Performance Analysis of a Multiband OFDM UWB System in the Presence of Narrowband Interference

Francisco C. B. F. Müller and Aldebaro Klautau
Signal Processing Laboratory - www.laps.ufpa.br
Universidade Federal do Para
Belem, PA, Brazil
Email: {fmuller, aldebaro}@ufpa.br

Claudio R. C. M. da Silva
Wireless@VT
Virginia Tech
Blacksburg, VA, USA
Email: cdasilva@vt.edu

Abstract—This paper presents a performance analysis of an ultra wideband (UWB) communication system based on Multiband Orthogonal Frequency Division Multiplexing (MB-OFDM) in the presence of narrowband interference. Although OFDM-based systems are known to be capable of withstanding some level of narrowband interference, mitigation techniques may have to be used when interference levels are high. Furthermore, as typical UWB systems employ low resolution analog-to-digital converters, the effect of narrowband interference may be exacerbated in the process of analog-to-digital conversion. In order to improve the performance of MB-OFDM UWB systems in the presence of narrowband interference, we consider the use of analog notch filters. This paper proposes an analytical model and formulation that allows for the evaluation of the probability of symbol error of an MB-OFDM UWB system when an analog notch filter is used for narrowband interference mitigation. It is shown that using an analog notch filter can significantly improve the performance of MB-OFDM UWB systems in high-interference scenarios.

I. INTRODUCTION

As defined by the FCC [1], ultra-wideband (UWB) systems will operate in the 3.1 to 10.6 GHz band and coexist with different narrowband systems, such as WiMAX [2] and PCI cards [3]. Therefore, UWB systems must be able to withstand potential narrowband interference (NBI). Although OFDM-based systems are known to be capable of withstanding some level of NBI, mitigation techniques may have to be used when interference levels are high. Furthermore, as typical UWB systems employ low resolution analog-to-digital converters (ADCs), due to the wide bandwidth of their signals [4], the effect of NBI may be exacerbated in the process of analog-to-digital conversion.

In this paper, we propose an analytical model and formulation that allows for the evaluation of the probability of symbol error of an UWB system when an analog notch filter is used for NBI mitigation. In particular, we consider an UWB system based on the Multiband OFDM technology [5], [6]. The analysis derived here takes into account both intercarrier interference (ICI) and possible intersymbol interference (ISI) resulting from notch filtering. The proposed formulation is valid for any notch filter.

The use of both analog and digital notch filters was previously considered in [3], [7], [8], among others, in order to minimize the effects of NBI on OFDM-based systems. However, it was shown in [8] that digital mitigation techniques are not effective when the signal-to-interference ratio (SIR) is less than 0 dB. For this reason, the focus of this paper is on analog notch filters. Other NBI rejection techniques for UWB systems can be found in [9]-[13], among many others.

The remaining parts of this paper are organized as follows. In Section II, we define the system model and present a formulation for the signal after analog notch filtering. A novel performance analysis of an MB-OFDM system that employs an analog notch filter for NBI mitigation is proposed in Section III. Performance results are presented in Section IV, and Section V contains conclusions obtained from this study.

II. MB-OFDM SYSTEM DEFINITION

In this analysis, we consider an UWB system based on the MB-OFDM standard [6]. One of the main advantages of MB-OFDM systems over impulse radio implementations, for example, is the lower complexity of the receiver front-end [5]. Two modulation types are defined in the MB-OFDM standard, namely Quadrature Phase Shift Keying (QPSK) and Dual-Carrier Modulation (DCM). In this paper, due to space constraints, we only consider the QPSK mode. However, the qualitative results obtained in this paper with respect to the use of an analog notch filter by UWB receivers for NBI mitigation are believed to be valid for both modulation modes.

Additionally, since DCM modulation provides both diversity gain and coding gain over the QPSK modulation [14], the symbol error rates obtained in this paper upper bound those of a DCM system.

Block diagrams of the MB-OFDM transmitter and receiver considered in this paper are shown in Fig. 1. The waveform corresponding to the $i^{th}$ transmitted OFDM symbol is written as

$$s_i(t) = \frac{1}{\sqrt{T}} \sum_{n=0}^{N-1} c_{n,i} p(t - iT_S)e^{j2\pi \frac{S}{T}(t - iT_S)}, \quad (1)$$

where $c_{n,i}$ is the complex symbol transmitted in the $n^{th}$ subcarrier of the $i^{th}$ OFDM symbol and $p(t)$ is a rectangular window function defined as

$$p(t) = \begin{cases} 1, & 0 \leq t \leq T \\ 0, & otherwise \end{cases}. \quad (2)$$

Also in (1), $N$ is the number of subcarriers and $T_S = T + T_{ZPS}$ is the total period of each OFDM symbol, where $T$ is the OFDM symbol duration without zero padding and $T_{ZPS}$ is the zero-padding suffix duration.
The received signal $r'(t)$ in Fig. 1, using a conventional tapped delay line model of a frequency selective multipath channel, is given by

$$r'(t) = \sum_{i'=-\infty}^{\infty} y_{i'}(t) + v(t) + i(t),$$

where $y_{i'}(t)$ is given by

$$y_{i'}(t) = \frac{1}{\sqrt{T}} \sum_{n=0}^{N-1} c_{n,i'} \sum_{l=0}^{L-1} \alpha_l p(t-i'T - T_l) e^{2\pi i (t-i'T - T_l)},$$

where $L$ is the number of resolvable paths, $\alpha_l$ is the amplitude of the $l$th path, and $T_l$ is the delay of the $l$th tap. In (3), $v(t)$ is a zero-mean WSS complex Gaussian process with two-sided spectral density $N_0$, and $i(t)$ is the NBI, modeled as a zero-mean WSS complex Gaussian process with power spectral density (PSD) $S_I(f)$:

$$S_I(f) = \begin{cases} N_I, & \text{if } |f - f_l| \leq W_I/2 \\ 0, & \text{otherwise}. \end{cases}$$

(Note that it was observed in [2] that the interference of OFDM-based WiMAX systems on MB-OFDM receivers can be accurately approximated by a zero-mean Gaussian process.)

As shown in Fig. 1, the received signal $r'(t)$ is passed through an analog notch filter with impulse response $h_{af}(t)$. The output of this filter, denoted by $r(t)$, is written as

$$r(t) = \sum_{i'=-\infty}^{\infty} x_{i'}(t) + v(t) * h_{af}(t) + i(t) * h_{af}(t),$$

where $x_{i'}(t) = y_{i'}(t) * h_{af}(t)$ is given by

$$x_{i'}(t) = \frac{1}{\sqrt{T}} \sum_{n=0}^{N-1} c_{n,i'} \sum_{l=0}^{L-1} \alpha_l \int_{T_l}^{T_l + T} h_{af}(t-\lambda) e^{2\pi i (\lambda-i'T - T_l)} d\lambda \times \int_{T_l}^{T_l + T} h_{af}(t-\lambda-\lambda) e^{2\pi i (\lambda-i'T - T_l)} d\lambda.$$

The output $z_{m,i}$, corresponding to the detection of the interference of the $m$th subcarrier of the $i$th OFDM symbol, is given by

$$z_{m,i} = \frac{1}{\sqrt{T}} \int_{i'T_s}^{i'T_s + T} r(t) e^{-j2\pi \frac{i}{T_s} (t-i'T_s)} dt.$$
the $1 \times N$ input vector $c_i$ is defined as
\[
c_i = [c_{0,i}, \ldots, c_{N-1,i}],
\] and $\alpha$ is the $L \times 1$ vector $\alpha = [\alpha_0, \alpha_1, \ldots, \alpha_{L-1}]^T$.

The evaluation of (9), (10), and (12) can be simplified through Fourier transform properties. The procedure was previously used in the performance analysis of UWB systems in [11]. Solving the integral in $t$ first, (12) can be rewritten as
\[
D_{m,n,i,i',l} = \frac{1}{T} e^{-j2\pi(f-i'T_{S}+T)}
\]
\[
\times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(\lambda-i'T_{S}-T) e^{-j2\pi(f-i')\lambda} \times h_{\text{af}}(f) \times P(m/T-f) e^{j2\pi f i T_{S} df} d\lambda,
\]
where $P(f)$ and $h_{\text{af}}(f)$ are the Fourier transforms of $p(t)$ and $h_{\text{af}}(t)$, respectively. By inverting the order of integration and solving the integral in $\lambda$, we obtain
\[
D_{m,n,i,i',l} = \frac{1}{T} \int_{-\infty}^{\infty} h_{\text{af}}(f) P^{*}(f-m/T) P(f-n/T) \times e^{-j2\pi(f-i'T_{S}+T)} df.
\]

III. PERFORMANCE ANALYSIS

In this section, we derive the probability of symbol error for an uncoded QPSK-based MB-OFDM UWB system. The analysis presented here does not consider the frequency hopping (FH) mechanism of the MB-OFDM standard. However, our formulation can be used to estimate the error rate of an uncoded FH system by appropriately averaging the probability of symbol error corresponding to each band used in the hopping process.

Our formulation is based on a well-known approach proposed by Craig in [16] for the evaluation of the probability of error for two-dimensional signal constellations. For this analysis, let us define the vector $c$ to be the sequence of transmitted OFDM symbols $c = \{c_i\}$, where $c_i$ is given by (15). Assuming that the phase shift of the data component in (13) can be perfectly estimated, the probability of symbol error conditioned on $\alpha$ and $c$ is given in [16] as
\[
P[E|\alpha, c] = \frac{1}{\pi} \int_{0}^{3\pi/4} \exp \left(-\gamma \sin^2\frac{\phi}{2}\sin^2\phi\right) d\phi,
\]
where $\gamma$ is the instantaneous SNR defined as
\[
\gamma = \frac{X^2}{\sigma^2}.
\]
In (19), $\sigma^2$ is the variance of the interference plus noise component, (13). This variance is calculated in the Appendix.

Also in (19), $X$ is the absolute value of the data component in (13),
\[
X = \left| \sum_{i'=m}^{\infty} c_{i'} D_{m,i,i'} \alpha \right|.
\]

It is worth noting that (18) is valid because the thermal noise and NBI are modeled as Gaussian processes. Therefore, conditioned on $c$ and $\alpha$, $z_{m,i}$ is Gaussian. Also, note that the ICI and ISI components resulting from notch filtering are implicitly present in (20).

A. Rayleigh Channel Taps

By using the Total Probability Theorem, the probability of symbol error conditioned on $c$, $P[E|c]$, is obtained by averaging (18) over $p(\gamma|c)$, the conditional density function of $\gamma$,
\[
P[E|c] = \int_{0}^{\infty} P[E|\alpha, c] p(\gamma|c) d\gamma
= \frac{1}{\pi} \int_{0}^{3\pi/4} \left[ \int_{0}^{\infty} \exp \left(-\gamma \sin^2\frac{\phi}{2}\sin^2\phi\right) p(\gamma|c) d\gamma \right] d\phi.
\]
The integral inside the brackets in (21) is equal to the moment generating function (MGF) of $\gamma$ conditioned on $c$, $M_{\gamma}(s) = \int_{0}^{\infty} p(\gamma|c) e^{ss} d\gamma$, evaluated in $s = -\sin^2\frac{\phi}{2}/\sin^2\phi$.

By assuming that the channel coefficients $c_i$ in (4) are zero-mean complex Gaussian random variables, we have that, conditioned on $c$, $X$ in (20) is Rayleigh distributed and $\gamma$ in (19) is exponentially distributed [17]. Therefore, the conditional MGF of $\gamma$ is $M_{\gamma}(s) = (1 - s\gamma)^{-1}$ [17], where $\gamma = E[X^2]/\sigma^2$ is the average SNR. By substituting the MGF expression in (21), and after some algebraic manipulation, it is shown in [17] that $P[E|c]$ can be solved in closed form as
\[
P[E|c] = \frac{3}{4} \left\{ 1 - \sqrt{\frac{\gamma \sin^2\frac{\phi}{2}}{1 + \gamma \sin^2\frac{\phi}{2}} \frac{4}{3\pi}} \right\}
	imes \left[ \frac{\pi}{2} + \arctan \left( \frac{\gamma \sin^2\frac{\phi}{2}}{1 + \gamma \sin^2\frac{\phi}{2}} \right) \right].
\]

B. Quantization Noise

As previously mentioned, the impact of NBI to an UWB receiver may be exacerbated in the process of analog-to-digital conversion. This is because UWB receivers typically employ ADCs with low resolution, due to the very wide bandwidth of UWB signals. In order to obtain a first order estimate of the performance loss of an MB-OFDM system due to the combined effect of NBI and quantization errors, we use the approach presented in [8, Section III]. In this approach, the instantaneous SNR of the received signal is given by
\[
\gamma_q = \frac{X^2}{(X^2 + \sigma^2)/\gamma_q + \sigma^2},
\]
where $\gamma_q$ is the signal-to-quantization-noise ratio. The reader is referred to [8] for details of this model. Numerical results are obtained in Section IV for three different values of $\gamma_q$, namely 19.4, 24.6, and 29.8 dB, which correspond to ADCs with 4, 5, and 6 bits, respectively.
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.
where $$n$$ is the variance of the thermal noise. The variance $$V$$ is given by

$$V = \frac{1}{n} \int_{T}^{T_s+T} n(t) e^{-j2\pi f(t-T_s)} dt$$

Then, (9) can be rewritten as

$$\sigma_{V}^2 = E\left[\frac{1}{T} \int_{T}^{T_s+T} n(t) e^{-j2\pi f(t-T_s)} dt\right]$$

which is the PSD of the thermal noise. The variance $$\sigma_{V}^2$$ is shown to be equal to (25) with $$S_V(f)$$ replaced with $$S_I(f)$$, the PSD of the NBI process.