Modeling and design of control algorithms over wireless networks

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Abstract—A major problem in control over wireless networks is conceiving a joint control and communication system design to optimize a given cost function. To solve this problem we have recently proposed [1] a theoretical framework based on the Platform Based Design (PBD). The first step consists in mapping control specifications onto communication network specifications: this involves parameters such as quantization noise, coding and modulation scheme, and power level. Next, a communication scheme is chosen so that the control specifications are satisfied. Hybrid models are exploited in order to capture the intrinsic discrete-continuous behaviors, and the developed framework is able to account for safety specifications that often occur in industrial control systems. While in our previous work we have been concerned with modeling efforts, in this paper we focus on the development of control methodologies and provide an example of application within a test case that is currently considered within the project HYCON.

I. INTRODUCTION

While the design and operations of wireless networks have been addressed during the last two decades to support communication applications, only recently there has been a growing interest in research on the interaction between control and wireless communication, e.g. [12], [21]. In particular, distributed control where the communication infrastructure is offered by wireless networks and the application of control methods for the energy efficient operation of wireless networks are two general topics of research interest, [6], [13].

In our work, we develop a hybrid dynamical model for resource allocation over wireless links, with particular attention to wireless sensor and actuator networks used in control applications. We consider a basic feedback digital control scheme, where the controller and the plant exchange signals by means of a time-varying wireless link. We address the problem of linear controller synthesis with safety specifications that arises in many application domains, e.g. automotive control, manufacturing systems, and air traffic management systems. The design problem involves setting control structure and parameters so that the specifications are satisfied. To deal with these issues, we use the principles of Platform Based Design (PBD) (e.g. [20]), thus breaking the original controller synthesis problem into two main steps: the design a controller for the plant to fulfill the specifications, without considering non—idealities of the wireless link and, basing on an assume-guarantee approach, the selection of communication parameters so that the synthesized controller still works in the presence of non—idealities. Under the mild assumption of asymptotic stabilizability of the plant, a solution to the safety problem is guaranteed to exist, provided that the communication system is capable of maintaining the “size” of the non—idealities bounded in a range given by the safety problem characteristics and by the dynamical properties of the plant. The bounds are obtained by mapping the control system specifications to specifications at the communication-system level. To maintain the operation of the communication scheme within these bounds, we propose an adaptive control approach for the communication channel, which, in correspondence to a given choice of the parameters in the constraints, checks if the underlying wireless link setup (in terms of capacity and bit error rate) can satisfy the requirements of the control problem. If this is not possible, the supervisor forces the channel to switch to a different setup. Since, for a given wireless link bandwidth, a limited set of combinations are allowed, the supervisory design problem can be formulated as a Hybrid System (HS) control problem where a HS model describes the operations of the feedback control scheme in correspondence of each discrete state (identified by the wireless link setup). The ultimate goal of this approach is to identify the characteristics of the overall feedback loop and to develop algorithms for concurrent design of the control laws and of the supervisor for the wireless channel.

While in [1] we have been concerned with modeling efforts for our system setup, in this paper we focus on the development of control methodologies for a specific example that involves safety specification. The example is targeted to development of components within a test case that is currently considered within the European project HYCON. Furthermore, we approach an extension of the model for the wireless channel. The paper is organized as follows. In Section 2 we set the control scheme and wireless communication scheme. Section 3 illustrates the methodology to solve a safety problem via a wireless communication scheme. In Section 4 we have applied our procedure to a case study of trajectory tracking using a remote wireless controller, using a hybrid model for characterizing protocol stack dynamic configuration. Finally, we provide some concluding remarks.

II. SAFETY PROBLEM OVER WIRELESS NETWORKS

Given a plant \( P \) and a set of good states within which the plant should evolve, the safety problem deals with finding the set of all initial states (maximal safe set [7]) for which there exists a controller such that the closed—loop system never leaves the set. This control problem has been studied in the past few years, e.g. [14], [22], [2], [4], [23], [7] and more recently it has been addressed in [8] where the focus is on the particular class of digital controllers. We consider problems with safety specifications with digital control and we suppose that the plant and the controller, that are mobile, share information via a time—varying wireless channel. The controller is thus required to adapt its configuration so that
the controlled plant satisfies the safety constraints in all 
possible channel states. Formally, let \( P \) be the plant and \( C \) the controller. We assume that \( P \) is a continuous–time linear dynamical control system:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) \\
x(t) &\in \mathbb{R}^n, u(t) \in \mathbb{R}^m, y(t) \in \mathbb{R}^p, t \geq 0.
\end{align*}
\]

We associate to \( P \) the corresponding classical exponential discretization \( P_{\Delta} \), with sampling time \( T_s > 0 \) (see e.g. [5]):

\[
\begin{align*}
x(t + 1) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) \\
x(t) &\in \mathbb{R}^n, u(t) \in \mathbb{R}^m, y(t) \in \mathbb{R}^p, t \in \mathbb{N},
\end{align*}
\]

The controller \( C \) is a digital controller. The control scheme is shown in Figure 1, where \( C \) and \( P \) share information via a wireless link. Information coming from the controller is first uniformly quantized and then sent via a wireless link. Information coming from the plant is sampled, quantized and sent via the wireless link to the controller. The parameters that characterize the communication system are: sampling time \( T_s \), quantization threshold \( M \), quantization width \( \delta \), modulation scheme \( \text{mod} \) (e.g. ASK, PSK, FSK), transmitted power level \( p \), distance \( r \) between \( P \) and \( C \), disturbance \( n \) due to the communication channel. The controller synthesis problem is focused on the design of a digital controller \( C \) and of an adaptive communication system such that the safety specifications on the controlled plant \( P \) are satisfied. Formally, the problem is stated as follows.

**Problem 1:** Given a plant \( P \) of the form (1) and a set of good states \( \Omega \subset \mathbb{R}^n \) within which the state \( x \) of the plant \( P \) should evolve, find:

- a digital controller \( C \),
- a communication system (as in Figure 1), and
- the set of all initial states in \( \Omega \)

such that the closed–loop system satisfies the safety requirements, i.e.

\[
x(t) \in \Omega, \forall t \geq 0.
\]

**III. SOLVING THE SAFETY PROBLEM OVER WIRELESS NETWORKS: A PLATFORM BASED DESIGN APPROACH**

In this section, we outline the main steps for solving the controller synthesis problem and resort on the principles of PBD [20]: we consider the synthesis of the controller \( C \) subject to bounded disturbances that capture the non–idealities of the communication channels, then we design the communication system so that the disturbance bounds are never violated. In order to restate the control problem in a classical control scheme, non–idealities coming from the communication system can be modeled as additive continuous disturbances \( d_1 \) and \( d_2 \), while \( \tau_1 \) and \( \tau_2 \) model delay components in data transmission in the two wireless links, as shown in Figure 2. Then, there exists a digital controller \( C \) that solves the safety problem if

\[
d_1 \in D_1, \quad d_2 \in D_2, \quad \tau_1 \in [0, \tau_{1,max}], \quad \tau_2 \in [0, \tau_{2,max}],
\]

where \( D_1 \subset \mathbb{R}^m \) and \( D_2 \subset \mathbb{R}^p \) compact sets with \( 0 \in D_1 \) and \( 0 \in D_2 \). Since \( D_1 \) and \( D_2 \) depend on the parameters chosen in the communication system, we will make it explicit the dependence of these sets on the communication system parameters: \( (T_s, M, \delta, \text{mod}, p, r, n) \). Actually we do not deal with detailed schemes for error detection and recovery mechanisms; we model instead errors in transmission mechanisms by means of the additive disturbance signals \( d_1 \) and \( d_2 \). In the following, we only carry out our analysis for the uplink, since the downlink may be treated similarly. Moreover, we neglect the delays \( \tau_1, \tau_2 \), since we assume that the transmission time required to deliver a sample is such that

\[
(\tau_1 < \tau_{1,max}) \land (\tau_2 < \tau_{2,max}).
\]

For controller synthesis problems with time delays the reader is referred to [15] and the references therein. The presence of continuous disturbances in the control scheme of Figure 2 has to be taken into account when designing the controller \( C \) to satisfy safety requirements. In this scenario, a controller solving the safety problem must be **robust** with respect to the disturbances, thus we refer to a **robust safety problem**.

Robust safety problems are in general a hard task to solve and a complete theory is still missing. Some preliminary results are available e.g. in [7], [17], and the references therein. Here we view this controller synthesis problem from a slightly different perspective: our approach is based on an **assume-guarantee** reasoning, frequently used in the formal verification of reactive and timed systems. First we synthesize a controller for the control scheme as in Figure 2, where we assume:

\[
D_1 = \{0\}, \quad D_2 = \{0\}, \quad \tau_{1,max} = 0, \quad \tau_{2,max} = 0.
\]

Then we verify that the obtained controller solves the robust safety problem with constraints (2). From now on, we suppose that a controller \( C \) has been synthesized using classical eigenvalues assignment for satisfying safety requirements on the plant system \( P \) in a safe set, under constraints (3). This controller in general is not a solution to the robust safety problem. Since sets \( D_1 \) and \( D_2 \) depend on the communication scheme parameters, the robust safety problem translates into finding suitable communication system parameters for which the robust safety problem can be solved.

We show how to map the set \( D_1 \) on the space \( W_{\text{uplink}} \) generated by the parameters \( (T_s, M, \delta, \text{mod}, p, r, n) \) of the communication scheme. We want to associate the set \( D_1 \) to the set \( \Phi(D_1) \subset W_{\text{uplink}} \), i.e. we want to define, for a given set of disturbances \( D_1 \) that may be taken by \( |d_1| \), the set of
communication parameters \(T_s, M, \delta, \text{mod}, p, r, n\) that guarantee \(|d_1| \in D_1\). For the sake of simplicity, we assume here that: \(T_s\) and \(M\) are fixed a priori, \(\delta\) and \(\text{mod}\) can have a finite set of values, \(D_1 = [0, D_{1,\text{max}}]\) is mono-dimensional. Then, we need to define a function \(\Phi_{T_s, M, \delta, \text{mod}}\) from \(D_1\) to the space generated by the triple \((p, r, n)\) (see Figure 3) for each combination of the parameters \(T_s, M, \delta, \text{mod}\). Note that the number of these combinations is finite. The function \(\Phi\) maps the set of allowed disturbance values \(d_1 \in D_1\) to a subset of \(W_{\text{uplink}}\) that includes allowed communication parameters. Note that in Figure 3 the range of \(\Phi(D_1)\) is a projection of \(\Phi(D_1)\) for fixed parameters \(T_s, M, \delta, \text{mod}\) on the 3-dimensional space with coordinates \((p, r, n)\). Assume now that we are using \(m - \text{PAM}\) modulation, for some \(m = 2^1, 2^2, \ldots, 2^{n_{\text{mod}}}\). The number of information bits contained in a symbol is given by \(\log_2 m\). Furthermore, for a given combination of \(T_s, M, \delta, n\), the number of bits to be sent in a sampling time interval is given by \(\log_2(2^m\delta)\). To guarantee a maximum fixed delay in data transmission, we need a transmission bit rate:

\[ f_b \geq \frac{\log_2(2^m\delta)}{T_s}, \]

Since we encode \(m\) bits for each symbol, the symbol rate is given by:

\[ f_{\text{sym}} \geq \frac{\log_2(2^m\delta)}{T_s \log_2 m}. \]

Clearly, the capacity of the channel \(C\) must be larger than \(f_{\text{sym}}\). We assume now that Grey coding is used to map a symbol to a sequence of bits: more precisely, given two symbols \(m_i\) and \(m_{i+1}\), \(i = 0, 1, ..., m-1\), they are associated to neighbor values of the associated sample. Assume now that the number of bits for sample is a multiple of the number of bits per symbol: in this case, the assumption of using a Grey Coding implies that, if we send a symbol \(m_i\) and receive the neighbor symbol \(m_{i+1}\) or \(m_{i-1}\), the error on the received value of the measured sample is bounded by:

\[ \alpha(M, \delta, m) \triangleq \delta \left( 2m^2 - \log_2(2^m) - 1 \right). \]

We have shown in [1] that \(\Phi(D_1)\) can be obtained as follows:

\[ \Phi_{T_s, M, \delta, \text{mod}}(D_1) = \left\{ (p, r, n) : |n(r(t), t)| \leq \frac{p(2z + 1)}{2m} \right\} \]

where \(z\) is the maximum allowed number of symbol errors, that depends on the disturbance bound \(D_{1,\text{max}}\) as follows:

\[ z \leq \frac{2D_{1,\text{max}} - \delta}{2\alpha(M, \delta, m)}. \]

This equation translates the specifications given at the controller–plant level to specifications at the communication system level.

IV. MINI-CAR TRAJECTORY TRACKING ON WIRELESS CHANNEL

We consider in this section a robust safety problem where the plant is a remotely controlled mini-car moving along a straight path. We approximate the dynamic of the car by means of a mass point moving on a 2–dimensional plan \((s_x, s_y)\) with constant velocity \(V\), as shown in Figure 4. The reference straight path coincides with the \(s_x\) axis. The direction of the speed vector is influenced by the steering angle \(\theta\) of the car. In this scenario the robust safety problem can be formulated as follows: we require that the deviation of the car from the desired path \(s_y\) is confined in a region close to the trajectory, and that the velocity of the car is constant. Namely, \(|s_y| < \Delta_{\text{max}}\) and \(s_x^2 + s_y^2 = V^2\). The relation between the derivative of the deviation and the steering angle is controlled by a DC motor. We will assume the following values for the physical parameters:

- moment of inertia of the rotor \(J = 0.01 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}\)
- damping ratio of the mechanical system \(\beta = 0.1 \text{ N} \cdot \text{m} / \text{s}\)
- electromotive force constant \(k_e = k_t = 0.01 \frac{\text{N} \cdot \text{m}}{\text{A}}\)
- electric resistance \(R = 1 \text{ \Omega}\)
- electric inductance \(L = 0.5 \text{ H}\)
- input \(v\): source Voltage measured in \(V\)
- output \(\theta\): position of shaft measured in \(\text{rad}\)

The rotor and shaft are assumed to be rigid. The motor torque \(F\) is related to the armature current \(i\) by a constant factor \(k_i\). The back electromagnetic force \(e\) is related to the rotational velocity as \(F = k_i i + e = k_e \theta\). In SI units, \(k_i\) (armature constant) is equal to \(k_e\) (motor constant). We can write the following equations based on Newton’s law combined with Kirchhoff’s law:

\[ J \ddot{\theta} + b \dot{\theta} = k_i \]

\[ L \frac{di}{dt} + Ri = v - k_e \theta \]

We can merge the point mass dynamics and the DC motor dynamics to express the state-space form of the overall model. The continuous state \(x = (s_y, \theta, \dot{\theta}, i)^T\) consists of the deviation from the desired trajectory \(s_y\), the rotational position \(\theta\), the rotational speed \(\dot{\theta}\), and the electric current \(i\). The input is given by the voltage \(v\). The output \(y = (s_y, \theta)^T\) is given by the deviation and the rotational position.
Furthermore, we consider an additional uncontrollable input $|w| \leq 0.01$, that models a disturbance on the steering dynamics due to the irregularity of the terrain. The complete state-space model is the following:

$$
\dot{x} = 
\begin{bmatrix}
0 & V & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -\frac{b}{T} & \frac{K}{T} \\
0 & 0 & -\frac{J}{T} & -\frac{L}{T}
\end{bmatrix}
x + 
\begin{bmatrix}
0 \\
0 \\
1 \\
-1
\end{bmatrix}
v + 
\begin{bmatrix}
0 \\
0 \\
1 \\
0
\end{bmatrix}
w \tag{5}
$$

As discussed above, we first synthesize a controller for a plant that is not subject to disturbance: thus, we design a controller for system (5) where $w = 0$. The discretization of system (5) with a sampling time $T_s$ is the classical exponential discretization:

$$
P : (A, B, C) \rightarrow P_{T_s}, P_{T_s} : (A_d, B_d, C_d)
$$

We synthesize a controller using a state–feedback eigenvalues allocation. The state vector is reconstructed from $y$ using a Luenberger’s observer. The separation principle ensures that, under observability and reachability assumptions, it is possible to independently synthesize the controller and the observer, and it is also possible to assign the eigenvalues. In this case the discrete–time system $P_{T_s}$ satisfies observability and reachability conditions. Applying the classical algorithm for eigenvalues assignment, we obtain the gain matrix $K_C = [20618 149.6426 0.3663 - 0.2648]$. Notice that for the duality property, the observer’s gain matrix of a system $P_{T_s} : (A_d, B_d, C_d)$ is equivalent to the state-feedback gain matrix of the dual system defined as follows:

$$
P_{T_s}^* : (A_d^*, B_d^*, C_d^*) = (A_d^{T_s}, C_d^{T_s}, B_d^{T_s})
$$

Thus, we can apply to $P_{T_s}^*$ the algorithm to compute the action control in case of multiple inputs, for solving the problem of computing the observer’s gain matrix $K_O$ of the system $P_{T_s}$, that instead has multiple outputs. We obtain the following observer’s gain matrix:

$$
K_O = 
\begin{bmatrix}
0.0574 & 0.0020 \\
-1908.9 & 2.9326 \\
-1572300 & 1670.8 \\
-56109000 & 40165
\end{bmatrix}
$$

The controller $C$ assumes the form:

$$
\dot{x}(k + 1) = (A_d - B_dK_C - K_OC_d)x(k) + K_Oy(k) \\
u(k) = -K_C\dot{x}(k)
$$

We will assume here w.l.o.g. that $D_1 = D \times D, D_2 = D, D \subseteq \mathbb{R}$. Since the measures on the plant and the control signals can have a different order of magnitude, it is interesting to analyze the maximum allowed disturbance sets $D_{1,\text{max}}, D_{2,\text{max}}$ separately for the uplink and the downlink channel: this can be done with the same procedure that we use in the following. It is possible to obtain by simulations of the feedback control scheme derived above a disturbance set $D = [−D_{\text{max}}, D_{\text{max}}], D_{\text{max}} = 0.01$, that guarantees confinement of the deviation $s_g$ in the set $\Omega = [−0.1, 0.1]$. It is possible to obtain the corresponding set of communication system parameters that guarantee safety, following the procedure proposed in the previous section. We will assume here fixed sampling time $T_s = 10^{-3}$s, quantization threshold $M = 4 \cdot 10^{4}$, and quantization width $\delta = 0.005$. The number of bits to encode each sample is given by $b_s = \lceil \log_2 \frac{2M}{\delta} \rceil$.

The modulation scheme is assumed to be $2^{b_s} - PAM$. It is clear that the symbol transmission time is given by $T_s$. Moreover, we assume that it is possible to tune on-the-fly the power level $p$ in a finite set of admissible values $\{p_1 = 5, p_2 = 10, p_3 = 20\}$. We will assume that the current power level can only be incremented or decremented. The idea of adapting the communication parameters is widely studied in the communication community literature (e.g. [10], [11], [18]) and applied in communication systems (e.g. in UMTS).

In this paper, we use the mathematical formalism of HSs to derive a rigorous formulation of an adaptive communication system, which is an advantage of this theoretical approach. As a first step, we formally introduce a HS as a tuple $\mathcal{H} = (Q \times X, Y, \varepsilon, \Sigma, E)$, where $Q = \{q_1, \ldots, q_N\}$ is the discrete finite state space; $X = \mathbb{R}^n$ is the continuous state space; $Y = \mathbb{R}$ is the continuous output space; $\varepsilon$ associates dynamics $\dot{x} = f_q(x), y = g_q(x)$ to each discrete state $q \in Q$, where $x \in X$ is the continuous state and $y \in Y$ is the continuous output variable; $\Sigma = \{\sigma_1, \ldots, \sigma_M\}$ is the finite set of discrete input symbols; $E \subseteq Q \times \Sigma \times Q$ is a collection of directed and labeled edges.

The discrete and continuous dynamics of $\mathcal{H}$ is determined by the continuous dynamics $\varepsilon_q$ (continuous layer), and by the discrete dynamics of the underlying finite state machine defined by $(Q, \Sigma, E)$ (discrete layer). For a formal definition of semantics of hybrid systems we refer to [14]. We now define a hybrid system $\mathcal{H}_{c}$ to model an adaptive control of a communication system on a time varying channel. The discrete state $q \in Q$ is a value $p$ that defines the current transmitted power. We define the set $Q$ as the set of all power levels in the transmission scheme, namely $Q = \{p_1, p_2, p_3\}$. We define $\Sigma = \{p_{\text{up}}, p_{\text{down}}\}$ as the finite set of configuration commands to the physical layer to set the power level: $p_{\text{up}}$ and $p_{\text{down}}$ respectively increase and decrease the current power level. The set of edges $E \subseteq Q \times \Sigma \times Q$ depicted in figure 5 defines the admissible switchings of the communication scheme, that can be triggered by the discrete control input $\sigma \in \Sigma$. For instance, if the physical layer could accept configuration commands to set the transmitted power $p$ by means of power level value instead of increment/decrement, the corresponding set of arcs $E$ would generate a fully connected graph.

The continuous state $n \in \mathbb{R}$ is given by the disturbance introduced by the wireless channel. In particular, we assume here an Additive White Gaussian Noise (AWGN) model and account for path loss variations induced by variations of the distance $r$ between the transmitter (the remote control) and the receiver (the mini-car). The dynamics of $r$ is given by the differential equation that rules the mini-car. Later, we provide insights on how to account for fading phenomena and multi-user interference that typically occur in modern models of wireless shared channels. Moreover, for the control algorithm
we consider here, the distance $r$ is monotonically increasing with time, and affects the quality of the received signal according to path loss laws. The observed continuous output $y \in Y$ is the Signal-to-Noise Ratio (SNR) (i.e. Channel state information) at the receiver input, that we assume to estimate at the receiver and then be available at the controller. The SNR depends on the current hybrid state (namely the current communication configuration, the communication channel status, and the distance). For the AWGN model assumed at this stage, the SNR can be expressed as:

$$\text{SNR}(r) = \frac{p_l}{p_n} = G(r) \cdot \frac{p_s}{p_n}$$

where $p_l$ and $p_n$ respectively denote the received and noise power at the receiver input, while $p_s$ indicates the transmitted power level $p$. In the last expression $G(r)$ denotes the distance dependent path gain $G(r) \propto 1/r^2$. The current discrete state of $\mathcal{H}_c$, i.e. the current communication system configuration, is determined by the discrete input $\sigma \in \Sigma$ generated by the controller of the communication system $C_c$. This controller receives as input the continuous output of $\mathcal{H}_c$, as shown in Figure 6. The controller $C_c$ can not generate a control at each time instant $t \in \mathbb{R}^+$: we assume here that a command $\sigma$ can be generated only after a time interval $T_p = 100\text{ms}$, namely at time instants $kT_p$, where $k \in \mathbb{N}$. Thus, $C_c$ controls the discrete dynamics of $\mathcal{H}_c$ generating output symbols according to a strategy modeled by the function $\gamma : Y \rightarrow \Sigma \cup \{\varepsilon\}$, where $\sigma(kT_p) = \gamma(y(kT_p))$, $k \in \mathbb{N}$. When the controller does not generate any symbol (no control command), $\gamma(kT_p)$ is equal to the empty string $\varepsilon$.

The safety problem we address on system (5) has a solution if for any disturbance introduced by the channel fading and by the motion of the plant, there exists a control strategy $\gamma$ such that the communication parameters belong at each time instant to the set $\Phi(D)$. To guarantee that the safety specification on the plant is satisfied, and following the approach described in the previous section, we can define the set of parameters of the communication channel that satisfy our safety specification:

$$|n(r,t)| \leq N = \frac{p(2z+1)}{2m}$$

Using equation (7), one can compute threshold values $SNR_{\text{min}}(q) < SNR_{\text{max}}(q)$ for each discrete state $q \in Q$, that define the adaptation strategy $\gamma$ as follows:

$$\gamma(kT_p) = \begin{cases} p_{\text{up}} & \text{if } SNR(kT_p) < SNR_{\text{min}}(q(kT_p)) \\ p_{\text{down}} & \text{if } SNR(kT_p) > SNR_{\text{max}}(q(kT_p)) \end{cases}$$

In our application, the hypothesis of constant horizontal velocity of the car implies that the only switching surfaces that we have to calculate are $SNR_{\text{min}}(q)$. In fact, as the distance increases, the SNR decreases and so it will be necessary to adopt a strategy that pulls up the SNR increasing the transmitted power level. Using equation (7) and the expression of the signal to noise ratio (6), we can compute the time invariant thresholds $SNR_{\text{min}}(q), q \in Q$ that ensure fulfillment of the safety requirement. Recall that we are considering an AWGN channel in which noise arises in an additive way and the degradation of the received signal is modeled through a coefficient deriving from the path gain, namely: $s'(t) = \sqrt{G(r)}s(t) + n(t)$ where $s'(t)$ and $s(t)$ are respectively the received and the transmitted signals. It is possible to bring back to the original additive formula of the noise: $s'(t) = s(t) + n(r,t)$ defining $n(r,t) = \sqrt{G(r)} - 1 \cdot s(t) + n(t)$. In order to compute the set of allowed values of $SNR$ such that the communication parameters belong to the set $\Phi(D)$, we express the constraint $|n(r,t)| \leq N$ as a constraint over the signal to noise ratio. In this way, we find a time independent threshold for the $SNR$ that depends on the value of current power level $p_s$:

$$SNR \geq SNR_{\text{min}}(p_s) = \left(1 + \frac{N}{2\sqrt{p_s}}\right)\frac{p_s}{N^2}$$

$SNR_{\text{min}}(p_s)$ defines three switching surfaces for a control strategy $\gamma(kT_p)$, to avoid that the $SNR$ value falls down the threshold causing a performance lack of the control loop.

We consider now the constraint that the control strategy $\gamma(kT_p)$ can be applied at time instants $kT_p, T_p > T_s$. After a time $T_p$, the quality of the received signal can decay within a maximum bound that depends by the velocity along the desired trajectory direction. Moreover, since the channel fading is supposed to be slow w.r.t. the decay due to the motion of the car, it can be neglected in the choice of the thresholds. The hypothesis of constant horizontal velocity $V$ of the car allows to relate the distance $r(t)$ to the time $t$ through a proportionality coefficient $r(t) = Vt$. Thus, the dependence of the distance becomes a dependence on time. To calculate the switching surface associated to the generic transmitted power level $p_s$ and distance $r$, we define $SNR'(p_s, k)$ as the maximum decrement of the $SNR(p_s)$ in the time interval $[kT_p, (k+1)T_p]$ with transmitted power level $p_s$:

$$SNR'(p_s, k) = SNR(kT_p) - SNR((k+1)T_p)$$

The overall threshold $SNR_{\text{min}}^*(q, k)$ is given by the sum of a time independent contribution (8) and a time dependent contribution (9):

$$SNR_{\text{min}}^*(p_s, k) = SNR_{\text{min}}(p_s) + SNR'(p_s, k)$$

If the measured $SNR$ is under the overall threshold, then it is necessary to switch to an higher power level. In this way, we pass to a switching surface that guarantees the fulfillment of the safety constraints. The adaption strategy $\gamma$ is defined as follows:

$$\gamma(kT_p) = p_{\text{up}}, \text{ if } SNR(kT_p) < SNR_{\text{min}}^*(p_s, r)$$

The value of the overall threshold depends by the current state $q(kT_p)$ since $q(kT_p) = p_s(kT_p)$.

We performed simulations using MATLAB, with the communication parameters defined above. Figure 7 shows that the safety constraint is largely satisfied. Actually, the safe set results to be contained in the set $[-0.08, 0.08]$; this is due to the fact that, when translating the allowed disturbance set $D$ into a safe set $\Phi(D)$ of communication parameter,
we used conservative approximations. Figure 8 shows the adaptation of the communication power, namely the discrete state execution of the system $\mathcal{H}_c$. The control strategy $\gamma$ that solves the safety problem is not unique: it is thus possible to introduce a cost function to search for an optimal control strategy.

V. CONCLUSIONS AND FUTURE WORKS

In this paper, we proposed a methodology and a modeling paradigm for the robust design of control over wireless networks. To do so, we leveraged the Platform Based Design (PBD) methodology [20] and hybrid systems. We applied our procedure to a case study of trajectory tracking using a remote wireless controller. Future work is concerned with progresses along two major directions. The first one deals with further formal enhancements of controller design in the current setup. The second research line is intended to explore the formalism of stochastic hybrid systems in qualifying safety specifications. Along these lines we are also working towards extensions of the wireless channel setup to the multi-user case in fading phenomena by taking into account the presence of multiple plants that share the same communication channel for data exchange. While considering multiple plants, we could imagine two possible scenarios depending on the particular application: the first one arises if we assume all plants communicating with one controller, the other one is related to the case of multiple pairs plant-controller. In both cases it will be important to consider effects of the interference which is caused by frequency reuse while considering narrow-band systems and by the correlation between the pseudo noise sequences that are used in wide-band systems. For both scenarios we plan to use results obtained in [18], [19].

REFERENCES