A Class of Games with Coupled Constraints to Model International GHG Emission Agreements *

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Abstract

This paper deals with the design of equilibrium solutions with coupled constraints in dynamic games of greenhouse gas (GHG) emissions abatement. Self enforcing International Environmental Agreements (IEA) among different groups of countries call for Nash equilibrium solutions when the abatement strategies of the countries are defined. In this paper we study the effect of having another party, like e.g. the United Nations which would impose to all players a coupled constraint on the total emissions allowed over the 21st century, or on the concentration of carbon reached at the end of the century. We show, using different formulations of environmental game, that the normalized equilibria obtained under a coupled constraint on emissions or concentration is close to Pareto optimality. This gives a clue on the way the post Kyoto negotiations could yield an agreement which could be close to efficiency.

1 Introduction

The aim of this paper is to model as a dynamic game the negotiation of a self-enforcing agreement on global greenhouse gas (GHG) emission reduction, for a group of nations in different states of development. The paper

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is a continuation of [Haurie et al. (2006)] where this differential game formalism has been introduced and of [Bahn et al. (2008)] where the problem of optimal timing of climate policies was formalized in a stochastic control framework. In the present paper we extend the optimal timing model to a game theoretic formalism and we provide a numerical illustration for a two-region version of the model.

The model aims at identifying the pace of GHG emission abatement to be negotiated in an international environmental agreement by different groups of countries having different levels of economic development. The Kyoto protocol, which has entered into force and will last until 2012 for a group of developed countries, is typical of a type of agreement that stipulates a cap and trade system. Such a system consists for each participating country (the so-called Annex-B countries, in the Kyoto Protocol) to agree on a cap on its own emissions and to participate in an international emissions trading system to achieve the global emission reduction aim at minimal global cost. In the negotiation of a post-Kyoto agreement it is probable that an international cap and trade system will also be established and will involve emerging economies such as Brazil, India and China (BIC). It is also clear from the emerging scientific consensus that one needs to curb drastically GHG emissions over the 21st century in order to avoid disastrous climate changes. Therefore a global constraint should be imposed on GHG concentration resulting from the cumulated emissions of all countries. The distribution and profiles over time of the emission caps for the emerging and developing countries (EDCs) and different groups of developed countries will then be the result of negotiations subject to this global constraint. The post-Kyoto agreement will therefore have to find an appropriate trade-off between the development needs and the limit to emissions imposed to the EDCs. Furthermore, to be self-enforcing, an agreement should be a Nash equilibrium [Nash (1950a)], since the stability of the agreement calls for each national abatement policy to be the best reply to the policies decided by the other nations. However the international agreement implies that this best reply will still satisfy the global environmental constraint. This calls naturally for the definition of the agreement as a normalized equilibrium, according to the definition proposed by J.B. Rosen [Rosen (1965)], for games where the strategies of players are coupled by a common constraint.

The use of Rosen’s normalized equilibrium concept in the modeling of environmental negotiations has been proposed in [Haurie (1995)] and [Haurie & Zaccour (1995)] and the mathematical theory of open-loop infinite-horizon differential games with coupled constraints has been fully developed in [Carlson & Haurie (2000)]. Application of the concept to games described
by distributed parameter systems has been considered in [Carlson (2002)]. Application of the concept to the modeling of local pollution problems have been proposed in [Haurie & Krawczyk (1997)] and further explored in [Krawczyk (2005)].

The combination of a general equilibrium model determining the growth paths of different nations and of a non-cooperative game structure to decide the abatement policies of the different (groups of) nations has been first proposed in [Nordhaus & Yang (1996)]. A more recent implementation of noncooperative game solutions in a multi-country economic growth framework has been proposed by [Bosetti et al. (2006)], [Bosetti et al. (2007)], who developed WITCH, a regional integrated assessment hard-link hybrid model. In [Helm (2003)] the concept of a two-level hierarchical game has been proposed, where nations decide their allowances while an international emissions trading system is organized. This approach has been extended in [Carbone et al. (2003)] to a multi-country general equilibrium framework. A similar two-level hierarchical game structure has been used in [Haurie & Viguier (2003)] to represent the dynamic competition between Russia and China in the exploitation of “hot air” in the Kyoto protocol. Another two-level hierarchical game based on the computable general equilibrium model GEMINI-E3 has been used in [Viguier et al. (2005)] to represent the strategic behavior of European countries in the allocation of allowances for the implementation of the Kyoto protocol.

In [Drouet et al. (2008)] a two-stage stochastic game model involving three groups of industrialized countries and one group of developing countries where the payoffs are obtained from simulations of GEMINI-E3 has been proposed and normalized equilibria have been computed numerically using an oracle based method applied to variational inequalities.

In the present paper we develop a dynamic multi-region economic growth model similar to those proposed by [Nordhaus (1994), Nordhaus & Yang (1996), Bosetti et al. (2006)], with a description of the technology choices similar to the one used in [Bahn et al. (2008)]. In this model the concept of normalized equilibrium is applied to characterize long term sustainable climate policies when it is possible to implement an international emissions trading scheme.

The paper is organized as follows: in Section 2, we recall the definition of a normalized equilibrium in a GHG abatement game stripped to its bare essentials which is similar to the game studied in [Drouet et al. (2008)]. We explain also how the normalized equilibrium concept can be implemented in particular cases of other GHG abatement games and how it would lead to particular ways of undertaking the post-Kyoto negotiations. In Section 3, we propose a multi-country economic growth model which captures the tran-
sition from a carbon intensive to a clean (carbon-free) economy in order to cope with global climate change. This model has also a two-level structure with the implementation of a global emission trading scheme at each period. In Section 4, we characterize the solutions of the game under different optimality concepts and, in Section 5, we compare via numerical simulations the different economic and environmental paths corresponding to Pareto, Nash equilibrium and Rosen normalized equilibrium, respectively. In Section 6, we conclude by stressing the relevance of this type of analysis in the context of post-Kyoto climate change negotiations.

2 Coupled constraints in GHG emission abatement games

2.1 The game of sharing an emissions budget

[Drouet et al. (2008)] studied a simple two-period stochastic game of GHG abatement involving 3 groups of nations where the payoffs were obtained from simulations of world general economic equilibria. The strategic variables were the carbon allowances, or caps, given to each group of countries at each period. Normalized equilibrium solutions were obtained numerically, using oracle based optimization techniques for variational inequalities. To summarize this game structure, which we call “game of sharing an emission budget”, in a deterministic framework, let us consider \( T \) periods \( t = 0, \ldots, T - 1 \) and a set \( M \) of \( m \) groups of countries hereafter called players which must decide on the caps (allowances) they impose on their respective global emissions of GHGs in each period. We denote \( \bar{e}_j(t) \) the cap decided by player \( j \) for period \( t \) and we assume that a global limit \( \bar{E} \) is imposed on the cumulative emissions of all periods \( t = 0, \ldots, T - 1 \). Therefore the following coupled constraint, representing a global emission budget, is binding all players together:

\[
\sum_{j \in M} \sum_{t=0}^{T-1} \bar{e}_j(t) \leq \bar{E}. \tag{1}
\]

Let \( \bar{e}(t) = \{ \bar{e}_j(t) \}_{j \in M} \) denote the vector of emission caps for all players in period \( t \) and let \( \bar{e}_t = \{ \{ \bar{e}_j(s) \}_{j \in M} : s = 0, \ldots, t \} \) be a sequence of such caps. Given these cap values, an international emission trading scheme is implemented at each period and a general economic equilibrium is computed for the \( m \) countries. This determines a welfare gain for each player,
hereafter called its payoff at period $t$ and denoted $W_j(e_t)$. Notice that this payoff may include all transfer payments due to the international emission trading scheme, the terms of trade effects and the loss of economic output due to global warming damages (which damages could be represented as a function of the cumulative emissions). For a choice of emission allowances $e = \{e(t) : t = 0, \ldots, T - 1\}$ the payoff to player $j$ is given by:

$$J_j(e) = \sum_{t=0}^{T-1} \beta_j^t W_j(e_t), \quad j \in M,$$

(2)

where $\beta_j$ is the periodic discount factor of player $j$.

2.1.1 Equilibrium definition

Let us call $E$ the set of emission caps $e$ that satisfy constraint (1). Denote also $[e^{*-j}, e_j]$ the emission cap program obtained from $e^*$ by replacing only the emission cap program $e^*_j$ of player $j$ by $\bar{e}_j$.

**Definition 1** The emission cap program $e^*$ is an equilibrium under coupled constraint (1) if the following holds for each player $j \in M$:

$$e^* \in E$$

(3)

$$J_j(e^*) \geq J_j([e^{*-j}, \bar{e}_j]) \quad \forall \ e_j \text{ s.t. } [e^{*-j}, \bar{e}_j] \in E.$$  

(4)

2.1.2 Normalized equilibrium

As proposed by [Rosen (1965)] we define a particular equilibrium under coupled constraint, called normalized equilibrium, using a fixed point condition on a “best reply” mapping defined as follows. Let $r = (r_j)_{j \in M}$ with $r_j > 0$ and $\sum_{j \in M} r_j = 1$ be a given weighting of the different players. Then introduce the combined response function:

$$\theta(e^*, e; r) = \sum_{j \in M} r_j J_j([e^{*-j}, \bar{e}_j]).$$

(5)

It is easy to verify that, if $e^*$ satisfies the fixed point condition:

$$\theta(e^*, e^*; r) = \max_{e \in E} \theta(e^*, e; r),$$

(6)

then it is an equilibrium under the coupled constraint.
**Definition 2** The emission program $\bar{e}^*$ is a normalized equilibrium if it satisfies (6) for a weighting $r$ and a combined response function defined as in (5).

The RHS of (6) defines an optimization problem under constraint. Assuming the required regularity we can introduce a Kuhn-Tucker multiplier $\lambda^o$ for each constraint $\sum_{t=0}^{T-1} \bar{e}_j(t,\omega) \leq \bar{E}(\omega)$ and form the Lagrangian:

$$L = \theta(\bar{e}^*, \bar{e}; r) + \lambda^o(\bar{E} - \sum_{j \in M} \sum_{t=0}^{T-1} \bar{e}_j(t)).$$

Therefore, by applying the standard K-T optimality conditions we can see that the normalized equilibrium is also the Nash equilibrium solution for an auxiliary game with a payoff function defined for each player $j$ by:

$$J_j(\bar{e}) + \lambda^j(\bar{E} - \sum_{j \in M} \sum_{t=0}^{T-1} \bar{e}_j(t)),$$

where:

$$\lambda^j = \frac{1}{r_j} \lambda^o, \quad \lambda^o \geq 0, \quad \lambda^o(\bar{E} - \sum_{j \in M} \sum_{t=0}^{T-1} \bar{e}_j(t)) = 0.$$ (9)

This characterization has an interesting interpretation in terms of negotiation for a climate change policy. A common “penalty” $\lambda^o$ is defined and applied to each player with an intensity $\frac{1}{r_j}$ that depends on the weight given to this player in the global response function.

**Remark 1** [Rosen (1965)] gives conditions for existence and uniqueness of a normalized equilibrium associated with a weighting $r$. There will then be a manifold of normalized equilibria indexed over the weighting $r$.

### 2.2 Equilibrium with coupled constraints in particular classes of models

In this section, we explore the properties of normalized equilibria in two special cases of models.
2.2.1 Linear models

Bottom-up energy-environment models such as MARKAL, TIMES and TIAM [Labriet & Loulou (2003)] use linear programming to solve partial equilibrium problems. When different countries are represented within a linear programming framework, the Nash equilibrium solutions are easily obtained through reduced size linear programs (LPs). This is demonstrated in [Labriet & Loulou (2003)] where the damage functions for each (group of) country (countries) is approximated by a linear function of the cumulative global emissions. The Nash equilibrium in a cost-benefit (CB) framework often exhibits a “prisoner’s dilemma” outcome. By contrast when one uses a cost-effectiveness (CE) approach and when one computes a normalized equilibrium, the equilibrium solution will coincide with a Pareto optimal solution. This is shown below for a game with $m$ players ($I = 1, \ldots, m$).

**Nash equilibrium in CB:** Let $c_j x_j$ and $A_j x_j = b_j$ be the payoff and the local constraints of country $j = 1, \ldots, m$, respectively. Let $\sum_{i=1}^{m} D_i^j x_i$ be the damage cost function for country $j$. In a Nash equilibrium solution, each player $j$ solves an LP defined as:

$$\max_{x_j : A_j x_j = b_j} c_j x_j - \sum_{i=1}^{m} D_i^j x_i, \tag{10}$$

which is equivalent to:

$$\max_{x_j : A_j x_j = b_j} c_j x_j - D_j^j x_i. \tag{11}$$

The part of the damage cost due to the other emitters is a fixed cost that does not enter into the decision making. This explains the large inefficiency of a Nash equilibrium solution often observed in linear energy-environment models like MARKAL or TIAM.

**Normalized Nash equilibrium in CE:** Let $c_j x_j$ and $A_j x_j = b_j$ be the payoff and the local constraints of country $j = 1, \ldots, m$, respectively. Let $\sum_{j=1}^{m} B_j x_j = e$ be the coupled constraint. The optimality conditions for a normalized equilibrium, with weighting $r = (r_i : i \in I)$, are equivalent to
solving the following problems:

\[
\max_{x_i \geq 0; A_i x_i = b_i} \quad c_i x_i + \frac{1}{r_i} \lambda_0^T \left( \sum_{j=1}^{m} B_j x_j - e \right), \quad i \in I, \quad (12)
\]

\[
0 = \lambda_0^T \left( \sum_{j=1}^{m} B_j x_j - e \right), \quad (13)
\]

\[
0 \leq \lambda_0. \quad (14)
\]

Because of the linearity of the coupled constraint, the maximization in (12) can be replaced by

\[
\max_{x_i \geq 0; A_i x_i = b_i} \quad r_i c_i x_i + \frac{1}{r_i} \lambda_0^T B_i x_i, \quad i \in I, \quad (15)
\]

\[
0 = \lambda_0^T \left( \sum_{j=1}^{m} B_j x_j - e \right), \quad (16)
\]

\[
0 \leq \lambda_0. \quad (17)
\]

This is exactly the optimality conditions for a Pareto optimal solution obtained with the weighting \( r_i > 0, \quad i \in I \). The LP framework pushed to the extreme the difference between Nash equilibrium in a CB context and normalized equilibrium in a CE context. The first one is very inefficient and the second one is Pareto optimal.

### 2.3 Sharing an emission budget

Consider now an \( m \)-player concave game a la Rosen with payoff functions:

\[
\psi_j(x_1, x_2, \ldots, x_m), \quad x_j \in X_j, \quad j = 1, \ldots, m, \quad (18)
\]

and a coupled constraint. When the latter is separable among players, i.e. when it takes the form:

\[
\sum_{j=1}^{m} \varphi_j(x_j) = e, \quad (19)
\]

the coupled equilibrium can be interpreted in an interesting way.
Case 1: \( e \) is scalar. Consider \( e \) as being a global allowance and call \( \varpi_j \geq 0 \) the fraction of this allowance given to player \( j \), with \( \sum \varpi_j = 1 \). Then define the game with payoffs and decoupled constraints:

\[
\psi_j(x_1, x_2, \ldots, x_m), \quad x_j \in X_j, \quad \varphi_j(x_j) \leq \varpi_j e, \quad j = 1, \ldots, m. \tag{20}
\]

A Nash equilibrium for this game is characterized, under the usual regularity conditions, by the following conditions:

\[
\max_j \psi_j(x_1, \ldots, x_j, \ldots, x_m) - \lambda_j \varphi_j(x_j) \quad \tag{21}
\]

\[
\lambda_j \geq 0, \tag{22}
\]

\[
0 = \lambda_j (\varphi_j(x_j) - \varpi_j e). \tag{23}
\]

Now assume that at the equilibrium solution all the constraints are active and hence all the \( \lambda_j \) are > 0. Since the multipliers are scalars, they can be written in the form:

\[
\lambda_j = \frac{\lambda_0}{r_j}, \tag{24}
\]

by taking:

\[
\lambda_0 = \sum_{j=1}^{m} \lambda_j, \tag{25}
\]

\[
r_j = \frac{\lambda_0}{\lambda_j}, \quad j = 1, \ldots, m. \tag{26}
\]

Therefore the conditions for a normalized coupled equilibrium are met since:

\[
\lambda_0 > 0 \quad \text{and} \quad \sum_{j=1}^{m} \varphi_j(x_j) - e = 0. \tag{27}
\]

Case 2: \( e \) is a vector. When the constraint is non scalar we cannot check for a normalized equilibrium, but we can show that the conditions for a coupled equilibrium still hold. Indeed, assume that \( (x_1^*, x_2^*, \ldots, x_m^*) \) is an equilibrium for the decentralized game associated with the repartition \( \varpi \), and assume that this strategy vector is not an equilibrium for the game with coupled constraint. This means that, for at least one player \( j \) there exists a strategy \( x_j \in X_j \) such that:

\[
\varphi_j(x_j^*) + \sum_{i \neq j} \varphi_i(x_i^*) \leq e \tag{28}
\]
and for which:

$$\psi_j(x^*_1, \ldots, x^*_j, \ldots, x^*_m) > \psi_j(x^*_1, \ldots, x^*_j, \ldots, x^*_m).$$  

(29)

However, since all the constraints are supposed to be active in the decentralized equilibrium, the condition (28) becomes: $$\varphi_j(x^*_j) \leq \varpi je.$$ Therefore one has now a contradiction with the decentralized equilibrium condition.

### 2.3.1 Negotiations to form stable IEA

Based on the insights previously given, a successful international climate policy could involve the following steps:

1. First, through international negotiations, groups of countries could agree on:
   - the total level of cumulative GHG emissions allowed over the 21st century, for instance based on a precautionary principle involving the overall temperature increase not to be exceeded;
   - the distribution of this cumulative emission budget among the different groups, for instance using some concepts of equity such as an egalitarian principle (same per-capita emission rights).

2. Second, an international emission trading scheme would have to be implemented.

3. Each group of countries would then allocate its global allowance over time in order to reach an equilibrium.

This would result in a stable IEA given the observance of the coupled (global) constraint on cumulative GHG emissions.

In a more sophisticated approach the sharing of a global emission budget could be replaced by a coupled constraint on the GHG concentration to be reached at the end of the century. In the remainder of this paper, we develop a multi-region economic growth model with accumulation of GHG and we apply the normalized equilibrium concept to characterize stable IEA.

### 3 A dynamic multi-region economic growth model

In this section we propose a dynamic game of GHG abatement between $m$ regions (groups of countries) that will be used to simulate numerically and compare different economic and environmental paths associated to different
solution concepts. Each region is described by an economic growth model a la Ramsey [Ramsey (1928)] with two types of technological organization, as proposed in [Haurie et al. (2006)] and [Bahn et al. (2008)]. Each region produces an homogenous good, that can either be consumed or invested, using three factors, labor, physical capital and fossil energy which generates carbon emissions. Production occurs in two types of economy: a “carbon economy” where a high level of carbon emissions is necessary to obtain output and a more expensive “clean economy” where a much lower level of emissions is necessary to produce the economic good.

3.1 Variables

The following variables enter in the description of the economic model, where $j$ is the index of each of the $m$ regions and $t$ corresponds to the model running time:

$C(j,t)$: total consumption in region $j$ at time $t$, in trillions ($10^{12}$) of dollars;

$ELF(t)$: economic loss factor due to climate changes at time $t$, in %;

$E_1(j,t)$: yearly emissions of GHG (in Gt–$10^9$ tons–carbon equivalent) in the carbon economy of region $j$ at time $t$;

$E_2(j,t)$: yearly emissions of GHG in the clean economy of region $j$ at time $t$, in GtC;

$I_i(j,t)$: investment in capital $i = 1, 2$ in region $j$ at time $t$, in trillions of dollars;

$K_1(j,t)$: physical stock of productive capital in the carbon economy of region $j$ at time $t$, in trillions of dollars;

$K_2(j,t)$: physical stock of productive capital in the clean economy of region $j$ at time $t$, in trillions of dollars;

$L_1(j,t)$: part of the (exogenously defined) labor force $L(j,t)$ of region $j$ allocated at time $t$ to the carbon economy, in millions ($10^6$) of persons;

$L_2(j,t)$: part of the labor force of region $j$ allocated at time $t$ to the clean economy, in millions of persons;

$M(t)$: atmospheric concentration of GHG at time $t$, in GtC equivalent;

$WRG(j)$: discounted welfare of region $j$;
$W$: total discounted welfare;

$Y_1(j,t)$: economic output in the carbon economy of region $j$ at time $t$, in trillions of dollars;

$Y_2(j,t)$: economic output in the clean economy of region $j$ at time $t$, in trillions of dollars.

### 3.2 Equations

In each region $j$, a social planner is assumed to maximize social welfare ($WRG$), given by the sum over $T$ periods of 10 years of discounted utility from available per capita consumption with the discount rate $dr(j,t)$:

$$WRG(j) = \sum_{t=0}^{T-1} 10 dr(j,t) L(j,t) \log[ELF(t) C(j,t)/L(j,t)].$$

Total labor ($L$) is divided between labor allocated to the carbon economy ($L_1$) and labor allocated to the clean economy ($L_2$):

$$L(j,t) = L_1(j,t) + L_2(j,t).$$

Capital stock ($K_i$) evolves according to the choice of investment ($I_i$) and depreciation rate $\delta_K$ as follows:

$$K_i(j,t+1) = 10 I_i(j,t) + (1 - \delta_K(j))^{10} K_i(j,t) \quad i = 1, 2.$$  

Economic output ($Y$) occurs in the two economies according to an extended Cobb-Douglas production function in three inputs, capital ($K$), labor ($L$) and energy (which use is measured through emission level $E$):

$$Y(j,t) = A_1(j,t) K_1(j,t)^{\alpha_1(j)} (\phi_1(j,t) E_1(j,t))^{\theta_1(j,t)} L_1(j,t)^{1-\alpha_1(j)-\theta_1(j,t)} + A_2(j,t) K_2(j,t)^{\alpha_2(j)} (\phi_2(j,t) E_2(j,t))^{\theta_2(j,t)} L_2(j,t)^{1-\alpha_2(j)-\theta_2(j,t)},$$

where $A_i$ is the total factor productivity in the carbon (resp. clean) economy (when $i = 1$, resp. $i = 2$), $\alpha_i$ is the elasticity of output with respect to capital $K_i$, $\phi_i$ is the energy conversion factor for emissions $E_i$ and $\theta_i$ is the elasticity of output with respect to emissions $E_i$.

Economic output is used for consumption ($C$), investment ($I$) and the payment of energy costs:

$$Y(j,t) = C(j,t) + I_1(j,t) + I_2(j,t) + \pi_1(j,t) \phi_1(j,t) E_1(j,t) + \pi_2(j,t) \phi_2(j,t) E_2(j,t),$$

for $i = 1, 2$. 

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where $\pi_i$ is the energy price in the carbon (resp. clean) economy (when $i = 1$, resp. $i = 2$).

The accumulation $M$ of GHG in the atmosphere evolves according to:

$$M(t+1) = 10\beta \sum_{j=1}^{m} (E_1(j,t) + E_2(j,t)) + (1 - \delta_M) M(t) + \delta_M M_p, \quad (35)$$

where $\beta$ is the marginal atmospheric retention rate (the fraction of emissions that remains in the atmosphere in the short run), $\delta_M$ is the natural atmospheric elimination rate (the rate of transfer from the atmosphere to the oceans) and $M_p$ is the preindustrial level of atmospheric concentration.

Finally, increasing atmospheric GHG concentrations yield economic losses (due to climate changes) that reduce the available consumption determined using the scaling factor $ELF$ computed by:

$$ELF(t) = 1 - \left(\frac{M(t)}{cat_M}\right)^2, \quad (36)$$

where $cat_M$ corresponds to a “catastrophic” concentration level.

### 3.3 Optimality conditions

In this section, we give the necessary optimality conditions for a Pareto optimal solution and for a normalized equilibrium in model (30)–(36).

#### 3.3.1 Pareto optimality

Suppose that one gives each region $j$ a weight $rg(j) > 0$, such that $\sum_{j=1}^{m} rg(j) = 1$. A Pareto equilibrium associated with this weighting is obtained when one optimizes the criterion:

$$\sum_{j=1}^{m} rg(j)W RG(j), \quad (37)$$

under constraints (30)–(36). Define now the costate variables:

- $\lambda_i(j,t)$: costate variable for capital stock $i = 1, 2$ in country $j$ at time $t$;
- $\lambda_M(t)$: costate variable for GHG stock at time $t$;

and the Lagrange multipliers:

- $p_i(j,t)$: marginal value of production of type $i$ in region $j$ at time $t$;
$p_C(j, t)$: marginal value of consumption in region $j$ at time $t$;

$p_E(t)$: marginal economic loss due to GHG concentration at time $t$;

$p_L(j, t)$: marginal value of labor in region $j$ at time $t$.

**Lagrangean:**

$$L = \sum_{t=0}^{T-1} \left[ 10 \sum_{j=1}^{m} r g(j) dr(j, t) L(j, t) \log [ELF(t) C(j, t) / L(j, t)]
- \sum_{i=1}^{2} \sum_{j=1}^{m} \lambda_i (j, t + 1) (10 I_i(j, t) + (1 - \delta_K(j)))^{10} K_i(j, t) - K_i(j, t + 1))
- p_i(j, t) \left( A_i(j, t) K_i(j, t)^{\alpha_i(j)} (\phi_i(j, t) E_i(j, t))^{\theta_i(j, t)} L_i(j, t)^{1-\alpha_i(j)-\theta_i(j, t)} - Y_i(j, t) \right) \right]$$

$$- \lambda_M(t + 1) (10 \beta \sum_{i=1}^{m} \sum_{j=1}^{m} E_i(j, t) + (1 - \delta_M) M(t) + \delta_M M - M(t + 1))$$

$$- \sum_{j=1}^{m} p_C(j, t) \left( C(j, t) + \sum_{i=1}^{2} (I_i(j, t) + \pi_i(j, t) \phi_i(j, t) E_i(j, t) - Y_i(j, t)) \right)$$

$$- p_E(t) \left( 1 - \left( M(t) / cat \right) - ELF(t) \right)$$

$$- \sum_{j=1}^{m} p_L(j, t) \left( \sum_{i=1}^{2} L_i(j, t) - L(j, t) \right) \tag{38}$$

**Optimality conditions:** For an interior solution the following should hold:

$$\frac{\partial L}{\partial C(j, t)} = 10 r g(j) \, dr(j, t) \, L(j, t) / C(j, t) - p_C(j, t) = 0; \tag{39}$$

$$\frac{\partial L}{\partial ELF(t)} = \sum_{j=1}^{m} 10 r g(j) \, dr(j, t) \, L(j, t) / ELF(t) + p_E(t) = 0; \tag{40}$$

$$\frac{\partial L}{\partial Y_i(j, t)} = p_C(j, t) + p_i(j, t) = 0; \tag{41}$$

$$\frac{\partial L}{\partial L_i(j, t)} = -p_i(j, t) (1 - \alpha_i(j) - \theta_i(j, t)) A_i(j, t) K_i(j, t)^{\alpha_i(j)}$$

$$\left( \phi_i(j, t) E_i(j, t) \right)^{\theta_i(j, t)} L_i(j, t)^{-\alpha_i(j)-\theta_i(j, t)} - p_L(j, t) = 0; \tag{42}$$
\[
\frac{\partial L}{\partial K_i(j,t)} = -p_i(j,t)\alpha_i(j)A_i(j,t)K_i(j,t)^{\alpha_i(j)-1}
\]
\[
+ (\phi_i(j,t)E_i(j,t))^\theta_i(j,t) L_i(j,t)^{1-\alpha_i(j)-\theta_i(j,t)}
\]
\[
- \lambda_i(j,t+1)(1 - \delta_K(j))^{10} + \lambda_i(j,t)
\]
\[= 0; \quad (43)\]

\[
\frac{\partial L}{\partial E_i(j,t)} = -p_i(j,t)\theta_i(j,t)\phi_i(j,t)A_i(j,t)K_i(j,t)^{\alpha_i(j)}
\]
\[
+ (\phi_i(j,t)E_i(j,t))^\theta_i(j,t)-1 L_i(j,t)^{1-\alpha_i(j)-\theta_i(j,t)}
\]
\[
- pc(j,t)\pi_i(j,t)\phi_i(j,t) - \lambda_M(t+1)10\beta
\]
\[= 0; \quad (44)\]

\[
\frac{\partial L}{\partial I_i(j,t)} = -\lambda_i(j,t+1) 10 - pc(j,t) = 0; \quad (45)\]

\[
\frac{\partial L}{\partial M(t)} = -\lambda_M(t+1)(1 - \delta_M) + \lambda_M(t) + 2pE(t)M(t) / (cat_M)^2
\]
\[= 0; \quad (46)\]

\[
\lambda_M(T) = 0; \quad (47)\]
\[
\lambda_i(j,T) = 0. \quad (48)\]

**Interpretation:** Let us concentrate on Eq. (44) which determines the emissions levels in the two types of economies, for each player. It shows that the emissions levels will be such that the marginal productivity of emissions minus the cost of energy associated with these emissions expressed as \(pc(j,t)\pi_i(j,t)\phi_i(j,t)\) is equal to a common price given by \(-\lambda_M(t+1)10\beta\).

### 3.3.2 Normalized equilibrium

Suppose next that there is an environmental constraint that must be satisfied by all the regions together. We have considered the following coupled constraint on atmospheric GHG concentration at the end of the model horizon:

\[M(T) \leq \bar{M}, \quad (49)\]

where \(\bar{M}\) is a pre-specified concentration level not to be exceeded.

Suppose that one gives each region \(j\) a weight \(rg(j) > 0\), such that \(\sum_{j=1}^{m} rg(j) = 1\). Define the costate variables:
\(\lambda_i(j, t)\): costate variable for capital stock \(i = 1, 2\) in country \(j\) at time \(t\);

\(\lambda_M(j, t)\): costate variable for GHG stock in country \(j\) at time \(t\);

and the Lagrange multipliers:

\(p_i(j, t)\): marginal value of production of type \(i\) in region \(j\) at time \(t\);

\(p_C(j, t)\): marginal value of consumption in region \(j\) at time \(t\);

\(p_E(j, t)\): marginal economic loss due to GHG concentration at time \(t\);

\(p_L(j, t)\): marginal value of labor in region \(j\) at time \(t\);

\(\nu_M(T)\): common K-T multiplier for terminal coupled constraint on GHG concentration.

**Lagrangean for Player \(j\):**

\[
\mathcal{L}_j = \sum_{t=0}^{T-1} \left[ 10 \cdot d r(j, t) \cdot L(j, t) \cdot \log(\text{ELF}(t) \cdot C(j, t) / L(j, t)) \right] \\
- \sum_{i=1}^{2} \left[ \lambda_i(j, t + 1) \cdot (10 \cdot I_i(j, t) + (1 - \delta_K(j))^{10} \cdot K_i(j, t) - K_i(j, t + 1)) \right] \\
- p_i(j, t) \left( A_i(j, t) \cdot K_i(j, t)^{\alpha_i(j)} \cdot (\phi_i(j, t) \cdot E_i(j, t))^{\theta_i(j, t)} \cdot L_i(j, t)^{1 - \alpha_i(j) - \theta_i(j, t)} - Y_i(j, t) \right) \\
- \lambda_M(j, t + 1) \cdot (10 \cdot \beta_M \sum_{i=1}^{2} E_i(j, t) + (1 - \delta_M) \cdot M(t) + \delta_M \cdot M_p - M(t + 1)) \\
- p_C(j, t) \left( C(j, t) + \sum_{i=1}^{2} (I_i(j, t) + \pi_i(j, t) \cdot \phi_i(j, t) \cdot E_i(j, t) - Y_i(j, t)) \right) \\
- p_E(j, t) \left( 1 - (M(t) / \text{cat}_M)^2 - \text{ELF}(t) \right) \\
- p_L(j, t) \left( \sum_{i=1}^{2} L_i(j, t) - L(j, t) \right) - \frac{\nu_M(T)}{rg(j)} \cdot (\bar{M} - M(T)) \tag{50}
\]

**Optimality conditions:** For an interior solution the following holds:

\[
\frac{\partial \mathcal{L}_j}{\partial C(j, t)} = 10 \cdot r g(j) \cdot d r(j, t) \cdot L(j, t) / C(j, t) - p_C(j, t) = 0; \tag{51}
\]

\[
\frac{\partial \mathcal{L}_j}{\partial \text{ELF}(t)} = 10 \cdot r g(j) \cdot d r(j, t) \cdot L(j, t) / \text{ELF}(t) + p_E(j, t) = 0; \tag{52}
\]

16
\[
\frac{\partial L_j}{\partial Y_i(j,t)} = p_C(j,t) + p_i(j,t) = 0; \quad (53)
\]

\[
\frac{\partial L_j}{\partial E_i(j,t)} = -p_i(j,t)(1 - \alpha_i(j) - \theta_i(j,t)) A_i(j,t) K_i(j,t)^{\alpha_i(j)} \\
(\phi_i(j,t) E_i(j,t))^{\theta_i(j,t)} L_i(j,t)^{-\alpha_i(j) - \theta_i(j,t)} - p_C(j,t) p_i(j,t) = 0; \quad (54)
\]

\[
\frac{\partial L_j}{\partial K_i(j,t)} = -p_i(j,t) \alpha_i(j) A_i(j,t) K_i(j,t)^{\alpha_i(j)-1} \\
(\phi_i(j,t) E_i(j,t))^{\theta_i(j,t)} L_i(j,t)^{1-\alpha_i(j) - \theta_i(j,t)} - \lambda_i(j,t + 1)(1 - \delta_K(j))^{10} + \lambda_i(j,t) = 0; \quad (55)
\]

\[
\frac{\partial L_j}{\partial I_i(j,t)} = -\lambda_i(j,t + 1) 10 - p_C(j,t) = 0; \quad (56)
\]

\[
\frac{\partial L_j}{\partial M(t)} = -\lambda_M(j,t + 1)(1 - \delta_M) + \lambda_M(j,t) + 2p_E(t) M(t) / (cat_M)^2 = 0; \quad (58)
\]

\[
\lambda_M(j,T) = \frac{\nu_M(T)}{r g(j)}; \quad (59)
\]

\[
\lambda_i(j,T) = 0. \quad (60)
\]

**Interpretation:** Again we focus on Eq. (56). We see again that the emissions levels are determined by equalizing the marginal productivity of emissions, net of the cost of energy input in the economies to an implicit price given by \(-\lambda_M(j,t + 1) 10\). However now, the evolution of this price is determined by Eqs. (58) and (59). So the price is different for each player and, according to (59), the higher the weight given to a player, the lower will be the price and hence the higher will be the emissions level, since the productivity of emissions decreases with their levels.
3.4 An extended two-level model with cap and trade

In this section, we extend our previous model to allow for the implementation of an international emission trading system. Among the strategic decision variables one finds now the emission quotas (or emission caps) which are to be negotiated among the different regions.

The information structure which is implied in this modeling can be described as follows: In each region a decision maker selects the caps, the investment rates in the two economies and allocates labor, knowing that the global output will be maximized, given the price of emissions on the international market.

3.4.1 New variables

The extended model requires the following new variables:

Cap\((j,t)\): total emission cap agreed by region \(j\) for period \(t\), in GtC.

\(\pi_C(t)\): carbon price on international market at period \(t\), in dollars.

3.4.2 New equations

To characterize the equilibrium on the international market of emission permits, the following equations are added. The first equilibrium condition is a complementarity condition at each time \(t\) between carbon price and the excess of total caps over total actual emissions:

\[
\pi_C(t) \left( \sum_{j=1}^{m} \text{Cap}(j,t) - \sum_{j=1}^{m} (E_1(j,t) + E_2(j,t)) \right) = 0. 
\] (61)

The last two conditions express that the carbon price at each time \(t\) must correspond to the marginal production of energy in each economy (carbon economy and clean economy):

\[
\pi_C(t) = \frac{\partial Y_1(j,t)}{\partial E_1(j,t)} - \pi_1 \phi_1(j,t) \\
= \theta_1(j,t) \frac{Y_1(j,t)}{E_1(j,t)} - \pi_1 \phi_1(j,t) 
\] (62)

\[
\pi_C(t) = \frac{\partial Y_2(j,t)}{\partial E_2(j,t)} - \pi_2 \phi_2(j,t) \\
= \theta_2(j,t) \frac{Y_2(j,t)}{E_2(j,t)} - \pi_2 \phi_2(j,t) 
\] (63)
Besides these three equilibrium conditions, constraint (35) is re-written as follows:

$$M(t + 1) = 10\beta \sum_{j=1}^{m} \text{Cap}(j, t) + (1 - \delta_M) M(t) + \delta_M M_p,$$

(64)

**Remark 2** The conditions (62)-(63) should be compared with Eq. (44), where we see a similar structure.

### 3.4.3 New Payoffs

Finally, the extended model with caps and trade involves new payoffs as regional consumption (on which regional welfare is based) includes also transfer payments due to the trade of emission permits. Equation (34) is thus replaced by:

$$C(j, t) = Y(j, t) - I_1(j, t) - I_2(j, t) - \pi_1(j, t) \phi_1(j, t) E_1(j, t)$$

$$- \pi_2(j, t) \phi_2(j, t) E_2(j, t) - \pi_C(t)(E_1(j, t) + E_2(j, t) - \text{Cap}(j, t)).$$

(65)

### 4 Solution concepts for the extended model

#### 4.1 Collaborative solutions

Let again \( r_g(j) > 0 \) be the respective weight given to each player (region) \( j \), such that \( \sum_{j=1}^{m} r_g(j) = 1 \). A collaborative solution associated with this weighting is obtained when one optimizes again criterion (37):

$$\sum_{j=1}^{m} r_g(j) W_{RG}(j),$$

under constraints (30)–(33), (36), (61)–(65).

#### 4.2 Nash equilibrium solutions

A Nash equilibrium is obtained when each region has chosen an investment path and a sequence of caps which consist in the best reply to the choices made by the other regions. The differential game has the open-loop information structure. A strategy \( s_j \) for region \( j \) consists of:

- a sequence of investments \( \{I_1(j, t), I_2(j, t) : t = 0, 1, ..., T - 1\} \) in capital \( K_1 \) and \( K_2 \) respectively;
• a sequence of labor allocations \( \{L_1(j, t), L_2(j, t) : t = 0, 1, ..., T - 1\} \) in the carbon economy and the clean economy respectively;
• a sequence of caps \( \{\text{Cap}(j, t) : t = 0, 1, ..., T - 1\} \).

Because of the carbon market equilibrium conditions (61), (62) and (63), the payoff (30) to region \( j \) will depend on the strategy choices of other regions. We shall denote it \( \Psi_j(s_j, s^{-j}) \), where \( s^{-j} \) represents the strategies chosen by the regions other than \( j \). We shall also denote \( s = (s_j : j = 1, \ldots, m) \) the strategy vector of all the regions. Let \( S \) denote the set of all admissible strategy vectors.

Consider the so-called reply function defined as follows over the product set \( S \times S \):

\[
\theta(r : s', s) = \sum_{j=1}^{m} r(j) \Psi_j(s'_j, s^{-j}), \forall s' \in S, s \in S, \tag{66}
\]

where \( r(j) \) is any positive weight given to player \( j \).

A Nash equilibrium is a fixed point of the optimal reply mapping:

\[
\Theta(r : s) = \{s^o \in \text{argmax}_{s' \in S} \theta(r : s', s)\}. \tag{67}
\]

To compute a fixed-point solution, we have implemented a cobweb approach. Although it is not guaranteed to converge, when it does, it does defines an equilibrium.

### 4.3 Equilibrium under coupled constraint

When a coupled constraint must be satisfied by all the players together one can look for a normalized equilibrium as defined by [Rosen (1965)]. We have considered again the coupled constraint (49) on atmospheric GHG concentration at the end of the model horizon.

Let us call \( E \subset S \) the subset of all strategy vectors that will satisfy the coupled constraint. A normalized equilibrium is obtained using the weighted reply function defined over the product set \( E \times E \) as follows:

\[
\theta(r : s', s) = \sum_{j=1}^{m} r(j) \Psi_j(s'_j, s^{-j}), \forall s' \in E, s \in E. \tag{68}
\]

A normalized equilibrium under the coupled constraint (49) is a fixed point of the optimal reply mapping:

\[
\Theta(r : s) = \{s^o \in \text{argmax}_{s' \in E} \theta(r : s', s)\}. \tag{69}
\]
that is:

\[ s^* \in \Theta(r : s^*) \subset E. \]  

(70)

Indeed the optimum in (70) is taken under the constraint \( s' \in E \) and therefore, there will be a manifold of equilibrium solutions indexed over the weighting \( r(j) : j = 1, \ldots, m) \).

5 Numerical illustration

In this section we compute numerically the cooperative (Pareto) solution, Nash equilibrium and Rosen normalized equilibrium, respectively, for the two models presented above (with or without caps and trade), in the case of two coalitions of regions \((m = 2)\).

In this simple setting, we have divided the one-region world model presented in [Bahn et al. (2008)] in two regions: the first one (region \(j = 1)\) representing all Annex I countries of the Kyoto Protocol (the developed countries), the second region \(j = 2)\) representing all other non-Annex I countries (the developing countries).

The model presented in this paper still uses mostly parameter values from the DICE model [Nordhaus (1994)]\(^1\), but we have adapted in the spirit of the RICE model [Nordhaus & Yang (1996)] some regional parameter values as follows. The two regions have different population levels and energy prices, and we assume for region 2 a higher elasticity of output with respect to capital as well as a lower initial value for energy efficiency:

\[ L(j, 0) \]: initial value for population level of region \(j\), in millions of persons;  
\( L(1, 0) = 2204.5; L(2, 0) = 4204.5; \)

\[ \pi_i(j, 0) \]: initial energy price in the carbon economy \((i = 1)\), resp. clean economy \((i = 2)\) in region \(j\);  
\( \pi_1(1, 0) = 0.35; \pi_1(2, 0) = 0.3; \pi_2(1, 0) = 0.6; \pi_2(2, 0) = 0.7; \)

\[ \alpha_i(j) \]: elasticity of output with respect to capital \(K_i (i = 1, 2)\) in region \(j\);  
\( \alpha_1(1) = 0.3; \alpha_1(2) = 0.35; \)

\[ \phi_i(j, 0) \]: initial value for energy conversion factor in the carbon economy \((i = 1)\), resp. clean economy \((i = 2)\) in region \(j\);  
\( \phi_1(1, 0) = 1.1; \phi_1(2, 0) = 0.9; \phi_2(1, 0) = 6.0; \phi_2(2, 0) = 4.0. \)

\(^1\)More precisely the DICE-2007 version; see:  
5.1 The numerical method

To compute numerically the Pareto optimal solutions we have used the GAMS modeling language with the optimizer CONOPT. To compute numerically equilibrium solutions we have implemented a cobweb approach to find a fixed point to the reply functions (5). At each step GAMS-CONOPT was used to find the optimal reply. Indeed, as already stated, this method is not guaranteed to converge, but when it does, it finds an equilibrium solution.

5.2 Model without caps

Table 1 reports first on the evolution of atmospheric GHG concentration in our different scenarios. Besides the three equilibrium solutions (Pareto, Nash and Rosen), we report here also on a counterfactual baseline (denoted BaU), where the two players do not suffer from climate change related damages, and thus do not implement any GHG emission control policy.

<table>
<thead>
<tr>
<th></th>
<th>BaU</th>
<th>Pareto</th>
<th>Rosen</th>
<th>Nash</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>808.80</td>
<td>808.80</td>
<td>808.80</td>
<td>808.80</td>
</tr>
<tr>
<td>2015</td>
<td>845.34</td>
<td>824.47</td>
<td>831.64</td>
<td>831.77</td>
</tr>
<tr>
<td>2025</td>
<td>887.63</td>
<td>839.05</td>
<td>856.62</td>
<td>856.93</td>
</tr>
<tr>
<td>2035</td>
<td>936.65</td>
<td>853.50</td>
<td>883.25</td>
<td>883.79</td>
</tr>
<tr>
<td>2045</td>
<td>993.17</td>
<td>868.40</td>
<td>911.12</td>
<td>912.03</td>
</tr>
<tr>
<td>2055</td>
<td>1,057.99</td>
<td>883.98</td>
<td>939.95</td>
<td>941.42</td>
</tr>
<tr>
<td>2065</td>
<td>1,131.87</td>
<td>900.42</td>
<td>969.58</td>
<td>971.89</td>
</tr>
<tr>
<td>2075</td>
<td>1,215.61</td>
<td>918.04</td>
<td>997.63</td>
<td>1,003.62</td>
</tr>
<tr>
<td>2085</td>
<td>1,310.03</td>
<td>924.54</td>
<td>1,018.44</td>
<td>1,037.17</td>
</tr>
<tr>
<td>2095</td>
<td>1,415.87</td>
<td>925.77</td>
<td>1,021.71</td>
<td>1,056.23</td>
</tr>
<tr>
<td>2105</td>
<td>1,533.72</td>
<td>926.15</td>
<td>1,020.38</td>
<td>1,072.49</td>
</tr>
<tr>
<td>2135</td>
<td>1,952.79</td>
<td>1,014.51</td>
<td>1,014.51</td>
<td>1,212.81</td>
</tr>
</tbody>
</table>

Table 1: Atmospheric GHG concentrations (GtC)

In the BaU scenario concentration reaches 1,475 GtC by 2100\(^2\) (or equivalently 702 ppm); this corresponds approximately to the SRES A1B scenario of the IPCC [Nakicenovic & Swart (2000)]. When taking into account climate damages, and assuming that the two regions are collaborating to reach

\(^2\)Although our model runs until 2135 (by steps of 10 years from 2005), we comment here results from 2100 to enable an easy comparison with [IPCC (2007)].
a Pareto equilibrium by optimizing equation (37) with \( r_g(1) = r_g(2) = 0.5 \), GHG concentration is much lower, down to 926 GtC by 2100 (or equivalently 441 ppm). Optimal losses due to climate changes, computed on the basis of a cost-benefit approach, amount then to 20% of total regional consumption. For the Nash equilibrium, as each player optimizes own welfare without consideration of the other player’s welfare, the environmental situation deteriorates with GHG concentration levels slightly higher than the Pareto solution: 1,064 GtC by 2100 (or equivalently 507 ppm). Consequently, climate damages are also higher, namely 27% of total regional consumption by 2100. In our Rosen case, we have assumed that each player have the same weight \( (r_g(1) = r_g(2) = 0.5) \) and that the coupled constraint imposes that atmospheric GHG concentration at the end of the model horizon (2135) does not exceed the Pareto level (1,014.51 GtC). It is however interesting to note that the concentration trajectory remains very close to the Nash trajectory until 2075, after which it deviates to meet the target at the end of the model horizon.

Table 2 reports next on the corresponding GHG emission trajectories in the two regions for our different scenarios.

<table>
<thead>
<tr>
<th>Year</th>
<th>Pareto 1</th>
<th>Pareto 2</th>
<th>Rosen 1</th>
<th>Rosen 2</th>
<th>Nash 1</th>
<th>Nash 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>1.01</td>
<td>2.67</td>
<td>1.98</td>
<td>2.82</td>
<td>2.00</td>
<td>2.82</td>
</tr>
<tr>
<td>2015</td>
<td>0.60</td>
<td>2.99</td>
<td>2.06</td>
<td>3.21</td>
<td>2.08</td>
<td>3.21</td>
</tr>
<tr>
<td>2025</td>
<td>0.42</td>
<td>3.24</td>
<td>2.16</td>
<td>3.50</td>
<td>2.20</td>
<td>3.50</td>
</tr>
<tr>
<td>2035</td>
<td>0.36</td>
<td>3.45</td>
<td>2.27</td>
<td>3.73</td>
<td>2.33</td>
<td>3.74</td>
</tr>
<tr>
<td>2045</td>
<td>0.36</td>
<td>3.64</td>
<td>2.39</td>
<td>3.92</td>
<td>2.47</td>
<td>3.93</td>
</tr>
<tr>
<td>2055</td>
<td>0.39</td>
<td>3.84</td>
<td>2.50</td>
<td>4.10</td>
<td>2.63</td>
<td>4.11</td>
</tr>
<tr>
<td>2065</td>
<td>0.42</td>
<td>4.08</td>
<td>2.24</td>
<td>4.27</td>
<td>2.80</td>
<td>4.31</td>
</tr>
<tr>
<td>2075</td>
<td>0.46</td>
<td>2.40</td>
<td>1.07</td>
<td>4.48</td>
<td>3.00</td>
<td>4.57</td>
</tr>
<tr>
<td>2085</td>
<td>0.51</td>
<td>1.56</td>
<td>0.74</td>
<td>2.18</td>
<td>3.24</td>
<td>2.25</td>
</tr>
<tr>
<td>2095</td>
<td>0.57</td>
<td>1.38</td>
<td>0.67</td>
<td>1.56</td>
<td>3.54</td>
<td>1.63</td>
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<tr>
<td>2105</td>
<td>0.63</td>
<td>2.33</td>
<td>0.67</td>
<td>1.43</td>
<td>3.92</td>
<td>2.93</td>
</tr>
</tbody>
</table>

**Table 2:** Total emission levels (in GtC) for the two regions (1,2)

In the Pareto solution, the reduction effort is mostly borne by the rich countries (region 1), whereas the developing countries (region 2) are still allowed to emit a significant amount of GHGs, at least until 2065. By contrast, in the Nash equilibrium where each region acts selfishly, they all
emit more than in the Pareto case, but especially region 1 whose cumulative emissions (between 2005 and 2105) are then almost as high as those of region 2. Besides, when comparing with the BaU emission trajectories, region 2 would have to do a reduction effort almost twice as high as region 1. In the Rosen case, although cumulative emissions in region 2 are about twice those of region 1 (compared with a factor 5.5 higher in Pareto), reduction effort compared to the baseline are now similar in both regions.

Table 3 reports finally on the accumulation of the physical stock $K_2$ of productive capital in the clean economy in the two regions for our different scenarios.

<table>
<thead>
<tr>
<th></th>
<th>Pareto</th>
<th>Rosen</th>
<th>Nash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>2005</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
</tr>
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<td>2015</td>
<td>54.59</td>
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<td>0.02</td>
</tr>
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<td>2025</td>
<td>78.46</td>
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<td>0.01</td>
</tr>
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<td>2035</td>
<td>94.26</td>
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<td>2045</td>
<td>109.64</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2055</td>
<td>126.98</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2065</td>
<td>146.73</td>
<td>0.00</td>
<td>27.42</td>
</tr>
<tr>
<td>2075</td>
<td>168.68</td>
<td>180.37</td>
<td>135.43</td>
</tr>
<tr>
<td>2085</td>
<td>191.98</td>
<td>310.31</td>
<td>186.94</td>
</tr>
<tr>
<td>2095</td>
<td>216.43</td>
<td>384.57</td>
<td>218.71</td>
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<tr>
<td>2105</td>
<td>240.60</td>
<td>346.32</td>
<td>243.36</td>
</tr>
</tbody>
</table>

Table 3: Stock of capital $K_2$ (in trillions of dollars) in the two regions (1,2)

The accumulation of capital $K_2$ in the clean economy is related to the GHG reduction effort carried out by each region. In the Pareto case, region 1 starts accumulating clean capital as early as 2015, whereas region 2 does so only by 2075 at a time when it significantly curbs its GHG emissions. The higher accumulation in region 2 from 2075 onward is due to an overall economy (as measured by total economic output) larger than the one of region 1. In the Nash case, it is interesting to note that the clean economy does not lift off in region 1. Compared to the baseline, the small increase in GHG emissions is there achieved only through a smaller growth of the (carbon) economy. Whereas in region 2, clean capital starts again accumulating toward the end of the 21st century, more precisely here by 2085 where GHG emissions are approximately halved compared to the previous period.
Finally in the Rosen case, overall GHG emission reduction effort is somehow in between the one in the Nash and Pareto cases for both regions. However the accumulation of capital $K_2$ is the highest by 2105 in the Rosen case, to enable the two regions to decrease atmospheric GHG concentration from 2105 onward so as to meet the concentration target at the end of the model horizon (2135), see Table 1.

5.3 Model with caps and trade

We now consider the extended model where an international emission trading scheme is implemented and where the two regions negotiate their initial emission endowments. Notice that by default, both in the Pareto and Rosen cases, we have assumed again that each player has the same weight ($rg(1) = rg(2) = 0.5$).

Table 4 reports on the dynamics of atmospheric GHG concentration in our different scenarios. Notice that we have imposed again as coupled constraint in the Rosen approach a concentration limit that corresponds to the Pareto level of 2135.

<table>
<thead>
<tr>
<th>Year</th>
<th>BaU</th>
<th>Pareto</th>
<th>Rosen</th>
<th>Nash</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>808.80</td>
<td>808.80</td>
<td>808.80</td>
<td>808.80</td>
</tr>
<tr>
<td>2015</td>
<td>845.34</td>
<td>820.00</td>
<td>820.00</td>
<td>830.91</td>
</tr>
<tr>
<td>2025</td>
<td>887.63</td>
<td>834.29</td>
<td>834.53</td>
<td>855.66</td>
</tr>
<tr>
<td>2035</td>
<td>936.65</td>
<td>850.91</td>
<td>851.52</td>
<td>882.57</td>
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<tr>
<td>2045</td>
<td>993.17</td>
<td>869.31</td>
<td>870.60</td>
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</tr>
<tr>
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<td>1,057.99</td>
<td>889.19</td>
<td>891.76</td>
<td>941.36</td>
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<tr>
<td>2065</td>
<td>1,131.87</td>
<td>906.16</td>
<td>909.11</td>
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<tr>
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<td>923.12</td>
<td>926.12</td>
<td>1,005.69</td>
</tr>
<tr>
<td>2085</td>
<td>1,310.03</td>
<td>925.91</td>
<td>928.85</td>
<td>1,040.60</td>
</tr>
<tr>
<td>2095</td>
<td>1,415.87</td>
<td>925.13</td>
<td>927.98</td>
<td>1,058.76</td>
</tr>
<tr>
<td>2105</td>
<td>1,533.72</td>
<td>924.12</td>
<td>926.87</td>
<td>1,073.27</td>
</tr>
<tr>
<td>2135</td>
<td>1,952.79</td>
<td>1,000.33</td>
<td>1,000.33</td>
<td>1,211.08</td>
</tr>
</tbody>
</table>

Table 4: Atmospheric GHG concentrations (GtC) in the different scenarios

The evolution of concentrations in the Pareto and Nash cases follows rather closely those produced by our model without caps. The Rosen framework exhibits however a distinct behavior. Instead of following the Nash trajectory during the earlier decades as in the model without caps, concentration levels remain here very close to the Pareto ones. As a sensitivity
analysis, we have also considered a concentration limit of 1,100 GtC by 2135. In that case, the concentration trajectory remains very close to the Nash trajectory until 2065, after which it deviates to meet the new target at the end of the model horizon.

Table 5 displays next the emission caps negotiated by the two regions in our different scenarios.

<table>
<thead>
<tr>
<th></th>
<th>Pareto</th>
<th>Rosen</th>
<th>Nash</th>
</tr>
</thead>
<tbody>
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<td>2</td>
<td>1</td>
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<tr>
<td>2005</td>
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<td>2.98</td>
<td>0.00</td>
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<tr>
<td>2015</td>
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<td>3.53</td>
<td>0.00</td>
</tr>
<tr>
<td>2025</td>
<td>0.00</td>
<td>3.97</td>
<td>0.00</td>
</tr>
<tr>
<td>2035</td>
<td>0.00</td>
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<tr>
<td>2075</td>
<td>0.00</td>
<td>2.31</td>
<td>0.00</td>
</tr>
<tr>
<td>2085</td>
<td>0.00</td>
<td>1.77</td>
<td>0.03</td>
</tr>
<tr>
<td>2095</td>
<td>0.00</td>
<td>1.73</td>
<td>0.00</td>
</tr>
<tr>
<td>2105</td>
<td>0.00</td>
<td>1.85</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 5: Cap levels (GtC) for the two regions in the different scenarios

As for their GHG emissions in the model without caps, the two regions have contrasted behavior depending on the equilibrium framework considered. In the Pareto solution, all caps are allocated to the developing countries (region 2), an amount of 35.92 GtC over 100 years. Notice that about 2/3 are used by region 2 to cover its own emissions (24.2 GtC), the remaining is sold to region 1 to allow for some GHG emissions especially until 2045. By contrast in the Nash equilibrium, the caps are almost equally divided among the two regions: over 100 years, 31.91 GtC (48%) to region 1 and 34.69 GtC (52%) to region 2. In that case, region 2 becomes a net buyer of emission permits to allow for an extra 3.47 GtC emissions over 100 years. In terms of caps allocation, the Rosen results are again very similar to the Pareto ones. As a sensitivity analysis, we have also increased the weight \( rg \) allocated to player 1. When \( rg(1) = 0.7 \), the Rosen framework yields then a contrasted sharing of caps somehow a la Nash with 18.56 GtC (56%) allocated to region 1 and 14.49 GtC (44%) to region 2.

Table 6 gives next the accumulation of clean capital \( K_2 \) in the two regions
for our different scenarios.

<table>
<thead>
<tr>
<th>Year</th>
<th>K₁</th>
<th>K₂</th>
<th>K₁</th>
<th>K₂</th>
<th>K₁</th>
<th>K₂</th>
</tr>
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<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>2015</td>
<td>0.02</td>
<td>0.02</td>
<td>1.91</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>2025</td>
<td>0.01</td>
<td>0.01</td>
<td>3.27</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
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<td>0.00</td>
<td>7.09</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
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<td>0.00</td>
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<td>0.00</td>
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<td>0.00</td>
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<td>0.00</td>
</tr>
<tr>
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<td>136.26</td>
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<td>0.00</td>
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<td>192.19</td>
<td>322.64</td>
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<tr>
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<td>388.33</td>
<td>216.72</td>
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<td>0.00</td>
<td>381.52</td>
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<tr>
<td>2105</td>
<td>241.10</td>
<td>438.10</td>
<td>239.91</td>
<td>437.77</td>
<td>0.00</td>
<td>360.27</td>
</tr>
</tbody>
</table>

Table 6: Stock of capital $K_2$ (in trillions of dollars) in the two regions (1,2)

The accumulation of clean capital $K_2$ is related to the emission reduction effort carried out by each region, as in the model without caps. In the Pareto case, region 1 starts accumulating clean capital only by 2055\(^3\) where GHG emissions are approximately halved compared to the previous period, as it relies before on emission permits bought from region 2. The latter accumulates clean capital (again) only from 2075 at a time where it significantly curbs its GHG emissions. The higher accumulation in region 2 from 2075 onward is due on the one hand to an overall economy larger than the one of region 1 (as in the model without caps) and on the other hand to the opportunity to sell unused emission permits to region 1. In the Nash case one observes behaviors similar to the ones of the model without caps: the clean economy lifts off only in region 2 toward the end of the 21st century. In the Rosen case, results are again comparable with Pareto, except for a small capital accumulation in region 1 earlier on.

Figure 1 displays finally the efficient (welfare) frontier for our Pareto and Rosen scenarios, obtained by varying the weights $rg(j)$ assigned to each player, together with the Nash situation.

The striking result is that welfare reached by both regions in the Rosen framework is very close to the Pareto optimum. In other words, if there is a supranational body like the United Nations with enough authority to

\(^3\)Versus 2015 in the model without caps.
impose to all regions a limit on atmospheric GHG concentration not to be exceeded, and if the regions negotiate then how to respect such a constraint (through a system of emission caps and trade), the resulting environmental agreement would not only be stable but also close to efficiency.

6 Conclusion

In this paper we have shown that a new class of differential game models can contribute to the design of self-enforcing international environmental agreement in the context of post-Kyoto negotiations on climate change. The model incorporates the two-level decision structure which is implicit in the design of international emission trading schemes. It shows clearly that the establishment of such markets introduces additional interdependencies among players and, therefore, may induce strategic behavior among the different groups of countries involved in the negotiations.

The numerical model that we have used to illustrate the solution concept embodies the fundamental economy/environment trade-off that is faced by the different economies in the world. The main policy design decision will consist in the definition of the speed with which the regional economies will switch from a carbon emitting economy to a much cleaner but more expensive carbon-free economy.

An important element that has been neglected in this modeling study concerns the treatment of uncertainty. A stochastic control approach has
been proposed by [Bahn et al. (2008)] for a model similar to the one developed here but without the two-level decision structure and with a single criterion optimization. A natural extension of this present paper would thus be to consider explicitly uncertainty along with the strategic decision of each player.

References


