Communication via Quantum Neural Networks

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Abstract—In this paper, a quantum teleportation protocol based on quantum neural networks is presented. We studied the relation between the network's weight and the fidelity of transferring information among neurons. It has been found that as the network's weight increases the accuracy of the transformed information increases. Preliminary results of a practical example are given in this paper.

Keywords: Artificial Neural Networks, Quantum Computing, Quantum Neural Networks, Quantum Teleportation

I. INTRODUCTION

Quantum information, QI is one of the most important topics which has considerable progress theoretically and experimentally in recent years [1][2]. To handle QI tasks such as sending, coding, etc., one needs entangled quantum channels. There are several efforts have been done to generate entangled channels demonstrated in different types of systems, like photons [3][4], ions [5], atoms [6] and charged qubit [7]. All these types of quantum channels have been used to perform different QI tasks, such as; quantum teleportation, dense coding and cryptography [8][9][10].

One of the most promising types of quantum channels is the artificial neural networks (ANN) [11][12]. ANN can compute any computable functions; therefore, it is classified as neuro-computer. It can be viewed as parallel computational models made up of densely interconnected adaptive processing units, usually called neurons [13].

Kak [14] and Perus [15] drew the attention to the similarity between the description of neuron states and that of quantum mechanical states and discussed a notion of quantum neural computing in accordance with quantum theory. In other words, the basic components of neural cytoskeleton are very likely to possess quantum mechanical due to their size and structure [16]. For example, the tubuline protein has the ability to flip from one conformation to another, which acts as two basis states of the system. Also the system can lie in a superposition of these two states, which gives a plausible mechanism for creating a coherent state in the brain [17]. This yields which known as quantum neural network (QNN) as a type of ANNs that study the human brains [18-21]. There are different models of QNN have been investigated. For example, kouda et al. [22] and Mori et al. [23] have proposed a model that is a multilayered neural network and investigated its characteristic features, such as the effect of quantum superposition and probabilistic interpretation. A model of recurrent quantum neural network has been introduced by Behera et al. [24], where they used a linear neural circuit to set up the potential field in which the quantum brain is dynamically excited. A novel learning method has been proposed for quantum neural network by Kinjo et al. [25]. Shafee [26][27] has suggested another model by replacing the classical neurons by quantum bits (qbit) and quantum operators in place of the classical action potentials observed in biological contexts. Panchi and Shiyong [28] constructed a quantum neuron model based on the universality of single qubit rotation gate and two-qubit controlled-Not gate. The QNN models can be used in a wide range of pattern recognition applications (e.g. [24][29][30]). It is well known that, entanglement provides miracle performances in the transmission of quantum information, where it enables the transmission of quantum state by sending only classical information [8]. Recently, Hayashi [31] investigated the effect of the prior entanglement on quantum network coding. This motivated us to investigate the effect of non-maximal entangled neural networks on quantum teleportation. Namely, we consider a partially entangled QNN where each neuron entangled with its neighbor. This entangled network is used to send information from one neuron to another by means of teleportation.

As a practical application, we used the proposed protocol to investigate the ability of controlling the spread of swine flu virus by controlling the QNN's parameters. It has been showed here that the weight of the QNN and the degree of infection play an important role on transferring infection among different age groups, where as the weight of the QNN increases the accuracy of the transformed infection increases. This paper is organized as follows: the properties of QNN and the proposed teleportation protocol are provided in section 2. The given application and results are provided in section 3. Section 4 concludes the paper.

II. THE PROPOSED MODEL DESCRIPTION

A. Description of QNN

A neuron is defined as a two logical state of usual bit “Yes, No” or “False, true” or “of, on” and so on. Also, one can express the state of the neuron as a superposition of the false and true representation as

\[ |\psi\rangle = \alpha |n\rangle + \beta |y\rangle \] (1)

Which means that, with weight \( \alpha \) the neuron is in state \( |n\rangle \), i.e. it has “No” information (unfired), while it is in state \( |y\rangle \) with weight \( \beta \), i.e. “Yes” carries information (fired) and \( \alpha^2 + |\beta|^2 = 1 \) [26-27]. In QNN, a neuron receives \( N \) real input signals from many other neurons by network channel. Also,
the output of a neuron can “fan-out”, the number of the input signals that can be driven by a single output, to several other neurons. So, we can formulate the collective response of all neurons as:

\[ |\psi(t)\rangle = \sum_{i=0}^{N} \omega_{i} |\psi_{i}\rangle \]

(2)

where \( |\psi_{i}\rangle \) represents the states of the input neurons and \( \omega_{i} \) is the corresponding weight. The neural network processing can be implemented by an operator \( \Phi \) acts on the input states and propagates the information forward to calculate the output state of the neuron as:

\[ |\psi(t)\rangle = \Phi \left( \sum_{i=0}^{N} \omega_{i} |\psi_{i}\rangle \right) \]

(3)

where \( \Phi \) is unknown operator that can be implemented by the gates of the neural network. On the other hand, in quantum computation the evolutionary operators must be unitary. If we consider the case where \( \Phi = I \), i.e. the identity operator, the output of quantum perceptron is given by:

\[ |\psi(t)\rangle = \sum_{i=0}^{N} \omega_{i} |\psi_{i}\rangle \]

(4)

This means that quantum perceptron can be defined as a quantum system consists of \( n \) neuron input register and in this case we have only considered quantum neural network.

To achieve quantum computation, the logical function can be performed by applying a group of unitary operations on the neuron states. These operations are called logic gates, which transform the information of the input neuron states to the output ones. The most basic universal gates is the single rotation gate, controlled not gate (cNOT), entangling gate and Hadamard gate. Since we are interested to employ the QNN into the quantum teleportation, we will describe the cNOT and Hadamard gates into our notations. The effect of the cNOT operation is defined by:

\[ \text{cNOT} |nn\rangle = |nn\rangle, \quad \text{cNOT} |ny\rangle = |ny\rangle \]

(5)

\[ \text{cNOT} |yn\rangle = |yy\rangle, \quad \text{cNOT} |yy\rangle = |yn\rangle \]

while Hadamard is,

\[ H |n\rangle = \frac{1}{\sqrt{2}} (|n\rangle + |y\rangle), \quad H |y\rangle = \frac{1}{\sqrt{2}} (|n\rangle - |y\rangle) \]

(6)

B. Quantum Teleportation

In this section, the entangled neural network is used to perform the original quantum teleportation protocol [5]. To achieve this task we assume that the neural network is built on the postulates given in [22-23]. In this consideration the neuron is represented by qubit such that the state of the neuron is described by (1). Let us assume that the neural network is partially entangled. This means that the state of each two neighboring neurons is non maximal entangled states. Assume that the state of the quantum neural networks, which consists of two connected neurons, is defined

\[ \rho = \frac{1+q}{2} |nn\rangle\langle nn| + \frac{p}{2} (|nn\rangle\langle yy| + |yy\rangle\langle nn|) + \frac{1-q}{2} |yy\rangle\langle yy|, \]

(7)

where \( q^2 + p^2 = 1 \) and \( 0 \leq p \leq 1 \) and \( 0 \leq q \leq 1 \) [32-33]. The following steps describe the implementation of the Bennett protocol [8].

1. Step one: A neuron \( B \) has got information from another neuron \( A \) (which can be order, computation, decision, etc..). The aim of neuron \( B \) is transferring these information to another neuron \( C \), which is a member in the network. Let us assume that the unknown information coded in the neuron state \( |\psi_{A}\rangle = \alpha |n\rangle + \beta |y\rangle \) and the neurons \( B \) and \( C \) share an entangled neuron state defined by \( \rho_{AC} \) (7), then the total state of the system is given by \( \rho_{S} = \rho_{A} \otimes \rho_{AC} \) where \( \rho_{A} = |\psi_{A}\rangle\langle \psi_{A}| \).

2. Step two: Neuron \( B \) performs local operations on its own state and the neuron \( \rho_{\tilde{A}} \). These local operations include the cNOT operation which is a two neurons gate as defined by (5). After this operation we have:

\[ \rho_{S}^{(1)} = (\text{cNOT} \otimes I) \rho_{S} (I \otimes \text{cNOT}) \]

(8)

where \( I \) is a unitary operator effects on the neuron \( C \). The second operation is the Hadamard operation \( \text{(6)} \) which transforms the state \( \rho_{S}^{(1)} \) into

\[ \rho_{S}^{(2)} = (\mathcal{H} \otimes I \otimes I) \rho_{S} (I \otimes I \otimes \mathcal{H}) \]

(9)

3. Step three: Alice makes measurements on the basis \( |nn\rangle, |ny\rangle, |yn\rangle \) and \( |yy\rangle \) randomly on her own neuron and the state which has got from the neuron \( A \). Alice gets a two classical bit of information and sends them to Bob.

4. Step four: As soon as Bob gets the classical data from Alice, he performs suitable rotations on his neuron to obtain the final information as shown in Table I, please see appendix.

Fig.1 below shows the fidelity of the teleported state, where we assume the case that Alice measures \( |pp\rangle \langle pp| \). From this figure, it is clear that when \( p = 0 \), then the quantum network is classically correlated and given by \( \rho = |nn\rangle\langle nn| \). As one increases the value of the parameter \( p \) on the expance of \( q \), where \( p = \sqrt{1-q^2} \), the quantum network becomes a partially entangled. The corresponding fidelity \( F \) increases as \( p \) increases. For \( p = 1 \), the network is maximally entangled and given by:

\[ \rho = \frac{1}{2} (|nn\rangle\langle yn| + |yn\rangle\langle nn|) \]
and the fidelity of the teleported state $\mathcal{F} = 1$ (maximum value). On the other hand, there is no effect on the type of the teleported state on the degree of the fidelity. So, one can teleport classical information as well as quantum information with the same efficiency. Then by controlling the devices which generate entangled network, one can send the information with high fidelity.

III. APPLICATION EXAMPLE

The prevalence of swine virus is one of the most difficult challenges facing the world nowadays. In this paper, we use the proposed QNN model, described in previous section in order to investigate the possibility of controlling the spread of the swine virus.

In the proposed treatment, we consider two categories of population with the same social and economic circumstances each of them contains different age groups. Let's assume that each one of these categories is described by a pair of information: the first part of which represents human characteristics such as young, old, rich, poor, have a medical culture and no medical culture. The second part represents information on the affecting factors such as: non-infected, infected, awareness and non-awareness has culture of health and has no culture of health. In fact, there are a lots of human and influence factors, so we consider some of them to illustrate the basic idea of this protocol.

Let us assume that the human properties are coded in the phase, while the influence factors coded in the amplitude. So, if the neuron is described by $|\psi\rangle = \alpha |y\rangle + \beta |n\rangle$, then with probability $|\alpha|^2$, the neuron represents the pair (young, non-infected), while with probability $|\beta|^2$, it represents the pair (young, infected). In this description, we set the positive phase (+) for the young and (-) for the old and the amplitude 1 for non-infected and 0 for infected. Table II, gives a complete description to some of these factors.

Assume that we have a class of human cases of pathological, described by the neuron $|\psi\rangle = \alpha |n\rangle + \beta |y\rangle$.

According to the definitions given in Table II, this state describes a young infected with probability $|\alpha|^2$ and it is non-infected with probability $|\beta|^2$. In view of the friction between the two entangled tranches of the community, the information which coded in the neuron $|\psi\rangle$, where the virus can be transmitted through it, is transformed to the other segment of the community. Also, the virus can be transmitted to another person in the same category. Since there is an environmental pollution, the transformed information interacts with this unhealthy environment and there may be a transfer of swine flu virus. For this example, we find with probability 25% the virus can be transformed as shown in Table III.

### Table II

<table>
<thead>
<tr>
<th>Human factor</th>
<th>Influence factor</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young</td>
<td>Non-infected</td>
<td>1</td>
</tr>
<tr>
<td>Old</td>
<td>Infect</td>
<td>0</td>
</tr>
<tr>
<td>Rich</td>
<td>Awareness</td>
<td>1</td>
</tr>
<tr>
<td>poor</td>
<td>Non-awareness</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table III

<table>
<thead>
<tr>
<th>Age group</th>
<th>Healthy case</th>
<th>Probability</th>
<th>Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old</td>
<td>Infected</td>
<td>$</td>
<td>\beta</td>
</tr>
<tr>
<td>Old</td>
<td>Non-infected</td>
<td>$</td>
<td>\alpha</td>
</tr>
<tr>
<td>Young</td>
<td>Infected</td>
<td>$</td>
<td>\alpha</td>
</tr>
<tr>
<td>Old</td>
<td>Infected</td>
<td>$</td>
<td>\beta</td>
</tr>
<tr>
<td>Young</td>
<td>Non-infected</td>
<td>$</td>
<td>\alpha</td>
</tr>
</tbody>
</table>

The values of $A_i$ and $B_i$, where $i = 1, 2, 3, 4$, are given by:

\[
\begin{align*}
A_1 &= \frac{p\sqrt{1-\alpha^2}}{\sqrt{(1-q)(1-\alpha^2) + p^2(1-\alpha^2)}}, \\
B_1 &= \frac{(1-q)\alpha}{\sqrt{(1-q)(1-\alpha^2) + p^2(1-\alpha^2)}}, \\
A_2 &= \frac{p\alpha\sqrt{1-\alpha^2}}{\sqrt{(1+q)(1-\alpha^2) + p^2(1-\alpha^2)}}, \\
B_2 &= \frac{(1+q)(1-\alpha^2)}{\sqrt{(1+q)(1-\alpha^2) + p^2(1-\alpha^2)}}, \\
A_3 &= \frac{(1+q)\alpha}{\sqrt{(1+q)(1-\alpha^2) + p^2(1-\alpha^2)}}, \\
B_3 &= \frac{p\sqrt{1-\alpha^2}}{\sqrt{(1+q)(1-\alpha^2) + p^2(1-\alpha^2)}}, \\
A_4 &= \frac{p\alpha\sqrt{1-\alpha^2}}{\sqrt{(1+q)(1-\alpha^2) + p^2(1-\alpha^2)}}, \\
B_4 &= \frac{(1+q)\alpha}{\sqrt{(1+q)(1-\alpha^2) + p^2(1-\alpha^2)}}.
\end{align*}
\]
Table III describes the output as a result of two types of measurements that can be done by the person who receives the transmitted information. In the upper part of Table III, we assume that the recipient measured the teleported state in the basis $|yy\rangle\langle yy|$. As a result of these measurements there are four possibilities: the first is infected old, the second is non-infected old, the third is infected young and the fourth is non-infected young. The bottom part of Table III represents the output of the second type of measurements, where the receiver measures the teleported state in the basis $|yn\rangle\langle yn|$. Also, one gets similar possibilities with different probabilities.

It is clear that, both the probabilities $|A|^2$ and $|B|^2$ for $i = 1, 2, 3, 4$ depend on the strength of the network ($p=\sqrt{1-q^2}$) and the initial state setting which is described by the parameters $a$ and $b$. On the other hand, the fidelity of transmitted information increases as the channel strength $p$ increases, this is clear from Fig. 1. Therefore, if we are able to control the laboratory devices to generate entangled channel to minimize the fidelity, one can reduce the rate of infections. Also, from theoretical point of view, if the receiver makes some kind of rotations the rate of infections between young is reduced.

IV. CONCLUSION

In this paper we employed the quantum neural network to achieve the quantum teleportation protocol. In our treatment we assumed that the neurons are connected through imperfect network. The sensitivity of the fidelity of the transmitted information is investigated for the network's parameter as well as for the structure of the input information. For large values of the network parameters, the network becomes a maximum entangled one and, consequently, the fidelity is maximum.

The proposed quantum teleportation protocol is applied in investigating the possibility of controlling the spread of swine flu virus. In this context, we showed that the virus can be transformed to a similar segment in the society with different probabilities. Also, it showed that the age of patient (old/young) has no effect on the spread of the virus in some situations. In some other situations, the degree of infection (or weight) plays a central role, while for some other situations the weight and patient's age play an equivalent role. On the other hand, by local rotations for the transformed infection young people can avoid swine flu virus infection.

REFERENCES

APPENDIX

### TABLE I: EXAMPLE OF INFORMATION CODING IN A NEURON

<table>
<thead>
<tr>
<th>Alice Deity</th>
<th>Bob state $\rho_b$</th>
<th>Bob Deity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>nn\rangle\langle nn</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>ny\rangle\langle ny</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>yn\rangle\langle yn</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>yy\rangle\langle yy</td>
<td>$</td>
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</tbody>
</table>