13. Figs. 14–16 show the zooms of the rail pressure and duty-cycle signals during three pressure steps. In all cases, it is possible to appreciate the excellent matching of the simulated rail pressure with the measured signal. On the other hand, a presence of a constant error at 100 (bar) confirms the previous considerations on pressure model (6) at higher pressure.

### VII. CONCLUSIONS

The modeling of a CR fuel injection system of a GDI engine has been developed in this paper. Furthermore a qualitative analysis of the ripple pressure generated by the HP pump in steady conditions has been presented. The synthesized model predicts satisfactorily the CR pressure both in stationary and transient conditions showing a good accuracy and reliability in all test cases. The model can be effectively used for control purposes and for real time simulation applications (e.g. hardware/software-in-the-loop). In future work, the model will be in fact adopted to design an injection pressure regulator to be used by engine control tasks.

### REFERENCES


Fig. 13. Model validation: the first plot depicts the comparison between simulated (dotted line) and experimental (solid line) data of common rail pressure. The second plot depicts the duty-cycle input.

Fig. 14. Model validation: the first plot depicts the comparison between simulated (dotted line) and experimental (solid line) data of pressure step by \(10\) to \(50\) (bar). The second plot depicts the duty-cycle input.

Fig. 15. Model validation: the first plot depicts the comparison between simulated (dotted line) and experimental (solid line) data of pressure step by \(50\) to \(80\) (bar). The second plot depicts the duty-cycle input.

Fig. 16. Model validation: the first plot depicts the comparison between simulated (dotted line) and experimental (solid line) data of pressure drop by \(100\) to \(30\) (bar). The second plot depicts the duty-cycle input.


