Maximum Likelihood Estimation and Correction of Carrier Frequency Offset in OFCDM Systems

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Abstract—In this paper, a Carrier Frequency Offset (CFO) correction scheme is proposed for a downlink Variable Spreading Factor (VSF) OFCDM system based on the maximum likelihood principle. We first derive the likelihood function for VSF-OFCDM system with CFO, and then propose to use a gradient algorithm to estimate and minimize the effect of CFO in a tracking mode. The maximal ratio combining receiver is considered for detection. Our results show that the BER performance in the low SNR environment can be improved significantly with few number of iterations.

I. INTRODUCTION

Orthogonal Frequency and Code Division Multiplexing (OFCDM) system has been proposed for future broadband wireless communications [1]. OFCDM uses data spreading in both time and frequency domain, where each data stream is segmented into multiple substreams and spread over multiple subcarriers and several OFCDM symbols, which provides both frequency and time diversity. The variable Spreading Factor (VSF) OFCDM, which changes the Spreading Factor (SF) in both the time and frequency domain, was proposed in [2]. The total spreading factor (SF) is the product of time domain spreading factor ($SF_{time}$) and frequency domain spreading factor ($SF_{freq}$). In VSF-OFCDM, both $SF_{time}$ and $SF_{freq}$ are adaptively controlled as per the cell structure, channel load and radio link conditions.

OFCDM, similar to other multicarrier systems, is sensitive to Carrier Frequency Offset (CFO). CFO causes a reduction in the desired signal amplitude as well as loss of orthogonality between subcarriers which results in InterCarrier Interference (ICI). The impact of CFO on multicarrier systems was investigated for MC-CDMA and MC-DS-CDMA systems in [3] and [4] respectively. Recently, there are some papers in the literature investigating the effect of CFO on OFCDM systems [5], [6] and on VSF-OFCDM systems [7].

Due to the adverse impact of CFO on VSF-OFCDM systems, a reliable estimation of the frequency offset is considered as a basic step in the coherent demodulation process. The CFO estimation process can be divided into two fundamental steps which are coarse acquisition and tracking. In this paper, we focus on the estimation and correction of the frequency offset in a tracking mode. For OFDM and MC-CDMA systems, similar algorithms have been proposed in [8] and [9] respectively.

In our work, a CFO correction scheme based on Maximum Likelihood (ML) estimation is proposed for OFCDM systems with different spreading factors using Maximal Ratio Combining (MRC) receiver. Our results show that the proposed method can minimize the normalized frequency offset residual error to less than $10^{-8}$ after only few iterations. This results in significant improvement in the system performance in terms of the BER.

This paper is organized as follows: Section II gives a brief description of the OFCDM system. In section III, the likelihood function for VSF-OFCDM system with CFO is derived. Using the gradient method to correct the frequency offset is proposed in section IV. In section V, some numerical results are presented. Finally, the paper is concluded in section VI.

II. SYSTEM MODEL

An OFCDM system with $K$ simultaneous users is considered. The total spectrum is divided into $G$ groups, each group has $M$ non-contiguous subcarriers that are equally spaced throughout the spectrum. This subcarrier grouping is used to maximize frequency diversity gains and minimize Multiple Access Interference (MAI) [10]. Fig. 1 shows the block diagram of the downlink OFCDM system.

The impulse response of the channel for all the users on the $t^{th}$ subcarrier of the $g^{th}$ group can be described as [11]:

$$h_{t,g}(t) = \alpha_{t,g}(t)e^{j\phi_{t,g}}, \quad (1)$$

where $\alpha_{t,g}$ are independent identically distributed (i.i.d.) Rayleigh random variables and the phase, $\phi_{t,g}$, is a uniformly distributed random variable over the interval $[0, 2\pi)$, which is assumed to be independent for every $l$ and $g$. We assume that each subcarrier experiences independent, frequency nonselective fading. Furthermore, the channel fading and phase shift variables are considered to be constant over a chip duration $T_c$.

The received signal for a downlink transmission, assuming BPSK modulation, for the users in group $g$ during the $j^{th}$ bit duration can be derived as

$$r(t) = \sum_{i} \sum_{g=1}^{G} \sum_{k=1}^{K_g} g_{j,g}^{(k)} \sum_{l=1}^{M} \alpha_{l,g} c_{j,l}^{(k)} e^{j(2\pi f_{l,g} t + \phi_{l,g})}$$

$$\times \sum_{n=1}^{N} c_{j,l,n} p(t - (iN + n)T_c) + n(t), \quad (2)$$

where $K_g$ is the total number of users in group $g$; $N$ is the length of the time domain PN code and it is assumed to be equal to the spreading factor in the time domain $SF_{time}$. The number of subcarriers $M$ in each group $g$ is assumed to be
equal to the spreading factor in the frequency domain $SF_{freq}$. $b_{j,g}^{(k)}$ is the BPSK signal for the $k^{th}$ user in the $g^{th}$ group during the $j^{th}$ transmitted bit. For the $k^{th}$ user, $c_{j,l}^{(k)}$ is the $l^{th}$ chip in the frequency domain spreading sequence and $c_{j,m,n}^{T(k)}$ is the $n^{th}$ chip in the time domain PN sequence on the $j^{th}$ subcarrier during the $j^{th}$ transmitted bit. Each chip belongs to the set $\{1, -1\}$. $p(t)$ is the rectangular pulse defined on the interval $[0, T_c]$. $f_d$ is the carrier frequency offset. $f_{t,g} = f_c + (g + (l - 1)G)$ is the frequency of the $l^{th}$ subcarrier of the $g^{th}$ group and $f_c$ is the carrier frequency. $n(t)$ is an Additive White Gaussian Noise (AWGN) with zero mean and double-sided power spectral density $N_0/2$.

After coherent demodulation, the decision variable of the $j^{th}$ transmitted bit of user 1 is given by

$$v^{(1)}_{j,1} = \frac{1}{T_c} \sum_{m=1}^{M} \sum_{n=1}^{N} \int_{(jN+n)T_c}^{(jN+n+1)T_c} r(t) \alpha_{m,n} c_{j,m}^{F(1)} c_{j,m,n}^{T(1)} P(t) \cos(2\pi f_{m,n} t + \phi_{m,1}) dt,$$

where $\phi_{m,1}$ denotes the estimated phase of the $m^{th}$ subcarrier in group 1 during the $j^{th}$ transmitted bit of user 1.

The average BER for MRC based on the Gaussian assumption of the noise and interference terms for BPSK modulation can be expressed as [7]

$$P_e = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E[2|S]}{2\pi\sigma_{total}^2}}\right).$$

where,

$$E[2|S] = \frac{N^2}{4} \sin^2(\pi f_d T_c) \left(\sum_{m=1}^{M} \alpha_{m,1}\right)^2$$

$$\sigma_{total}^2 = \frac{N}{8} (K_1 - 1) \sin^2(\pi f_d T_c) \left(\sum_{m=1}^{M} \alpha_{m,1}\right)^2$$

$$+ \frac{N\sin^2(\pi f_d T_c)}{8\pi^2} \sum_{g=2}^{G} \sum_{m=1}^{M} \sum_{l=1}^{l_G} \left(\frac{K_g(\alpha_{m,1})^2}{(f_d T_c + g - 1 + (l - m)G)^2} - \frac{N K_g \sin^2(\pi f_d T_c)}{8\pi^2} \sum_{m=1}^{M} (\alpha_{m,1})^2 \sum_{l=1}^{l_G} \left(\frac{K_g(\alpha_{m,1})^2}{(f_d T_c + l - m G)^2}\right)\right)$$

$$+ \frac{N N_0}{4 T_c} \sum_{m=1}^{M} (\alpha_{m,1})^2$$

(6)

Fig. 2 shows the BER of the VSF-OFCDM system vs. the normalized (with respect to the subcarrier spacing) frequency offset $(f_d T_c)$ at $E_b/N_o = 10$dB with 16 users and 128 subcarriers for different spreading factors ($SF = SF_{time} \times SF_{freq}$) with a fixed total $SF$ of 32. These results were obtained using equations (4)-(6) and Monte Carlo simulation. This figure shows that the VSF-OFCDM system is very sensitive to CFO. It also shows that the BER degradation is small when the normalized frequency offset is below a certain level for different $SF_{time}$ and $SF_{freq}$. For example, for a BER of $10^{-2}$, the performance starts to deteriorate at $f_d T_c = 0.1, 0.2, 0.25$ and 0.3 for a total $SF$ of 2x16, 4x8, 8x4 and 16x2 respectively. This means that if we can mitigate the frequency offset to less than that level for the different $SF_{time}$ and $SF_{freq}$, we can improve the system performance significantly. In the remainder of this paper, we will focus on the estimation and correction of the CFO for the VSF-OFCDM system.

III. LIKELIHOOD FUNCTION

The log likelihood function considered in our analysis is given by [12]

$$\Lambda = \frac{2}{N_o} \int_{T_o} \text{Re}\{r(t)\bar{r}(t)\} dt,$$

where $T_o$ is the observation period and it should satisfy $T_o \geq T_c$. 

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During the possible values of the data and fading parameters. In our case, the data dependence can be removed by averaging the log likelihood function over all the subcarriers. The data dependence can be considered for each subcarrier.

In our analysis, a perfect estimation of the fading amplitude and phase is considered for each subcarrier.

Our objective is to maximize the likelihood function based on the ML estimation principle. We will assume in our analysis that the receiver performs the coarse frequency offset correction with training sequences before data transmission and we will consider the frequency offset in the tracking process only [8].

The log likelihood function depends on the data sequence and the number of subcarriers. The data dependence can be removed by averaging the log likelihood function over all the possible values of the data and fading parameters. In our case, we will define the dependence on data and fading parameters during the \( n \)th chip of the \( j \)th bit as follows

\[
d_m,g = \sum_{k=1}^{K_j} \alpha_{m,g} b_{j,g}^{(k)} c_{j,m,n}^{(k)} T(k),
\]

where \( \tilde{f}_d \) is the estimate of the frequency offset error \( f_d \). In our analysis, \( \tilde{f}_d \) is the estimate of the frequency offset error with respect to the subcarrier spacing.

The log likelihood function becomes

\[
\Delta f_d = f_d - \tilde{f}_d.
\]

By making some mathematical approximations that are valid for low SNR [3], we obtain the likelihood function with the carrier frequency offset as follows

\[
\Lambda = \frac{1}{N_o} \left( \sum_{g=1}^{G} \sum_{m=1}^{M} d_{m,g} \times \text{Re}\{q_{m,g}\} \right)^2.
\]

where,

\[
q_{m,g} = \int_{0}^{T_c} r(t) e^{-i(2\pi f_{m,g} t + \phi_{m,g})} e^{-i(2\pi f_d t)} dt
\]

\[
= \sum_{g=1}^{G} \sum_{l=1}^{M} d_{l,g} \sum_{l=1}^{M} e^{i(2\pi (f_{l,g} - f_{m,g} + \Delta f_d) t + \phi_{l,g} - \phi_{m,g}) dt},
\]

and,

\[
\Delta f_d = f_d - \tilde{f}_d.
\]

Therefore,

\[
\text{Re}\{q_{m,g}\} = \sum_{g=1}^{G} \sum_{m=1}^{M} d_{m,g} \times \text{Re}\{q_{m,g}\}
\]

We remove the data dependence by averaging the likelihood function over all the values of the data and fading parameters \( d_{m,g} \). After some calculations and manipulations, we get

\[
E[\Lambda] = \frac{T_c^2 E[|d_l|^2]}{4\pi^2 N_o} \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{g=1}^{G} \sum_{l=1}^{M} \frac{1 - \cos(2\pi \Delta f_d T_c)}{(\Delta f_d + f_{l,g} - f_{m,g} + \Delta f_d) T_c},
\]

where \( \Delta f_d T_c \) is the normalized frequency offset error with respect to the subcarrier spacing.

Fig. 3 shows the normalized mean value of the likelihood function versus the normalized frequency offset error \( \Delta f_d T_c \) with 128 subcarriers. It can be seen from the figure that the likelihood function reaches a maximum value for \( \Delta f_d T_c = 0 \) and that the acquisition range is between -0.5 and 0.5 of the subcarrier spacing. The optimum value of \( \tilde{f}_d \) is therefore the one that maximizes the likelihood function. To find the maximum of this function, a non linear algorithm is employed as discussed in the next section.

IV. ESTIMATION AND CORRECTION OF THE FREQUENCY OFFSET

Due to the characteristic of the likelihood function (local convexity of the function with respect to the normalized frequency offset error), we propose to use the gradient method to maximize the likelihood function. The gradient method is chosen because it is easy to implement and it requires low computational complexity.

Averaging on data and fading parameters and then taking the derivative of the likelihood function in (15) with respect to \( \tilde{f}_d \) to obtain the ML estimation, we get

\[
\text{Re}\{q_{m,g}\} = \sum_{g=1}^{G} \sum_{l=1}^{M} d_{l,g} \sum_{l=1}^{M} e^{i(2\pi (f_{l,g} - f_{m,g} + \Delta f_d) t + \phi_{l,g} - \phi_{m,g}) dt},
\]

\[
\Delta f_d = f_d - \tilde{f}_d.
\]

Therefore,

\[
\text{Re}\{q_{m,g}\} = \sum_{g=1}^{G} \sum_{m=1}^{M} d_{m,g} \times \text{Re}\{q_{m,g}\}.
\]

where,

\[
q_{m,g} = \int_{0}^{T_c} r(t) e^{-i(2\pi f_{m,g} t + \phi_{m,g})} e^{-i(2\pi f_d t)} dt
\]

\[
= \sum_{g=1}^{G} \sum_{l=1}^{M} \sum_{l=1}^{M} d_{l,g} \sum_{l=1}^{M} e^{i(2\pi (f_{l,g} - f_{m,g} + \Delta f_d) t + \phi_{l,g} - \phi_{m,g}) dt},
\]

and,

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\Delta f_d = f_d - \tilde{f}_d.
\]

Therefore,

\[
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\]

We remove the data dependence by averaging the likelihood function over all the values of the data and fading parameters \( d_{m,g} \). After some calculations and manipulations, we get

\[
E[\Lambda] = \frac{T_c^2 E[|d_l|^2]}{4\pi^2 N_o} \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{g=1}^{G} \sum_{l=1}^{M} \frac{1 - \cos(2\pi \Delta f_d T_c)}{(\Delta f_d + f_{l,g} - f_{m,g} + \Delta f_d) T_c},
\]

where \( \Delta f_d T_c \) is the normalized frequency offset error with respect to the subcarrier spacing.

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\[
\text{Re}\{q_{m,g}\} = \sum_{g=1}^{G} \sum_{m=1}^{M} d_{m,g} \times \text{Re}\{q_{m,g}\}.
\]
For the estimation of the CFO, we use the gradient method and hence,

\[
\frac{\partial \bar{\Lambda}}{\partial f_d} = \frac{2CG}{N_2} \sum_{g=1}^{G} \sum_{m=1}^{M} \Re\{q_{m,g}\} \times p_{m,g},
\]

where,

\[
p_{m,g} = \frac{\partial \Re\{q_{m,g}\}}{\partial f_d}.
\]

By differentiating (14), we get

\[
p_{m,g} = 2\pi \sum_{g=1}^{G} \sum_{l=1}^{M} d_{l,g} \times \int_{0}^{T_r} t \sin(2\pi(f_{l,g} - f_{m,g} + \Delta f_d)t + \phi_{l,g} - \phi_{m,g}) dt
\]

\[
= 2\pi \sum_{g=1}^{G} \sum_{l=1}^{M} d_{l,g} \left[\frac{-T_r \cos(2\pi f_{l,g}T_r + \phi_{l,g} - \phi_{m,g})}{2\pi(f_{l,g} - f_{m,g} + \Delta f_d)}
\right]
\]

\[
+ \frac{\sin(2\pi f_{l,g}T_r + \phi_{l,g} - \phi_{m,g}) - \sin(\phi_{l,g} - \phi_{m,g})}{(2\pi f_{l,g} - f_{m,g} + \Delta f_d)^2}.
\]

For the estimation of the CFO, we use the gradient method and hence,

\[
\tilde{f}_{d_{r+1}} = \tilde{f}_{d_{r}} + k\epsilon_{r},
\]

where \(k\) is a positive constant, \(\tilde{f}_{d_{r}}\) and \(\tilde{f}_{d_{r+1}}\) are the estimates of \(f_d\) at the \(r^{th}\) and \((r+1)^{th}\) iteration respectively and \(\epsilon_{r}\) is given by the following expression

\[
\epsilon_{r} = \sum_{g=1}^{G} \sum_{m=1}^{M} \Re\{q_{m,g}\} \times p_{m,g}.
\]

V. NUMERICAL RESULTS

In this section, we consider a VSF-OFCDM system with 128 subcarriers. Each of the configurations uses a total spreading factor \((SF = SF_{time} \times SF_{freq})\) of 32 to provide a suitable performance comparison. The user data rate is identical in each analysis. We first investigate the convergence speed and sensitivity of the gradient method with different spreading factors with 16 users simultaneously using the channel. Different spreading factors can be used to provide different levels of frequency diversity, or to minimize MAI in the event of high channel loads [10]. In Figs. 4(a) and 4(b), the number of iterations are plotted versus the square of the normalized CFO error. In Fig. 4(a), we use a total \(SF\) of 2x16 which represents a case with higher \(SF_{freq}\) with respect to \(SF_{time}\) with different values of the constant \(k\). The same is done in Fig. 4(b) but with a total \(SF\) of 16x2 which represents a case with higher \(SF_{time}\) and the constant \(k\) takes different values. From Figs. 4(a) and 4(b), we notice that with different values of the constant \(k\), the algorithm converges to the same floor of the normalized frequency offset error \((\Delta f_d T_r)\) which is around \(10^{-8}\) but at different convergence speeds. Also, different constants will result in different error variances after reaching the error floor. This is because different values of \(k\) result in different step sizes of converging to the optimal value (when there is no frequency offset) which affects the rapidity of convergence. Smaller values of \(k\) will result in smaller error variance but the algorithm will require more iterations to converge to the optimal value. Similar figures can be obtained.

![Fig. 3. Average of the likelihood function vs. the normalized CFO error.](image)

![Fig. 4. CFO correction vs. number of iterations for different \(k\)](image)
for the different spreading factors for different values of $k$. From Figs. 4(a) and 4(b), it is clear that different $SF_{time}$ and $SF_{freq}$ will require different step sizes to converge at the same speed. Figs. 5(a) and 5(b) show the BER vs. the number of users with 128 subcarriers and initial normalized frequency offset (with respect to the subcarrier spacing) of 0.3 for different number of iterations with total $SF$ of 2x16 and 16x2 respectively. The BER is calculated using (4)-(6) together with the results of the normalized CFO error after a fixed number of iterations obtained from Figs. 4(a) and 4(b), for a total $SF$ of 2x16 and 16x2 respectively. The value of the constant $k$ is appropriately adjusted such that after the same number of iterations, the frequency offsets converge to the same value in all the cases. It was shown in [7] that, when the total spreading factor is fixed to 32, the OFCDM system with higher $SF_{freq}$ is more sensitive to frequency offset than that with lower $SF_{freq}$. This effect can be seen in Figs. 5(a) and 5(b), where lower $SF_{freq}$ converges faster to the optimal value. The figures show that within only 20 iterations, we are close to the optimal value when the normalized frequency offset can be minimized to the case with negligible offset. This significantly improves the BER performance for different number of users and different spreading factors.

VI. Conclusions

VSF-OFCDM system suffers from performance degradation even with small values of frequency offsets. The BER performance of VSF-OFCDM can be improved significantly when the effect of frequency offset is mitigated. In this paper, we presented a maximum likelihood estimation of the CFO in a VSF-OFCDM system for the case of low SNR. Our results show that the likelihood function reaches a maximum value when the normalized frequency offset is zero. Using the characteristics of the likelihood function and the ML principle, a gradient algorithm was used to minimize the CFO for different spreading factors. Numerical results show that it is possible to estimate and correct the frequency offset with a normalized residual error of lower than $10^{-8}$ for different spreading factors. Different spreading factors in time and frequency domain require different step sizes to converge at the same speed. The results also show that the BER of the VSF-OFCDM system can be improved significantly for different spreading factors after only a few number of iterations.

REFERENCES