A Fair Subcarrier Allocation Algorithm for Cooperative Multiuser OFDM Systems with Grouped Users

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Abstract—Dynamic resource allocation improves the performance of multiuser OFDM systems by exploiting multiuser diversity. Cooperative diversity is a technique where multiple users share their resources to realize a spatial diversity gain through cooperation. In this paper, the problem of downlink subcarrier allocation in a cooperative multiuser system is investigated. We assume a single-cell case where the base station has perfect knowledge of subchannel gains and all the mobile users are paired in cooperative groups. The mobile users in one cooperative group relay their partner data stream which is received from base station using a time division protocol. Based on the capacity contribution from the relaying terminal, a new parameter called cooperation coefficient is introduced. Considering the cooperation among users in assigning the subcarriers, a new subcarrier allocation algorithm is proposed. The performance of the proposed algorithm is then evaluated for different values of cooperation coefficients and is shown to maintain the same level of fairness but higher data rates compared with a similar algorithm which does not consider cooperation.

Index Terms: Cooperative wireless network, subcarrier allocation, orthogonal frequency division multiple access, fairness.

I. INTRODUCTION

The random fading nature of wireless channel has drawn many researchers to propose new techniques to increase the diversity order of the wireless system. Increasing the diversity order of the system can be achieved by transmitting the signal over independent fading paths. Appropriate combining of the signals received from these different paths at the receiver realizes diversity gain and leads to a more reliable system. One of the diversity techniques, known as multiuser diversity, benefits from the independence of fading process among multiple users in the system. In a multicarrier system, the total bandwidth is divided into $N$ subchannels and a portion of these subchannels can dynamically be allocated to each user. It is quite unlikely that a subchannel is in deep fade for all the users in the system simultaneously. Therefore, the subchannels can be assigned dynamically to the users based on their instant subchannel gains to benefit from diversity and, hence to provide robust performance.

Many dynamic resource allocation algorithms and optimization techniques have been proposed in the literature for the downlink of a non-cooperative multiuser OFDM system. The ultimate goal of all these algorithms is either to achieve the highest possible throughput with constraints on total transmit power (rate adaptive) [1], [2] or to achieve the minimum total transmit power with the data rates as constrains (power adaptive) [3]. There is also a third category of rate adaptive dynamic resource allocation algorithms [4], [5], which are developed to support variable bit rate services with fairness in the system. In this category, the objective is to maximize the total throughput within the power budget with the goal of maintaining the proportionality between the users’ data rates according to proportional constraints rather than reaching a specific requested data rate. A subcarrier allocation algorithm based on the sensitivity of the users was recently proposed in [6] and shown to achieve performance improvement.

Cooperative diversity is another diversity technique which uses other relaying terminals to achieve diversity gain [7], [8]. Using nearby mobile users for relaying was proposed in [9], [10] for total throughput enhancement. The main idea is that, after selecting a partner from the in-cell mobile users, each user detects a faded and noisy version of the partner’s transmitted signal and combines this information with its own data to construct its transmitted signal. Different relaying schemes have been proposed to increase the bandwidth efficiency of relaying protocols. Based on Alamouti space-time codes, a distributed space-time coding technique was proposed for a system with one or two assisting relays [11], [12]. It was shown that this design achieves a diversity gain over a point-to-point system with the same bandwidth. Based on the transmission protocol in [7], linear dispersion codes were applied to a wireless system with $R$ relays in [13]. It was theoretically shown that for high SNR regime the system with $R$ relays has the same diversity as a multiple-antenna system with $R$ antennae.

Assuming that there is an efficient cooperation scheme in the physical layer, we can move forward to other issues in the cooperative network and investigate the effect of cooperation. Cooperative resource allocation is an interesting idea to be explored. How to assign the subcarriers to cooperative users is the focus of this paper. We formulate the the problem of subcarrier allocation in downlink of a cooperative wireless network based on a TDMA-based relaying protocol [14]. We assume that all the users in the system are paired to cooperate. Each user in a group relays its partner’s signal based on an amplify-and-forward relaying scheme in different timeslots. The focus of this paper is not the cooperation protocol itself, rather to observe the effect of cooperation on subcarrier allocation in multiuser OFDM systems. One of the contributions of this paper is the introduction of a new parameter called cooperation coefficient which quantifies the cooperation level among users in a group. Based on the capacity of the cooperation protocol, a fair subcarrier allocation algorithm is proposed and the performance of this algorithm is compared for different cooperation coefficients with Max-Min algorithm [4] first proposed for non-cooperative wireless networks and modified appropriately for cooperative networks.
The rest of the paper is organized as follows: In section II, system model is introduced and the related analysis for capacity of each user is presented. The subcarrier allocation optimization problem for a cooperative network is discussed in section III. The modified algorithm for cooperative network and the proposed algorithm are explained next in section IV. Simulation results are presented in section V. The paper is concluded in section VI.

II. COOPERATIVE WIRELESS SYSTEM MODEL AND ANALYSIS

The problem of resource allocation in a cooperative multi-user OFDM system with $N$ subcarriers is considered. Total bandwidth of the system is $B$. Therefore, each subcarrier has bandwidth of $\frac{B}{N}$. The wireless channel is modeled as frequency selective Rayleigh fading. The bandwidth of each subchannel is much smaller than the coherence bandwidth of the channel and the channel status does not change within a block of transmission; therefore, flat fading channel is assumed for all subchannels. Additive white Gaussian noise (AWGN) is present with single-sided noise power spectral density (PSD) level of $N_0$ for all subcarriers and all users. For simplicity of analysis, we assume a flat power allocation. The total transmit power at the base station is assumed to be $P$ over the whole bandwidth; therefore, transmit power over each subcarrier is $\frac{P}{N}$. There are $G$ groups of cooperative users in the cell. In each group there are $K$ users cooperating according to the cooperation protocol. In the following, we describe the system model for $K=2$ and derive the capacity of the system.

A. Relaying-Based System Model

The cooperation protocol is explained for a single-group consisting of two users cooperating in a single-cell system. We intend to find the capacity of each user.

![Fig. 1. The cooperative system model, K=2.](image)

For simple analysis, we have chosen a TDMA-based amplify-and-forward cooperation protocol. The cooperating users and base station are shown in Fig. 1. MS$_{j,1}$ and MS$_{j,2}$ are in the $j$th group ($j = 1, 2, ..., G$) and have their own data stream to receive from base station. Transmission protocol has two phases: broadcast and cooperation. In broadcast phase (the first timeslot), base station broadcasts to MS$_{j,1}$ and MS$_{j,2}$. In cooperation phase (second and third timeslot), MS$_{j,1}$ and MS$_{j,2}$ after receiving the other user’s signal in the first timeslot, amplify and forward it in the second and third timeslot respectively. The sequence of transmission is shown in Table I.

<table>
<thead>
<tr>
<th>Timeslot</th>
<th>BS</th>
<th>MS$_{j,1}$</th>
<th>MS$_{j,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>transmits</td>
<td>listens</td>
<td>listens</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>transmits</td>
<td>listens</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>listens</td>
<td>transmits</td>
</tr>
</tbody>
</table>

It is assumed that different subcarriers are assigned to MS$_{j,1}$ and MS$_{j,2}$; i.e., none of the users are sharing subcarriers. According to Fig. 1, the $n$th subcarrier is allocated to MS$_{j,1}$ and the $n$th subcarrier to MS$_{j,2}$. In the first timeslot, each user’s data stream is transmitted over the allocated subcarrier. In the second and third timeslots, users relay their partners signal on the same subcarrier that they have received their partner’s signal from. It can also be assumed that both users relay their partner’s signal on the same subcarrier.

The channel gains between BS and MS$_{j,1}$, MS$_{j,2}$ are shown in Fig. 1. $h_{j,k,n}$ is the channel gain between BS and the $k$th user of $j$th cooperative group. Therefore, $h_{j,1,m}$ and $h_{j,2,n}$ are BS-MS$_{j,1}$ and BS-MS$_{j,2}$ subcarrier gains allocated to MS$_{j,1}$ and MS$_{j,2}$ respectively. $h_{j,1,n}$ and $h_{j,2,m}$ are the channel gains for BS-MS$_{j,1}$ and BS-MS$_{j,2}$ when users are listening to their partner’s subcarrier respectively. In the second and third timeslots, the received signal from the other user in the first timeslot is amplified and forwarded over the same subcarrier that users have been listening to. The inter-user channel is denoted as $h_{r_{j,1,m}}$ and $h_{r_{j,2,n}}$, where the former is the subcarrier gain from MS$_{j,1}$ to MS$_{j,2}$ and the latter is the subcarrier gain from MS$_{j,2}$ to MS$_{j,1}$. Each user uses a maximal ratio combining scheme to combine two signals received from direct and indirect paths.

B. Data Rate Analysis and Cooperation Coefficient

The received signal at MS$_{j,1}$ and MS$_{j,2}$ in the first timeslot can be written as:

$$y_{j,1,m} = \frac{PT}{N} h_{j,1,m} x_{j,1} + n_{j,1,m}$$

$$y_{j,1,n} = \frac{PT}{N} h_{j,1,n} x_{j,2} + n_{j,1,n}$$

$$y_{j,2,n} = \frac{PT}{N} h_{j,2,n} x_{j,2} + n_{j,2,n}$$

$$y_{j,2,m} = \frac{PT}{N} h_{j,2,m} x_{j,1} + n_{j,2,m}$$

TABLE I

| Transmission protocol for a cooperative group with two cooperative users. |
|------------------|------------------|------------------|
| Timeslot 1 | BS | MS$_{j,1}$ | MS$_{j,2}$ |
| 1 | transmits | listens | listens |
| 2 | - | transmits | listens |
| 3 | - | listens | transmits |
where \( y_{j,k,m} \) is the received signal at MS\(_{j,k} \) \((k = 1, 2)\) on the \(m\)th subcarrier. \( n_{j,k,m} \) is AWGN added to the received signal at MS\(_{j,k} \) on the \(m\)th subcarrier. \( \frac{PT}{N} \) is the transmit energy of MS\(_{j,k} \) over one symbol period, \( T \), and \( x_{j,k} \) is the data stream of MS\(_{j,k} \) of the \(j\)th group.

Each user MS\(_{j,k} \) normalizes the received signal of the other user and retransmits it in the second and third timeslot with the same energy of \( \frac{PT}{N} \) to its partner. The received signal, \( y_{r,j,k,m} \), in the second and third timeslots at MS\(_{j,k} \) is then:

\[
y_{r,j,1,m} = \sqrt{\frac{PT}{N}} h_{r,j,2,m} \frac{y_{j,2,m}}{\sqrt{E\{ |y_{j,2,m}|^2 \}}} + n_{r,j,1,m} \tag{3}
\]

\[
y_{r,j,2,n} = \sqrt{\frac{PT}{N}} h_{r,j,1,n} \frac{y_{j,1,n}}{\sqrt{E\{ |y_{j,1,n}|^2 \}}} + n_{r,j,2,n} \tag{4}
\]

where \( n_{j,k,m} \) is the additive noise at MS\(_{j,k} \) on the \(m\)th subcarrier, \( E\{ |y_{j,k,m}|^2 \} \) is the average energy of the received signal at MS\(_{j,k} \) in the first timeslot.

The maximum error-free data rate of MS\(_{j,1} \) and MS\(_{j,2} \), \( r_{j,1,m} \) and \( r_{j,2,n} \), can be written as [14]:

\[
r_{j,1,m} = \frac{B}{3N} \log_2 \left( 1 + \frac{P}{N_0 B} \left( |h_{j,1,m}|^2 + \frac{|h_{r,j,2,m}|^2 |h_{j,2,m}|^2}{1 + \frac{P}{N_0 B} + |h_{r,j,2,m}|^2} \right) \right) \tag{5}
\]

\[
r_{j,2,n} = \frac{B}{3N} \log_2 \left( 1 + \frac{P}{N_0 B} \left( |h_{j,2,n}|^2 + \frac{|h_{r,j,1,n}|^2 |h_{j,1,n}|^2}{1 + \frac{P}{N_0 B} + |h_{r,j,1,n}|^2} \right) \right) \tag{6}
\]

We notice that the data rates are functions of the direct paths and indirect paths. Therefore, \( r_{j,1,m} \) and \( r_{j,2,n} \) can be rewritten as:

\[
r_{j,1,m} = \frac{B}{3N} \log_2 \left( 1 + \frac{P}{N_0 B} \left( |h_{j,1,m}|^2 + a_{j,2,m} |h_{j,2,m}|^2 \right) \right) \tag{7}
\]

\[
r_{j,2,n} = \frac{B}{3N} \log_2 \left( 1 + \frac{P}{N_0 B} \left( |h_{j,2,n}|^2 + a_{j,1,n} |h_{j,1,n}|^2 \right) \right) \tag{8}
\]

where \( a_{j,k,m} \) is the contribution level from MS\(_{j,k} \) for the data rate of the user to which the \(m\)th subcarrier is allocated. We call this parameter cooperation coefficient. Therefore, \( a_{j,1,n} \) and \( a_{j,2,n} \) are:

\[
a_{j,1,n} = \frac{|h_{r,j,1,n}|^2}{1 + \frac{P}{N_0 B} + |h_{r,j,1,n}|^2} \tag{9}
\]

\[
a_{j,2,n} = \frac{|h_{r,j,2,n}|^2}{1 + \frac{P}{N_0 B} + |h_{r,j,2,n}|^2} \tag{10}
\]

It should be noted that the cooperation coefficient in an amplify-and-forward protocol can not exceed one; i.e., the impact of cooperation in the capacity from the indirect path is always less than the direct path. Also, if MS\(_{j,1} \) and MS\(_{j,2} \) use the same channel for relaying, cooperation coefficient would be the same for both users. We could also use any other amplify-forward cooperation protocol, but the final data rate equations would be similar to (7) and (8).

### III. Subcarrier Allocation Optimization Problem

The problem of resource allocation in a cooperative multi-user OFDM system with \( N \) subcarriers and \( G \) groups of \( K \) cooperative users is to determine the elements of subcarrier allocation matrix: \( C = \{c_{j,k,n}\} \times K \times N \) specifying which subcarrier should be assigned to which user of which group. \( c_{j,k,n} = 1 \), if and only if subcarrier \( n \) is allocated to user \( k \) in group \( j \); otherwise it is zero. None of the users shares subcarrier, so in case \( c_{j,k,n} = 1 \) then \( c_{i,l,n} = 0 \) for all \( i \neq j \) and \( l \neq k \).

We investigate the optimization problem for the special case of \( K = 2 \). According to the derivation presented earlier, the total data rate of the two users in the \(j\)th group, \( R_{j,1} \) and \( R_{j,2} \), after subcarrier allocation can be written as:

\[
R_{j,1} = \sum_{n=1}^{N} c_{j,1,n} r_{j,1,n} \tag{11}
\]

\[
R_{j,2} = \sum_{n=1}^{N} c_{j,2,n} r_{j,2,n} \tag{12}
\]

The optimization problem in a cooperative network with equal rate constraints and flat power allocation can then be formulated as:

\[
\max_{\{c_{j,k,n}\}} \sum_{j=1}^{G} (R_{j,1} + R_{j,2}) \tag{13}
\]

Subject to:

- \( C1 : c_{j,k,n} \in \{0, 1\}, \forall j, k, n \)
- \( C2 : \sum_{j=1}^{G} \sum_{k=1}^{2} c_{j,k,n} = 1, \forall n \)
- \( C3 : R_{1,1} = R_{1,2} = \ldots = R_{G,1} = R_{G,2} \)

Constraints \( C1 \) and \( C2 \) state that subcarrier sharing is not considered and \( C3 \) means equal data rate requirement for all users.

### IV. Subcarrier Allocation Algorithms

Based on the theorem proved in [1], the data rate of a multiuser OFDM system is maximized when each subcarrier is assigned to only one user that has the best channel gain for that subcarrier. The results shown in [1] and [4] indicate that a flat transmit PSD would hardly reduce the total data rate of a multiuser OFDM system provided that the proper subcarrier allocation is applied. In this paper, the focus is mainly on how to fairly allocate the subcarriers in the cooperative system to increase the total capacity. We have selected Max-Min algorithm from the literature [4] and modified it for the cooperative network. We propose our algorithm by defining a new objective parameter in assigning the subchannels to the users. Both the modified and proposed algorithms are described in the next section. These algorithms are described for \( K = 2 \) and their performance are compared. The proposed algorithm can be easily generalized to larger group sizes as well.
A. Modified Max-Min Algorithm

The subcarrier allocation algorithm proposed in [4] is modified for a cooperative network. At the first step, one subcarrier is allocated to each user where the channel gain is the highest for that user. In the next step, the user with the lowest data rate is prioritized for subcarrier allocation. The procedure continues until all the subcarriers are allocated. As the algorithm tries to maximize the data rate of the user with minimum data rate at each step, it is also called Max-Min subcarrier allocation algorithm. We refer to this algorithm as “modified algorithm” throughout this paper. The modified algorithm for cooperative network can be summarized as:

- Initialization
  \[ c_{j,k,n} = 0, \forall j, k, n \]
  \[ R_{j,k} = 0, \forall j, k \]
  \[ A = \{1, 2, ..., N\} \]

- Subcarrier Allocation
  - for \( k = 1 \) to \( 2 \) and \( j = 1 \) to \( G \)
    (a) find \( n \) satisfying \( |h_{j,k,n}| \geq |h_{j,k,m}| \) \( \forall m \in A \)
    \[ c_{j,k,n} = 1 \]
    (b) update \( R_{j,k} \) with \( R_{j,k} = \sum_{n=1}^{N} c_{j,k,n} r_{j,k,n} \)
    \[ A = A - \{n\} \]
  - while \( A \neq \emptyset \)
    (a) find \( j \) and \( k \) satisfying \( R_{j,k} \leq R_{i,l} \)
    \( \forall i = 1, 2, ..., G \) and \( l = 1, 2 \)
    (b) for the found \( j \) and \( k \), find \( n \) satisfying
      \( |h_{j,k,n}| \geq |h_{j,k,m}| \) \( \forall m \in A \)
      \[ c_{j,k,n} = 1 \]
    (c) update \( R_{j,k} \) and \( A \) with \( j, k, n \)
      \[ R_{j,k} = \sum_{n=1}^{N} c_{j,k,n} r_{j,k,n} \]
      \[ A = A - \{n\} \]
  - End

B. The Proposed Algorithm

Based on the modified algorithm, we propose an algorithm which allocates subcarriers to users considering the cooperation between them. By looking at the capacity equation (7) and (8), it is seen that the data rate is not only a function of user’s channel gain but also depends on user’s partner channel gain. Therefore, the objective parameter in assigning the subcarriers should include the partner’s channel gain in assigning the subcarriers. We choose the objective parameter \( |h_{j,1,n}|^2 + a_{j,1,n} |h_{j,2,n}|^2 \) for user 1 and \( |h_{j,2,n}|^2 + a_{j,1,n} |h_{j,1,n}|^2 \) for user 2 of the \( j \)th group respectively. The proposed algorithm is as follows:

- Initialization
  \[ c_{j,k,n} = 0, \forall j, k, n \]
  \[ R_{j,k} = 0, \forall j, k \]
  \[ A = \{1, 2, ..., N\} \]

- Subcarrier Allocation
  - for \( k = 1 \) to \( 2 \) and \( j = 1 \) to \( G \)
    (a) find \( n \) satisfying
      \( \text{obj}_{j,k,n} \geq \text{obj}_{j,k,m} \) \( \forall m \in A \)
      \[ c_{j,k,n} = 1 \]
    (b) update \( R_{j,k} \) with \( R_{j,k} = \sum_{n=1}^{N} c_{j,k,n} r_{j,k,n} \)
      \[ A = A - \{n\} \]
  - while \( A \neq \emptyset \)
    (a) find \( j \) and \( k \) satisfying \( R_{j,k} \leq R_{i,l} \)
      \( \forall i = 1, 2, ..., G \) and \( l = 1, 2 \)
    (b) for the found \( j \) and \( k \), find \( n \) satisfying
      \( \text{obj}_{j,k,n} \geq \text{obj}_{j,k,m} \) \( \forall m \in A \)
      \[ c_{j,k,n} = 1 \]
    (c) update \( R_{j,k} \) and \( A \) with \( j, k, n \)
      \[ R_{j,k} = \sum_{n=1}^{N} c_{j,k,n} r_{j,k,n} \]
      \[ A = A - \{n\} \]
  - End

C. Fairness

Fairness is an important issue in the system which indicates how equally the resources are distributed among the users and could be defined in terms of different parameters of the system. It can be defined in terms of bandwidth or power allocated to different users in the system. In this paper, we define the fairness index in terms of data rates. Assuming equal data rate requirements among the users, fairness index, \( F_p \) is defined as [15]:

\[
F_p = \left( \frac{\sum_{j=1}^{G} \sum_{k=1}^{2} R_{j,k}}{2G \sum_{j=1}^{G} \sum_{k=1}^{2} (R_{j,k})^2} \right)^{1/2},
\]

\( F_p \) is a real number in the interval \((0, 1]\) with the maximum value of one for the case that the achieved rate is equal for all the users.

V. PERFORMANCE EVALUATION: CAPACITY AND FAIRNESS

In this section, the performance of the proposed algorithm is compared with the modified Max-Min algorithm in terms of capacity and fairness.

A. Simulation Parameters

The channel is modeled as frequency selective with six independent multipaths and exponential profile. The total bandwidth of the system is \( B = 1\)MHz and the total transmit power is \( P = 1\)W in downlink. It is assumed that there are \( N = 128\) subcarriers to be allocated to the users cooperating in a group with \( K = 2\). Base station has perfect knowledge
of all subchannel gains. Equal cooperation coefficients are assumed among the users of different groups which implies equal channel gains for the relay channel of MS_{j,1} and MS_{j,2}. The three different values chosen for cooperation coefficient are \( a = 0.1, 0.5, 0.9 \). Total of 10000 channel realizations were used and the results were averaged. The data rates of users are assumed to be equal to the capacity of the subcarriers allocated to them. The performance of the modified algorithm is compared with the proposed algorithm in terms of total capacity and fairness for different values of cooperation coefficients.

**B. Simulation Results**

Fig. 2 compares the total data rate of the proposed and modified algorithm versus the average signal to noise ratio (SNR) for different values of cooperation coefficients. The number of groups is \( G = 8 \). As observed, the total data rate of the proposed algorithm is higher than the modified non-cooperative algorithm. Also, as the cooperation coefficient increases, the difference between the throughput of both systems increases almost linearly as well.

![Fig. 2. Total capacity versus average SNR for different cooperation coefficients. \( N = 128, G = 8 \)](image)

The fairness and total data rate of the system versus different number of users is shown in Figs. 3-5 for different cooperation coefficients. It is observed that for larger cooperation coefficients, performance of the cooperative algorithm improves significantly. The fairness index of both algorithms is almost one for different number of users in the system.

It should be noted that the performance improvement obtained in the proposed algorithm comes directly from the way we defined the objective parameter in assigning the subcarriers to different users. In other words, the proposed algorithm considers cooperation in subcarrier allocation which results in performance improvement in the system. To verify this, we have also compared the fairness index of both algorithms for different number of users and various cooperation coefficients.

As seen in Figs. 3-5, fairness of both algorithms are almost equal which means that we have not lost fairness in the system for increasing the total throughput of the system using the proposed subcarrier allocation algorithm. Also, for different number of users, the total capacity increases for higher cooperation coefficients.

**VI. CONCLUSIONS AND FUTURE TRENDS**

In this paper, the problem of subcarrier allocation in downlink of an OFDM cooperative system was presented and formulated. Simulation results indicated that by simply modifying the objective parameter in Max-Min subcarrier allocation algorithm, it is possible to achieve performance improvement in the cooperative wireless system. Compared to the non-cooperative subcarrier allocation algorithm, the proposed algorithm achieves higher data rates without losing fairness in the system. This work can be extended to other cooperation protocols and for more number of cooperating users in a group. Proportional data rates can be considered and the performance of the algorithm can be evaluated. Other algorithms proposed for non-cooperative system can also be modified and applied to the proposed model and their performance can be investigated for future work.

**REFERENCES**

Fig. 3. Throughput and fairness versus number of users for $a = 0.1$, $N = 128$, SNR = 30dB.

Fig. 4. Throughput and fairness versus number of users for $a = 0.5$, $N = 128$, SNR = 30dB.

Fig. 5. Throughput and fairness versus number of users for $a = 0.9$, $N = 128$, SNR = 30dB.