Local Shape Registration Using Boundary-Constrained Match of Skeletons

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Abstract

This paper presents a new shape registration algorithm that establishes “meaningful correspondence” between objects, in that it preserves the local shape correspondence between the source and target objects. By observing that an object’s skeleton corresponds to its local shape peaks, we use skeleton to characterize the local shape of the source and target objects. Unlike traditional graph-based skeleton matching algorithms that focus on matching skeletons alone and ignore the overall alignment of the boundaries, our algorithm is formulated in a variational framework which aligns local shape by registering two potential fields that are associated with skeletons. Also, we add a boundary constraint term to the energy functional, such that our algorithm can be applied to match bulky objects where skeleton and boundary are far away to each other. To increase the robustness of our algorithm, we incorporate M-estimator and dynamic pruning algorithm to form a feedback system that eliminates local shape outliers caused by nonrigid deformation, occlusion, and missing parts. Experiments on 2D binary shapes and 3D cardiac sequences validate the accuracy and robustness of this algorithm.

1. Introduction

Shape registration is of great interest in the domain of computer vision and medical image analysis. Shape registration deals with measuring the resemblance between two shapes, estimating the optimal transformation, and warping one shape onto the other using the estimated transformation. The most difficult part of a shape registration algorithm is probably the design of an appropriate matching measure. The characterization of a matching measure is an ill-posed problem in the sense that there is no unique solution. To handle this, it is necessary to introduce the concept “meaningful correspondence”, that takes into account the underlying geometric or physical property of shapes and allows the characterization of a unique matching measure.

What is “meaningful correspondence”? While some researchers hope to find a general theory of shapes, including the determination of an appropriate space with a metric and probabilistic structure, the definition of “meaningful correspondence” is actually dependent on specific applications. The definition used in this paper is motivated by the human vision system, that builds “meaningful correspondence” based on the local shape of objects [4]. Loosely speaking, this means that strongly curved regions of one curve are paired with strongly curved regions of the other and flat regions are paired with flat regions. This definition is useful in computer vision, such as tracking and multiview registration. Interestingly, some biological shapes also deform in a way that preserves their local shape. Take cardiac motion as an example. The deformation of myocardial surfaces can be recovered by tracking dense 3D trajectory of the surface points based on local shape properties over an entire cardiac cycle [15].

Some efforts have been made in local shape registration. Curvature-based algorithms, for example, determine the point-wise correspondence of two shapes by minimizing the difference between their curvatures [12]. Manay et al. [9] introduced an integral descriptor to to avoid computing curvature or other differential operators, thus becoming robust to noise. Hong et al. [8] applied this integral operator to the registration and segmentation of objects from potentially noisy data. These methods assume the complete correspondence of local shape between two shapes. In real world, however, local shape can be only partially correspondent. In other words, local shape outliers may be created by nonrigid deformation, occlusion, or missing parts, as illustrated later in this section.

In this paper, we propose to use skeleton as a local shape descriptor. We observe that skeleton corresponds to the local curvature peaks (see Figure 1). In other words, skeleton is able to capture local shape features, that are important for comparing two shapes. Thus, the problem of local shape registration is converted to the problem of skeleton matching. In 2D cases, the term skeleton is used interchangeably with medial axis. In 3D cases, however, two different skeletons exist: medial surface and medial axis (centerline). The distinction between medial surface and medial axis stems from the different definition of 3D simple points that are
essential for skeleton extraction. While medial axis is able to characterize branching structures in some cases like for example neurons with their axons and dendrites, it is not a good \textit{local shape} descriptor in general. For example, the medial axis of the object shown in figure 2(a) is a single line which is not correspondent to any \textit{local shape features}. Therefore, in this paper, we focus on medial surface for 3D cases.

A great number of efforts have been made in designing skeleton matching techniques. Most of the approaches in the literature opt to use graph matching theory. Siddiqi \textsl{et al.} \cite{16} build a graph from skeletal points by labeling them according to the radius of the bitangent circle inscribed in the shape, and convert the graph matching problem to a tree-search problem. Golland and Grimson \cite{5} use undirected graphs to model connectivity of the skeleton points, and minimize a boundary functional to find the optimal fit to a fixed model skeleton. Graph matching algorithms are useful in identifying similar shapes, such as object recognition and image database retrieval. Graph matching alone, however, is inadequate for shape registration, particularly for the registration of bulky objects. A shape and its skeleton has a “many-to-one” relation; in other words, many shapes share the same skeleton \cite{3} and therefore skeleton is not a complete representation of the original shape. This fact implies that the match of skeleton alone could still mean that the boundaries are not perfectly registered, which we will call the \textit{boundary misalignment problem}, as shown in Figure 3. Unlike traditional graph representation, in this paper, we propose to embed skeleton implicitly in an associated \textit{skeleton potential field}, and convert the problem of skeleton matching into the problem of potential field matching. Also, we add a \textit{boundary constraint} to solve the \textit{boundary misalignment problem}.

In real world, \textit{local shape} correspondence sometimes is partially satisfied. It happens in many situations, such as nonrigid deformation, occlusion, and missing parts. A shape undergoing nonrigid deformation may create new \textit{local shape features} and remove old ones, as illustrated in Figure 4. Occlusion and missing parts may also create skeleton outliers or remove stable skeletons, as shown in Figure 5. In this paper, we assume that \textit{local shape} of two objects are partially correspondent. To increase robustness of our algorithm, we adopt M-estimator to alleviate the impact of skeleton outliers, and introduce \textit{dynamic pruning algorithm} to iteratively prune unmatched skeletons.

\section{2. Algorithm Overview}

The algorithm proposed in this paper is a non-rigid shape registration algorithm that preserves the correspondence of \textit{local shape} by matching the skeletons of two objects with boundary constraint. The block diagram is shown in Figure 6. It starts with the extraction of skeleton using Augmented Fast Marching Method (AFMM) \cite{17}, followed by a two-stage global-to-local registration procedure to estimate the optimal transformation between the source and target shapes. The first stage is global alignment, aiming to recover a global transformation which brings the pose of the source shape as close as possible to that of the target shape. In the global alignment stage, only boundary information is considered. The second stage is non-rigid local registration, in which the extracted skeletons are used to generate a local deformation field that preserves the correspondence of \textit{local shape features}. Boundary constraint is added to solve the \textit{boundary misalignment problem}. To deal with skeleton outliers, our algorithm uses M-estimator to increase the robustness under such situations. Also, we compute the \textit{skeleton potential residual} to dynamically prune unmatched skeletons. This pruning process iterates until an optimal transformation field is achieved.
3. Skeletonization

The first step in skeleton-based registration is computing skeleton. Many methods of skeletonization have been proposed in the literature (see [18] for an overview). Most of skeletonization algorithms, however, are sensitive to small boundary deformations and the consequent formation of ligatures: segments of the skeleton that are not correspondent to any local shape features [2]. To deal with this problem, some robust approaches are proposed [17, 6, 7]. In this paper, we adopt AFMM [17] to compute skeletons. AFMM is easy to implement and does not require any training samples to learn the topology of the skeleton. Simply speaking, AFMM propagates a parameterized boundary in its normal direction, with constant speed, by solving the Eikonal equation \( |\nabla \varphi| = 1 \), with \( \varphi = 0 \) on the boundary. The resultant \( \varphi(\cdot) \) is a level set function with zero level sets on the boundary. By relating the skeleton branch back to its source boundary, AFMM removes the branches whose source boundary details are smaller than a given threshold, and retains those with larger source boundary details, as described below. Thus, it provides a robust mechanism to distinguish between stable skeletons and ligatures.

**Initialization:**

1. Accepted Points: Let \( \text{Accepted} \) be the set of all grid points on shape boundary and the value of \( \varphi \) for those points is assigned to be 0.

2. Narrow Band points: Let \( \text{NarrowBand} \) be the set of all grid points in the narrow band of \( \text{Accepted} \) Points. The value of \( \varphi \) for those points is evaluated as \( dy \).

3. Far Away Points: Let \( \text{Far Away} \) be the set of all the rest of the grid points. The value of \( \varphi \) is set equal to \( \infty \).

4. Parameterization Field: \( U(i,j) \): Let \( U : \mathbb{R}^2 \rightarrow [0, L] \) be the arc-length parameterization of an object’s boundary.

**Propagation:**

1. Begin Loop: Let \( (i_{\min}, j_{\min}) \) be the point in \( \text{NarrowBand} \) with the smallest value for \( \varphi \).

2. Add point \( (i_{\min}, j_{\min}) \) to \( \text{Accepted} \); remove it from \( \text{NarrowBand} \).

3. Tag as neighbors any points \( (i_{\min} - 1, j_{\min}), (i_{\min} + 1, j_{\min}), (i_{\min}, j_{\min} - 1), (i_{\min}, j_{\min} + 1) \) that are not \( \text{Accepted} \); if the neighbor is in \( \text{Far Away} \), remove it from that set and add it to the \( \text{Narrow Band} \) set.

4. Recompute \( \varphi \) at all neighborhoods according to the equation:

\[
\max \left( D_{i,j}^x \varphi, 0 \right)^2 + \min \left( D_{i,j}^x \varphi, 0 \right)^2 + \max \left( D_{i,j}^y \varphi, 0 \right)^2 + \min \left( D_{i,j}^y \varphi, 0 \right)^2 \right]^{1/2} = 1.
\]

5. Update the parameterization field \( U \) by taking the average of \( U \) over \( \text{Accepted} \) neighbors unless the maximal difference of \( U \) over \( \text{Accepted} \) neighbors is larger than 2, in which case \( U \) is set to be \( U(i_{\min}, j_{\min}) \).

6. Return to the top of the loop.

**Skeleton Extraction:**

1. For each grid point \( (i, j) \), if \( U(i,j) \) differs from the \( U \)’s of its 8-neighbors for more than a given threshold, grid point \( (i, j) \) is a skeletal point, otherwise not.

In [17], only 2D AFMM is discussed. However, it is easy to extend to 3D cases. In 3D cases, a surface can be parameterized by a vector \( (U, V) \). Thus, for any point \( (i, j, k) \) on the surface, there exists a mapping: \( (i, j, k) \rightarrow (U(i, j, k), V(i, j, k)) \), and 3D AFMM can be derived accordingly. The experiment results from AFMM will be shown in section 7.

4. Global Alignment

Shape registration can be decomposed into global transformations and local ones. The global-to-local registration strategy provides a robust solution to most shape registration problems. Assume the boundaries of shape \( O_1 \) and \( O_2 \) are implicitly represented by level set function \( \varphi_1 \) and \( \varphi_2 \) obtained from AFMM, as described in section 3. The global rigid transformation \( T_{\text{global}} \) between \( O_1 \) and \( O_2 \) involves three parameters \( (s, \theta, t) \), where \( s \) is the scaling factor, \( \theta \) is the rotation angle, and \( t \) is the translation vector. To be robust to unmatched skeletons, we use the well-known M-estimator to deal with outliers. Thus, the global alignment
functional is given by [11]

$$E(T_{\text{global}}) = \int_\Omega \rho (s \cdot \varphi_1(x) - \varphi_2 \circ T_{\text{global}}(x)) \, dx \quad (1)$$

where $\rho$ is a symmetric and positive definite function. One then can define the following influence function $\psi(r) = \frac{d}{dr} \rho(r)$.

We consider two well-known influence functions, the fair and the Cauchy given by

$$\rho_{\text{fair}}(r) = c^2 \left[ \frac{|r|}{c} - \log \left( 1 + \frac{|r|}{c} \right) \right]$$

$$\rho_{\text{cauchy}}(r) = \frac{c^2}{2} \log \left( 1 + \left( \frac{r}{c} \right)^2 \right)$$

To take advantages of both functions, we use the fair function until convergence and apply the Cauchy one to eliminate large errors. Gradient descent method is used to find the optimal solution for equation 1.

5. Local Shape Registration

5.1. Skeleton Potential Field Registration

Let $\Gamma : \Omega \rightarrow \{0, 1\}$ be the skeleton extracted in section 3. In particular, $\Gamma$ is defined as

$$\Gamma(x) = \begin{cases} 1 & \text{if skeleton} \\ 0 & \text{if not skeleton} \end{cases} \quad (2)$$

where $x \in \Omega$.

Let us also define the local transformation $T_{\text{local}}$ that maps shape $O_1$ onto shape $O_2$. Suppose $\Gamma_1$ and $\Gamma_2$ are skeletons of $O_1$ and $O_2$, we need to find the nonrigid transformation $T_{\text{local}}$ that maps $\Gamma_1$ to $\Gamma_2$. To establish a dense transformation field that relates skeleton and its boundary (see section 5.2), we introduce a smooth and differentiable potential field, that can be constructed from the skeleton.

A number of approaches are available to construct such a potential field. Considering the idea of level set function, we choose Fast Marching Method (FFM) [13] to build a distance potential field, denoted as $\Pi$ (see Figure 7).

It can be seen from figure 7 that the distance potential field of the skeleton is smooth and differentiable. Therefore, the problem of matching two skeletons is converted to the problem of matching their associated potential fields.

To register two potential fields, we minimize the functional given in the following form:

$$E_{\text{ske1}}(T_{\text{local}}) = \int_\Omega \rho (\Pi_1(x) - \Pi_2 \circ T_{\text{local}}(x)) \, dx \quad (3)$$

where $\Pi_1$ and $\Pi_2$ are potential fields from the skeletons.

5.2. Boundary Constraint

As mentioned in section 1, complete shape matching is not equivalent to pure skeleton matching. It is because perfect alignment of skeletons does not imply the overall agreement between shape boundaries (see Figure 8). While the skeleton captures local shape, it does not provide sufficient boundary information. To overcome this, we need to modify equation 3 by adding a boundary constraint.

$$E_{\text{bound}}(T_{\text{local}}) = \int_\Omega \rho (\varphi_1(x) - \varphi_2 \circ T_{\text{local}}(x)) \, dx \quad (4)$$

5.3. Energy Functional

Now that we have specified the energy functional for the skeleton matching term and the boundary constraint term. To estimate the transformation, we still need to define a regularization term that gives spatially-smooth transformation.
The regularization term writes:

\[ E_{\text{reg}} (T_{\text{local}}) = \int_{\Omega} |\nabla T_{\text{local}}|^2 d\mathbf{x} \quad (5) \]

Combining these three terms, we have the following energy functional:

\[ E (T_{\text{local}}) = E_{\text{reg}} (T_{\text{local}}) + \alpha_1 E_{\text{skele}} (T_{\text{local}}) + \alpha_2 E_{\text{bnd}} (T_{\text{local}}) \quad (6) \]

where \( \alpha_1 \) and \( \alpha_2 \) are fixed parameters that balance the contribution among the skeleton matching term, boundary constraint term, and the regularization term.

Minimization of the energy \( 6 \) by computing the Gateaux derivative (or first variation) leads to the well-known Euler-Lagrange equation as follows:

\[ \frac{\partial T_{\text{local}}}{\partial t} = - \frac{\delta E}{\delta T_{\text{local}}} = - \alpha_1 \frac{\delta E_{\text{skele}}}{\delta T_{\text{local}}} - \alpha_2 \frac{\delta E_{\text{bnd}}}{\delta T_{\text{local}}} \quad (7) \]

The Gateaux derivation of the regularization term leads to the well-known diffusion term \( \nabla^2 T_{\text{local}} \).

The Gateaux derivation of the skeleton term is given by:

\[ \frac{\delta E_{\text{skele}}}{\delta T_{\text{local}}} = - \nabla \Gamma_2 \circ T_{\text{local}} (\mathbf{x}) \cdot \psi (\Gamma_1 (\mathbf{x}) - \Gamma_2 \circ T_{\text{local}} (\mathbf{x})) \quad (8) \]

and the Gateaux derivation of the boundary constraint is given by:

\[ \frac{\delta E_{\text{bnd}}}{\delta T_{\text{local}}} = - \nabla \varphi_2 \circ T_{\text{local}} (\mathbf{x}) \cdot \psi (\varphi_1 (\mathbf{x}) - \varphi_2 \circ T_{\text{local}} (\mathbf{x})) \quad (9) \]

Equation \( 6 \) can also be interpreted as the combination of local shape matching and nearest point matching. Experiments show that \( \alpha_1 = 0.1, \alpha_2 = 0.2 \sim 0.3 \) is appropriate for most images. As \( \alpha_1 \to 0 \), equation \( 6 \) degrades to the nearest point based matching without considering local deformation. As \( \alpha_2 \to 0 \), equation \( 6 \) degrades to skeleton-based matching without boundary constraint. We will discuss in section \( 7 \) the effect of parameter \( \alpha_1 \) and \( \alpha_2 \).

6. Dynamic Pruning

As mentioned in section \( 1 \), nonrigid deformation, occlusion, and missing parts can produce skeleton outliers and remove stable skeletons. To overcome this, we propose to use dynamic pruning algorithm that iteratively prunes unmatched skeletal points.

Let \( T_{\text{local}} (\mathbf{x}) \) be the estimated local transformation from section \( 5.3 \), one can define the skeleton potential residual

\[ \mathcal{R}_{\text{skele}} (\mathbf{x}) = \Gamma_1 (\mathbf{x}) - \Gamma_2 \circ T_{\text{local}} (\mathbf{x}) \quad (10) \]

We observe that unmatched skeletal points has significantly larger potential residual than matched ones. By pruning the skeletal points whose absolute value of the skeleton potential residual is larger than a given threshold, dynamic pruning algorithm eliminates skeleton outliers and reconstructs a new skeleton potential field to feed into the local registration module and update the transformation field. Our experiments show that a threshold of top 5% of skeleton potential residual produces stable results for a large set of data. This dynamic pruning algorithm iterates until the difference of the largest and smallest skeleton potential residuals is within a given threshold.

Figure 9 illustrates dynamic pruning algorithm using the example given in Figure 4. It starts with the skeletons of two objects that are locally registered using the algorithm described in section \( 5 \). However, as shown in Figure 9(1)(a), they are not perfectly aligned due to the existence of skeleton outliers. These unmatched skeletons produce significant skeleton potential residual shown in Figure 9(1)(b). The histogram of the skeleton potential residual (Figure 9(1)(c)) has a broad range of values, which implies a large mismatch between the skeletons of two objects. By pruning the skeletons that have large skeleton potential residual, the remaining skeletons are fed into the local registration module to generate a refined local transformation, as shown in Figure 9(2)(a). A significant improvement is observed because the histogram is now biased toward the lower values, which implies smaller skeleton potential residual. Further improvement is made in the 3rd iteration where the histogram is peaked at the origin, which implies the successful removal of skeleton outliers. For most images, 3-6 iterations are need before convergence.

![Figure 9. Dynamic Pruning. (a) Skeleton. (b) Skeleton potential residual. (c) Histogram of skeleton potential residual. (1) First iteration. (2) Second iteration. (3) Third iteration.](image-url)
7. Experiments

7.1. 2D Binary Shape

We conducted experiments on LEMS database [13] that have 1024 categorized 2D shapes. We add boundary noise to the shapes to test the robustness of AFMM. Figure 10 demonstrates the performance of our algorithm with a few examples. The first column of Figure 10 are source shapes, and the second column are target shapes. The extracted skeletons are overlaid on both shapes. We observe that boundary perturbations do not produce ligatures. The third column shows the results from global alignment of the source (in red) and target (in blue) shapes. For global alignment, we studied five cases: (1) rotation (Figure 10(a)), (2) rotation + translation (Figure 10(b)), (3) rotation + missing parts (Figure 10(c)), (4) scaling (Figure 10(d)), and (5) rotation + scaling + missing parts (Figure 10(e)). In the fourth column, we show the correspondences between the source and target objects. The local shape outliers and their associated skeletons are pruned using dynamic pruning algorithm, and therefore do not significantly bias the local shape correspondence. The last column shows the source shapes overlaid on the target shapes. We observe that the source shapes do not overlay on the local shape outliers of the target shapes.

We compared the results from our algorithm with those from the algorithm presented in [11]. The algorithm presented in [11] only considers the boundary information of two shapes, which roughly speaking is equivalent to the case in our algorithm when \(\alpha_1 = 0\). Essentially, it is the nearest point-based matching that ignores the “meaningful correspondence” of local shape. Visual comparisons can be seen in Figure 11 where the correspondences between the tails of two birds are shown.

![Figure 11. Comparison with the nonrigid shape registration algorithm presented in [11]. (a) Results from our algorithm. (b) Results from the algorithm in [11].](image)

Also, we compared the results from our algorithm with those from integral-kernel based algorithm [8]. While the algorithm in [8] does encode the local shape of objects, it does not provide any mechanism to overcome shape outliers which may create unexpected results in some situations. As shown in Figure 12(b), the source image mistakenly matches the shape outlier in the target image. Our algorithm removes local shape outliers by using dynamic pruning algorithm and establishes the correct correspondence, as shown in Figure 12(a).

![Figure 12. Comparison with integral kernel based algorithm. (a) Results from our algorithm. (b) Results from integrated kernel based algorithm.](image)

7.2. Cardiac Motion Tracking

To further demonstrate our approach on real data, we perform experiments on the 3D canine cardiac MRI sequences. We validate the hypothesis that we can use local shape properties of myocardial surfaces to recover the myocardial motion trajectories.

The canine MRI sequences that we conducted experiments on were obtained on a GE signa 1.5-T scanner. The resulting image set consists of a matrix of \(100 \times 100 \times 16\) voxels per temporal frame, and 16 temporal 3D frames per cardiac cycle. Pairs of copper markers are implanted on the endocardial and epicardial surfaces, and the motion of these markers are used as the golden standard against which the validity of our method is evaluated (see [10] for details of the markers.)

For each 3D frame, the endocardial and epicardial surfaces were first segmented using BioImage Suite software [1], and represented using level sets. Next, the skeletons of endocardial and epicardial surfaces were extracted by using AFMM. Having the endocardial and epicardial surfaces and their skeletons on hand, we performed our local shape registration algorithm to recover the motion field of myocardial surfaces. Global registration was not involved because the inter-frame deformation is small and no explicit global transformation is observed. The resulted motion field is illustrated in Figure 13.

To quantify the accuracy of our algorithm, we measure the positional errors between implanted markers and algorithm-derived point positions for 12 MRI studies. The mean error and standard deviation for endocardial and epicardial surfaces are given in table 1.

![Table 1. Positional errors for endocardial and epicardial surfaces](image)
We observe that the positional error increase by 30\% when the skeleton information is not used ($\alpha_1 = 0$) or boundary constraint is not added ($\alpha_2 = 0$). To visualize the boundary misalignment problem, we check the Euclidean distance error between the warped endocardial surface and its successive frame to determine how well they are overlapped. Figure 14 shows that the error is uniformly less than $\frac{1}{5}$ of one pixel when $\alpha_1 = 0.1, \alpha_2 = 0.3$, and around one pixel in the case without boundary constraint.

8. Conclusion

In this paper, we present a new local shape registration algorithm with boundary-constraint match of skeletons. By taking advantage of the fact that an object’s skeleton is correspondent to its local shape peaks, our algorithm establishes the “meaningful correspondence” between the local shape of two objects. In real situations, however, the assumption of complete local shape correspondence is not always satisfied due to non-rigid deformation, occlusion, and missing parts. We assume that the source object and target object have partial local shape correspondence, and use M-estimator to increase the robustness of our algorithm. Also, we propose dynamic pruning algorithm to form a feedback system that iteratively eliminates local shape outliers. Experiments on 2D binary shapes and 3D MRI cardiac sequences show the validity of our algorithm.

Directions of future research include the joint segmentation and local shape registration from intensity images. The registration component proposed in this paper estimates a transformation field between two objects. The segmentation component propagates coupled deformable models in the source and target images according to image-derived information and the shape constraint from the registration.
component. Global shape information is used in the first couple of iterations, and local shape correspondence can be added when the coupled deformable models are close to the boundaries of the images.

References


