ABSTRACT
Automatic segmentation of the left ventricle (LV) from cardiac images remains an open problem. While current methods are already sufficient to outline endocardial (ENDO) surface automatically, these methods are problematic for finding reliable epicardial (EPI) surfaces. It is mainly due to the low myocardium/background contrast. In this paper, we propose a new algorithm that is motivated by the approximate incompressibility of myocardium during a cardiac cycle and takes it as an important constraint. We design in a probabilistic framework a deformable model that evolves according to the regional intensity distribution while maintaining the volume of myocardium. Experiments on 225 sets of volumetric cardiac MR images validate the accuracy and robustness of this method.

Index: Cardiac Image Segmentation, Incompressibility Constraint, Coupled Segmentation, Deformable Model, Maximum A Posterior

1. INTRODUCTION
The automatic segmentation of LV is a prerequisite for quantitative LV analysis, such as the measurement of myocardial dynamics and wall thickness, that can provide important diagnostic information for heart disease. While existing algorithms can extract ENDO surfaces from magnetic resonance (MR) images with sufficient accuracy, our experience found it still open to find EPI boundaries that are equally important in the assessment of the myocardial strain and strain rate with a biomechanical model.

Compared with ENDO surface segmentation, the segmentation of EPI surfaces is much more challenging. The location of the EPI surface is more ambiguous than that of the ENDO surface due to the low intensity contrast between myocardium and background (See Fig. 1). This low myocardium/background contrast, combined with various artifacts, such as partial volume effects, makes existing methods unreliable.

Several attempts have been made to the segmentation of EPI surfaces from MRI images. Mitchell used 3D Active Appearance Model (AAM) and Active Appearance Motion Model (AAMM) whose behavior is learned from a set of manually traced segmentation samples, to outline the ENDO- and EPI boundary from cardiac images [1]. Huang proposed a novel cubic B-spline based Free Form Deformable (FFD) model that encodes shape and appearance information from a large set of training examples, and applied it to segmenting MRI tagging images [2]. Montillo put forward a cardiac MRI tagging image segmentation framework that evolves a deformable model according to Gradient Vector Flow (GVF) of an edgemap of MRI tagging image [3].

Fig. 1. The EPI boundary is not well shown while the ENDO boundary is more clear.

Fig. 2. The short-axis view of the heart. Two dotted contours are ENDO- and EPI clear.

All of the above efforts, however, do not explicitly use the constraints due to myocardial structure, and therefore are limited for the purpose of detecting EPI surfaces. Our method described in this paper is motivated by the nearly constant volume of myocardium during the entire cardiac cycle and takes this property as an important constraint. By evolving ENDO- and EPI surfaces simultaneously, each is driven by its own image derived region force while maintaining the coupling, a final representation of the ENDO- and EPI surface is thus achieved.

2. METHODOLOGY
2.1. Incompressibility of Myocardium
The heart is a remarkably efficient and durable mechanical pump composed of complex biological material. In cardiac
mechanics, various mathematical models have been proposed to study the normal and pathophysiologic responses that arises as various disease processes affect the heart(see [4]) for an overview). In [5], we analyzed the deformation of heart by applying a biomechanical model with a Poisson ratio close to 0.5, which implies that the volume of myocardium is nearly incompressible during a cardiac cycle. A few studies haven been carried out to study the change of myocardium volume during a cardiac cycle [6, 7]. The common conclusion is that the change is less than 5%. This fact is used as an important cardiac structure constraint that is taken into account in our method, as will be detailed in section 2.4.

2.2. MAP Framework

Let \( I \) be a 3D cardiac image, a Maximum A Posteriori (MAP) framework that realizes coupled segmentation of ENDO and EPI surfaces with incompressibility constraint can be expressed as

\[
\left( \hat{S}_{\text{in}}, \hat{S}_{\text{out}} \right) = \arg \max_{S_{\text{in}}, S_{\text{out}}} P \left( S_{\text{in}}, S_{\text{out}} | I \right)
\]

where \( S_{\text{in}} \) is ENDO surface, and \( S_{\text{out}} \) is EPI surface. Equation 1 can be interpreted as the probability function that adheres to image data, modulated by the prior knowledge of nearly incompressibility property of myocardium. Since the myocardium is only nearly incompressible, it is more reasonable to define it within a probabilistic framework rather than applying a deterministic constraint.

2.3. Data Adherence Term

Region-based deformable models, due to their robustness, have been successfully applied to the segmentation of images with weak boundaries [8]. Here, in our approach, we evolve a three-phase region-based deformable model based on the statistical intensity distribution from cardiac MR images.

To evolve a region-based model, we first need to determine the intensity distribution of each region within the image. It is obvious that the entire image is partitioned by \( S_{\text{in}} \) and \( S_{\text{out}} \) into three regions: LV blood pool, myocardium, and background. The LV blood pool and myocardium are homogeneous, and therefore can be modeled with a single probability density function (pdf). The most common pdf for MR images is Gaussian (Normal) distribution which is expressed as follows

\[
P \left( I; \mu, \sigma \right) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ -\frac{(I - \mu)^2}{2\sigma^2} \right\}
\]

where \( \mu \) is the mean of Gaussian distribution, and \( \sigma \) is its deviation. Equation 2 describes the intensity distribution for LV when \( i = 1 \), and the intensity distribution for myocardium when \( i = 2 \).

The background, however, is inhomogeneous (see Fig 2) because it contains more than one tissues (RV blood pool, RV myocardium, and lung air), and therefore modeling it with a single distribution function would be insufficient because it contains a wide range of intensities. To handle this problem, we use a mixture model and invoke EM algorithm to fit the background histogram.

Under the mixture model, the background intensity distribution is given as

\[
\mathcal{P}_{\beta} \left( I; \mu_{\beta}, \sigma_{\beta} \right) = \sum_{k=1}^{M} \alpha_k \mathcal{P}_{3,k} \left( I; \mu_{3,k}, \sigma_{3,k} \right)
\]

where \( M \) is the number of components, \( \alpha_k \) is the mixture proportion of component \( k \) that satisfies \( \sum_{k=1}^{M} \alpha_k = 1 \), \( \mu_{3,k} \) and \( \sigma_{3,k} \) are the mean and deviation of its component distribution.

To estimate the parameters of Gaussian mixture model, we adopt EM algorithm which iterates the expectation step (E-step) and the maximization step (M-step) until convergence. Given an initial estimate \( \alpha^{(0)} \), \( \mu^{(0)} \), and \( \sigma^{(0)} \), EM algorithm can be described as

E-Step

\[
P^{(i)} \left( k | n \right) = \frac{\alpha_{3,k}^{(i)} P_{3,k} \left( I_n; \mu_{3,k}^{(i)}, \sigma_{3,k}^{(i)} \right)}{\sum_{m=1}^{M} \alpha_{3,m}^{(i)} P_{3,m} \left( I_n; \mu_{3,m}^{(i)}, \sigma_{3,m}^{(i)} \right)}
\]

M-Step

\[
\begin{align*}
\mu_{3,k}^{(i+1)} &= \frac{\sum_{n=1}^{N} P^{(i)} \left( k | n \right) I_n}{\sum_{n=1}^{N} P^{(i)} \left( k | n \right)} \\
\sigma_{3,k}^{(i+1)} &= \sqrt{\frac{\sum_{n=1}^{N} P^{(i)} \left( k | n \right) \left\| I_n - \mu_{3,k}^{(i+1)} \right\|^2}{\sum_{n=1}^{N} P^{(i)} \left( k | n \right)}} \\
\alpha_{3,k}^{(i+1)} &= \frac{1}{N} \sum_{n=1}^{N} P^{(i)} \left( k | n \right)
\end{align*}
\]

We show in Fig. 3 the histogram of each region with fitted distribution function.

Let \( \Omega \) be a bounded open subset of \( \mathbb{R}^3 \), and be partitioned by \( S_{\text{in}} \) and \( S_{\text{in}} \) into three regions, i.e. LV blood pool, LV myocardium, and background, which are denoted respectively as \( \Omega_1 \), \( \Omega_2 \), and \( \Omega_3 \). Thus, the data adherence term can be defined by a three-phase deformable model.
The maximization of equation 4 can be interpreted as the propagation of $S_{in}$ and $S_{out}$ that maximizes the piecewise homogeneities.

### 2.4. Incompressibility Constraint

It is mentioned in section 2.1 that the volume of myocardium changes less than 5% during a cardiac cycle. Therefore, we make an assumption that the volume of myocardium has a Gaussian distribution $N(V_0, \sigma_V)$

\[
\mathcal{P}(S_{in}, S_{out}) = \frac{1}{\sqrt{2\pi}\sigma_V} \exp\left\{-\frac{(V - V_0)^2}{2\sigma_V^2}\right\}
\]  

(5)

where $V = \int_{\Omega} d\mathbf{x}$ is the volume of myocardium enclosed by the deforming ENDO and EPI surfaces. Parameter $V_0$ is the average volume, that can be calculated from frame 1, which is carefully traced by experts. Since the variance of the volume is less than 5%, parameter $\sigma_V$ can be obtained by invoking $3 - \sigma$ rule.

\[
3\sigma_V = 0.025V_0
\]

\[
\sigma_V = \frac{V_0}{120}
\]

Thus, the incompressibility constraint is encoded into a probability function that favors small variation of the myocardium volume and penalizes large variation.

### 2.5. Level Set Implementation

Combining equation 1, 4, and 5, the maximization of posterior can be identified by the coupled Euler-Lagrange equations

\[
\frac{\partial S_{in}}{\partial t} = \left(\log\left(\frac{\mathcal{P}(I; \mu_1, \sigma_1)}{\mathcal{P}(I; \mu_2, \sigma_2)}\right) - \frac{V - V_0}{\sigma_V^2}\right) \mathbf{n}_{in}
\]  

(6)

\[
\frac{\partial S_{out}}{\partial t} = \left(\log\left(\frac{\mathcal{P}(I; \mu_1, \sigma_1)}{\mathcal{P}(I; \mu_2, \sigma_2)}\right) + \frac{V - V_0}{\sigma_V^2}\right) \mathbf{n}_{out}
\]  

(7)

where $\mathbf{n}_{in}$ and $\mathbf{n}_{out}$ are the normals of $S_{in}$ and $S_{out}$ respectively, and $t$ is the propagation time step. To implement the evolution of equation 6 and 7, we embed the surface $S_{in}$ and $S_{out}$ in two higher dimensional functions $\phi_1(\mathbf{x})$ and $\phi_2(\mathbf{x})$, which implicitly represents $S_{in}$ and $S_{out}$ as the zero level set, i.e. $S_{in} = \{\mathbf{x}|\phi_{in}(\mathbf{x}) = 0\}$ and $S_{out} = \{\mathbf{x}|\phi_{out}(\mathbf{x}) = 0\}$.

Thus, it is easy to derive the level set formation of the coupled evolution equations which reads

\[
\frac{\partial \phi_{in}}{\partial t} = \left(\log\left(\frac{\mathcal{P}(I; \mu_1, \sigma_1)}{\mathcal{P}(I; \mu_2, \sigma_2)}\right) - \frac{V - V_0}{\sigma_V^2}\right) |\nabla \phi_{in}| \quad (8)
\]

\[
\frac{\partial \phi_{out}}{\partial t} = \left(\log\left(\frac{\mathcal{P}(I; \mu_1, \sigma_1)}{\mathcal{P}(I; \mu_2, \sigma_2)}\right) + \frac{V - V_0}{\sigma_V^2}\right) |\nabla \phi_{out}| \quad (9)
\]

### 3. EXPERIMENTS

In Fig. 4, we compare the results with and without incompressibility constraint. We observe that, without incompressibility constraint, the evolution of EPI surface encounters leakage problem due to the low myocardium/background contrast, while ENDO surface still finds its correct boundary.

![Fig. 4. Detection EPI surface with (Left) and without (Right) incompressibility constraint.](image)

Fig. 5 represents the short axis view of automatically segmented ENDO and EPI surfaces on consecutive frames during ventricular systole. Also, we measured the volumes of myocardium during ventricular systole, and show them in Fig. 6.

Let $S_1$ and $S_2$ be two surfaces from the automatic and manual segmentation respectively. Suppose they are represented as point sets, i.e. $S_1 = \{u_1, u_2, ..., u_n\}$ and $S_2 = \{v_1, v_2, ..., v_m\}$, the difference between $S_1$ and $S_2$ is described by MAD ($S_1, S_2$) = \[
\frac{1}{2} \left\{ \frac{1}{n} \sum_{i=1}^{n} d(u_i, S_2) + \frac{1}{m} \sum_{j=1}^{m} d(v_j, S_1) \right\},
\]

where $d(u_i, S_2) = \min_{v_j \in S_2} |u_i - v_j|$. Let $\Omega_{S_1}$ and $\Omega_{S_2}$ be the region enclosed by $S_1$ and $S_2$, we define
Fig. 5. Short-axis view of ENDO- and EPI surfaces from cardiac MRI images during ventricular systole. Top row: ENDO, Bottom row: EPI.

Fig. 6. Comparison of the volume of myocardium using algorithms with and without incompressibility constraint.

PTP = \( \frac{\text{Volume}(\Omega_{S_1} \cap \Omega_{S_2})}{\text{Volume}(\Omega_{S_2})} \)

PFP = \( \frac{\text{Volume}(\Omega_{S_2}) - \text{Volume}(\Omega_{S_1} \cap \Omega_{S_2})}{\text{Volume}(\Omega_{S_1})} \)

ENO = 1 - \( \frac{(\text{Volume}(\Omega_{S_1}) + \text{Volume}(\Omega_{S_2}))}{2} \)

Table 1 presents the quantitative analysis of the segmentation results over 225 sets of volumetric data using MAD, PTP, PFP, and ENO. We observe a significant improvement of accuracy in locating EPI surfaces.

Table 1. Quantitative validation for the segmentation of EPI surfaces over 225 sets of volumetric data.

<table>
<thead>
<tr>
<th></th>
<th>with constraint</th>
<th>without constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAD(mm)</td>
<td>0.69 ± 0.08</td>
<td>6.54 ± 1.45</td>
</tr>
<tr>
<td>PTP(%)</td>
<td>97.6 ± 1.34</td>
<td>78.9 ± 5.43</td>
</tr>
<tr>
<td>PFP(%)</td>
<td>2.31 ± 1.35</td>
<td>10.9 ± 5.44</td>
</tr>
<tr>
<td>ENO(%)</td>
<td>4.24 ± 1.23</td>
<td>13.37 ± 5.42</td>
</tr>
</tbody>
</table>

4. CONCLUSION

This paper presents an automatic approach to extracting ENDO- and EPI contours from volumetric canine MR images. Motivated by the fact that the volume of myocardium is nearly constant during a cardiac cycle, we design a deformable model that propagates ENDO- and EPI surfaces according to image-derived information and incompressibility constraint that restraints the variation of volume with 5%. As seen from the experiments, our algorithm is automatic, accurate, and robust. Future work includes quantitative measurement of myocardium wall thickness, estimation of strain and strain rate by analyzing the myocardium deformation, and extension to echocardiography.

5. REFERENCES


