Three-Dimensional Acoustic Analysis of an Elliptical Chamber Muffler with a Side Inlet and a Side Outlet

A. Mimani and M.L. Munjal

Facility for Research in Technical Acoustics (FRITA),
Department of Mechanical Engineering, Indian Institute of Science, Bangalore-560 012, India

The acoustical behaviour of an elliptical chamber muffler having a side inlet and side outlet port is analyzed in this paper, wherein a uniform velocity piston source is assumed to model the 3-D acoustic field in the elliptical chamber cavity. Towards this end, we consider the modal expansion of the acoustic pressure field in the elliptical cavity in terms of the angular and radial Mathieu functions, subjected to the rigid wall condition. Then, the Green’s function due to the point source located on the side (curved) surface of the elliptical chamber is obtained. On integrating this function over the elliptical piston area on the curved surface of the elliptical chamber and subsequent division by the area of the elliptic piston, one obtains the acoustic pressure field due to the piston driven source which is equivalent to considering plane wave propagation in the side ports. Thus, one can obtain the acoustic pressure response functions, i.e., the impedance matrix ($Z$) parameters due to the sources (ports) located on the side surface, from which one may also obtain a progressive wave representation in terms of the scattering matrix ($S$). Finally, the acoustic performance of the muffler is evaluated in terms of the Transmission loss (TL) which is computed in terms of the scattering parameters. The effect of the axial length of the muffler and the angular location of the ports on the TL characteristics is studied in detail. The acoustically long chambers show dominant axial plane wave propagation while the TL spectrum of short chambers indicates the dominance of the transversal modes. The 3-D analytical results are compared with the 3-D FEM simulations carried on a commercial software and are shown to be in an excellent agreement, thereby validating the analytical procedure suggested in this work.

Keywords
elliptical chamber; angular and radial Mathieu functions; Green’s function; piston driven source

1. Introduction

A muffler chamber with a side inlet and side outlet is often found as an integral part of the present day commercially used automotive muffler system. Numerous works can be found in the literature which study the acoustic attenuation performance of circular cylindrical chamber mufflers with end inlet and end outlet (an expansion chamber or flow reversal end chamber type of configuration) using a 3-D analytical approach based upon the modal expansion techniques\textsuperscript{1-3}. However, relatively fewer works are found dealing with acoustic analysis of circular cylindrical expansion chambers with a side port using a 3-D analytical approach. Yi and Lee analyzed a circular chamber muffler system with a side inlet and side outlet\textsuperscript{4} and also with an end inlet and side outlet\textsuperscript{5} using a 3-D analytical approach based upon uniform piston driven model and the modal expansion.

There are still fewer papers dealing with the analysis of an elliptical chamber muffler using
3-D analytical approach based upon the modal expansion techniques. This can probably be attributed to the complexity of modal functions, i.e., the Mathieu functions, in contrast to the relatively simpler case of dealing with Bessel functions for the case of a circular cylindrical chamber muffler. Lawson and Baskaran evaluated the cut-on frequencies of the higher order circumferential modes of an elliptical duct by imposing the rigid wall condition whence the first derivative of the modified Mathieu functions was required to be zero at the boundary of the elliptical section. Hong and Kim reported a similar work dealing with the 3-D analytical solution of the acoustic wave equation in elliptical chamber in terms of the modal expansion. The first few mode shape patterns corresponding to the radial or circumferential modes along with the angular or non-axisymmetric modes were shown for the cases of hollow and confocal annular elliptic chambers. Denia et al. solved the 3-D wave propagation problem in a 2-port elliptical chamber having ports on the end faces i.e. an expansion chamber or a reversal chamber type of configuration through an analytical approach by means of Green’s function (assuming a point source). Similarly, Denia et al. investigated the acoustic behaviour of elliptical chambers with single end inlet and double end outlets located on the opposite end plate (a 3-port system and an expansion chamber type of configuration) using the principle of analytical mode matching and the results were compared with 3-D FEM based predictions.

The acoustic behaviour of short elliptical chamber with end centred inlet and end offset outlet or side outlet (2-port) system was studied in detail by Selamet and Denia using a 3-D FEM based approach. The length of the elliptical chambers was deliberately considered short in order to study the effects of the transversal modes on the acoustic attenuation characteristics. However, to the best of the authors’ knowledge, a full 3-D analytical procedure (based upon the uniform piston driven model and the modal expansion method, i.e. Mathieu function and modified Mathieu function) to evaluate the TL performance of a hollow elliptical chamber muffler having side inlet and side outlet does not exist in the literature as of today. Thus, with a view to bridge the aforementioned gap in the literature, the present work employs a uniform piston driven model via the Green’s function method to characterize side inlet and side outlet (a 2-port system) elliptical chamber mufflers by means of impedance \([Z]\) matrix, following which the scattering \([S]\) matrix is obtained from a simple matrix relation between the \(Z\) and \(S\) matrices. The \(S\) parameters are then used to compute the TL performance of the muffler system. The effects of variation of the axial length and relative angular location of the ports are studied in detail. The 3-D analytical results are validated against 3-D FEM based analysis carried on a commercial software package (SYSNOISE). Finally, a 2-D analytical model, based upon only the transverse modes and neglecting the axial modes, is presented for short elliptical chamber mufflers with good agreement with the 3-D model, thereby indicating that axially short chamber mufflers can be modelled by considering only the transverse modes.

2. Characterization of a two-port element and computation of TL

A muffler having two ports is characterized by means of an impedance \([Z]\) matrix wherein the relation between the acoustic pressures \((p_1\) and \(p_2)\) and the mass-velocities \((v_1\) and \(v_2)\) at the two ports are shown hereunder.

\[
\begin{bmatrix}
p_1 \\
p_2
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
\]  
(1)

The subscripts 1 and 2 in Eqs. (1), refer to the port 1 and port 2 respectively. The corresponding scattering matrix \([S]\) matrix relationship can be explicitly expressed in terms of the \(Z\) matrix parameters by the following equations:

\[
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2
\end{bmatrix}
\]  
(2a)

where the incident wave amplitudes at the ports are given by \(A_1\) and \(A_2\), while the reflected wave amplitudes are denoted by \(B_1\) and \(B_2\) and the \(S\) matrix parameters are given by
Consequently, one obtains the following expression for TL:

\[
TL = 10 \log_{10} \left( \frac{Y_2}{Y_1} \right) = 10 \log_{10} \left( \frac{1}{4Y_2} \left| \frac{(Z_{11} + Y_1)(Z_{22} + Y_2) - Z_{12}Z_{21}}{Z_{21}} \right| \right)
\]  

(3)

In Eqs. (2b) and (3), \( Y_1 \) and \( Y_2 \) denote the characteristic impedance of the ports 1 and 2 respectively.

3. Solution of the 3-D Helmholtz equation in elliptic cylindrical co-ordinates

The elliptic cylindrical co-ordinates \((\xi, \eta, z)\) and Cartesian co-ordinates \((x, y, z)\) are related as:

\[
x = h \cosh(\xi) \cos(\eta), \quad y = h \sinh(\xi) \sin(\eta), \quad z = z.
\]  

(4)

where, \( 0 \leq \xi < \infty \) and \( 0 \leq \eta \leq 2\pi \) in elliptical co-ordinates. The curves of constant \( \xi \) define a family of confocal ellipses with semi-major axis \( D_1 / 2 = h \cosh(\xi) \) and semi-minor axis \( D_2 / 2 = h \sinh(\xi) \), while curves of constant \( \eta \) denote the family of confocal hyperbolae. Each of these families of curves has common foci at \( x = \pm h \), where \( 2h \) is the inter focal distance. The scale factors for this co-ordinate system are given in references 11,12. The Helmholtz equation describing the acoustic wave propagation in a three-dimensional Cartesian co-ordinate system \((x, y, z)\) is given by the following equation\(^1\)\(^3\),\(^1\)\(^4\).

\[
\nabla^2 p(x,y,z) - \frac{1}{c_0^2} \frac{\partial^2 p(x,y,z)}{\partial t^2} = 0,
\]

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.
\]  

(5)

In Eq. (5), \( p \) is the acoustic pressure, \( c_0 \) denotes the sound speed and \( t \) denotes the time dependence. We assume a harmonic time dependence, i.e., \( p(x,y,x,t) = p(x,y,z)e^{j\omega t} \). On making use of Eqs. (4), we compute the scale factors and obtain the Helmholtz equation in the elliptic cylindrical co-ordinates (see for instance McLachlan\(^1\)).

\[
\frac{2}{h^2(\cosh(2\xi) - \cos(2\eta))} \left( \frac{\partial^2 p}{\partial \xi^2} + \frac{\partial^2 p}{\partial \eta^2} \right) + \frac{\partial^2 p}{\partial z^2} + k_0^2 p = 0
\]  

(6)

The modal solution of Eq. (6) for the acoustic pressure is obtained in terms of the Mathieu function (for the angular part, \( \eta \)), modified Mathieu function (for the radial part, \( \xi \)) of the Even-Even, Even-Odd, Odd-Even and the Odd-Odd type. The trigonometric functions describe the axial (\( z \) co-ordinate) dependence and the total solution is written as follows.\(^11\)

\[
p(\xi, \eta, z) = \sum_{m=0}^{\infty} C_m(\xi, q) c_m(\eta, q) \left( C_m^1 e^{-jk_0 z} + C_m^2 e^{jk_0 z} \right) + \sum_{m=0}^{\infty} S_m(\xi, q) s_m(\eta, q) \left( S_m^1 e^{-jk_0 z} + S_m^2 e^{jk_0 z} \right)
\]  

(7)

In Eqs. (7), \( c_m(\eta, q) \) and \( s_m(\eta, q) \) denote the Mathieu functions, while \( C_m(\xi, q) \) and \( S_m(\xi, q) \) denote the modified Mathieu functions of order \( m \). The \( q \) parameter in the solution of wave equation in elliptic cylindrical co-ordinates, i.e., Eq. (7), results from the separation of wave equation in the \( z \) direction and the transverse \((\xi, \eta)\) direction, and it is related to the wave number \( k_0 \) and the axial wave number \( k_0 \) by the following equation:

\[
4q + k_0^2 = k_0^2
\]  

(8)

This parameter \( q \) plays the same role as transverse or the radial wave number \( k_0 \) in the case of a circular duct. Indeed, the parameter \( q \) is crucial to us inasmuch as only discrete values of \( q \) are
allowed which correspond to the rigid wall condition.

A more detailed description of these functions is given below.\(^\text{(11)}\)

\[
\begin{align*}
ce_{2n}(\eta, q) &= \sum_{r=0}^{\infty} A_{2r}^n \cos(2r\eta), \\
se_{2n}(\eta, q) &= \sum_{r=0}^{\infty} B_{2r}^n \sin(2r\eta), \\
\end{align*}
\]

Eqs. (8-9) denote the Mathieu function of the Even-Even and Even-Odd type, while Eqs. (10-11) denote the Mathieu functions of the Odd-Even and Odd-Odd type. The expansion coefficients \(A_{2r}^n, B_{2r}^n\) in each of the Eqs. (8-11) depend upon the value of the parameter \(q\) and can be obtained by solving the finite matrix eigenvalue problem by making use of the recurrence relations between the coefficients of each type of solution shown in Eqs. (8-11). For a detailed study, the reader is referred to the work by Stamnes and Spjelkavik.\(^\text{(15)}\) Similarly, the radial or the modified Mathieu functions are expressed as rapidly converging Bessel function product series (not shown here for the sake of brevity). Final ly, we apply the rigid wall conditions in the transverse or the \(\xi\)-direction and the axial (\(z\)) direction and express the corresponding solution of homogeneous Helmholtz equation subject to the homogeneous rigid wall condition as follows.

\[
p(\xi, \eta, z) = \left\{ \sum_{p=0,1,2,...}^{\infty} \sum_{m=0,1,2,...}^{\infty} C_{p,m,n} Ce_m(\xi, q_{m,n}) e^{i(\eta, q_{m,n})} + \sum_{p=0,1,2,...}^{\infty} \sum_{m=1,2,...}^{\infty} S_{p,m,n} Se_m(\xi, q_{m,n}) e^{i(\eta, q_{m,n})} \right\} e^{i\omega t}, \quad (12)
\]

where, \(q_{m,n}\) and \(\bar{q}_{m,n}\) are the parametric zeros or the \(n^{th}\) root of the derivative of Even and Odd type of modified Mathieu function respectively of order \(m\) and corresponding to the rigid wall condition in the transversal direction. They are found by numerically solving the derivative of the modified Mathieu function at the boundary of the elliptical section. In other words, we have

\[
\left. \frac{dCe_m(\xi, q_{m,n})}{d\xi} \right|_{\xi=\xi_0} = 0, \quad \left. \frac{dSe_m(\xi, q_{m,n})}{d\xi} \right|_{\xi=\xi_0} = 0, \quad (13)
\]

where, \(\xi_0\) defines the value of the radial co-ordinate (in elliptic co-ordinate system) at the boundary of the elliptical section. This is related to the eccentricity (\(e\)) as follows.

\[
\xi_0 = \cosh^{-1}(1/e), \quad e = \sqrt{1 - (D_z/D_f)^2}. \quad (14, 15)
\]

The roots \(q_{m,n}\) and \(\bar{q}_{m,n}\) define a series of confocal ellipses, such that as one traverses along the \(\xi\) direction, pressure nodes are obtained along any of the confocal hyperbolae, i.e., at any arbitrary value of \(\eta = [0,2\pi]\). By setting \(k_z=0\) in Eq. (8), the non-dimensional cut-on frequency of the Even or Odd mode of order \(m\) and \(n^{th}\) parametric zero, i.e., \(q_{m,n}\) and \(\bar{q}_{m,n}\) respectively is obtained as follows.

\[
k_0(D_z/2)_{even} = 2\sqrt{q_{m,n} / e}, \quad k_0(D_z/2)_{odd} = 2\sqrt{\bar{q}_{m,n} / e}. \quad (16)
\]

Finally, we obtain the Green’s function response (due to a point source, located at an arbitrary point of the side surface, i.e., on \(\xi = \xi_0\)) as follows.

\[
p(\xi, \eta, z) = G(\xi, \eta, z; \xi_0, \eta_0, z_0) = G(R|R_0)\]

\[
= \rho_0 Q(jk_0c_0) \left\{ \sum_{p=0,1,2,...}^{\infty} \sum_{m=0,1,2,...}^{\infty} \sum_{n=1,2,...}^{\infty} \frac{\psi'(\xi, \eta, z, \xi_0, \eta_0, z_0)}{[P\pi/L]^2 + 4q_{m,n} / h^2 - k_0^2 |N_{m,n,p}|^2} + \sum_{p=0,1,2,...}^{\infty} \sum_{m=0,1,2,...}^{\infty} \sum_{n=1,2,...}^{\infty} \frac{\psi'(\xi, \eta, z, \xi_0, \eta_0, z_0)}{[P\pi/L]^2 + 4\bar{q}_{m,n} / h^2 - k_0^2 |\bar{N}_{m,n,p}|^2} \right\} \quad \text{(17)}
\]

where
\( \psi(\xi, \eta, z) = Ce_m(\xi, q_{m,n}) \cos\left( \frac{P \pi z}{L} \right) \),
\( \psi(\xi, \eta, z) = Ce_m(\xi, q_{m,n}) \cos\left( \frac{P \pi z}{L} \right) \),
while, \( N_{m,n,p} \) and \( \overline{N_{m,n,p}} \) denote the orthogonal mode shape values. These can be computed numerically by evaluating the following integrals:

\[
N_{m,n,p} = \int_{z=0}^{\infty} \int_{\xi=0}^{\infty} \int_{\eta=0}^{\infty} \frac{C e_m(\xi, q_{m,n}) \cos(2\xi - \cos(2\eta)) \cos\left( \frac{P \pi z}{L} \right)}{d\xi d\eta dz},
\]

In Eq. (17), \( \rho_0 \) is the ambient density, \( Q = U_{piston} S_{port} \) is the flow rate due to the side port approximated as a point source, while \( U_{piston} \) is the piston velocity and \( S_{port} \) is the cross-sectional area of the elliptic piston port located on the side surface. Also, \( L \) is the length of the elliptical chamber and the co-ordinates of the source are denoted by \( \xi_0, \eta_0, z_0 \) while the co-ordinates of the receiver point are denoted by \( \xi_r, \eta_r, z_r \). Now, we integrate the Green’s function over the elliptic piston area of the source \( S_{source} \), i.e., over the co-ordinates \( \eta_0, z_0 \) and divide by the same to obtain the pressure response function due to uniform piston driven model. Furthermore, this function is integrated over the receiver port co-ordinates on the curved surface, i.e., over \( \eta_r, z_r \) and divided by the receiver port area \( S_{receiver} \) to yield the average pressure response.

\[
p(\xi_0, \eta_0, z_0, \xi_r, \eta_r, z_r) = \frac{h^2}{2} \int_{S_{source}} \frac{1}{S_{port}} \int_{\eta=0}^{\infty} \int_{\xi=0}^{\infty} \frac{G(\xi_0, \eta_0, z_0, \xi, \eta, z) (\cosh(2\xi_0 - \cos(2\eta_0)) \cosh(2\xi_r - \cos(2\eta_r)) \cosh^2(2z_0) - \cosh^2(2z_r)) \cosh^2(2\eta_0) \cosh^2(2\eta_r)}{dz dz \eta}.
\]

We make use of Eq. (17-20) to evaluate the \( Z \) parameters. For obtaining the first column, source is located at port 1, the co-ordinates of which are denoted by \( \xi_0, \eta_1, z_1 \) while those of the receiver (port 2) are denoted by \( \xi_0, \eta_2, z_2 \), whereby one evaluates \( Z_{11} \) and \( Z_{21} \) as follows.

\[
Z_{11} = p(\xi_0, \eta_1, z_1, \xi_0, \eta_0, z_0)/(\rho_0 Q), \quad Z_{21} = p(\xi_0, \eta_2, z_2, \xi_0, \eta_0, z_0)/(\rho_0 Q).
\]

On applying the principle of acoustic reciprocity for a stationary medium with zero mean flow mach number, we obtain \( Z_{12} = Z_{21} \). Similarly, \( Z_{22} \) may be evaluated using the following expression.

\[
Z_{22} = p(\xi_0, \eta_2, z_2, \xi_0, \eta_0, z_0)/(\rho_0 Q).
\]

In Eqs. (22), the elliptic piston source is located at the port 2, the co-ordinates of which are denoted by \( \xi_0, \eta_2, z_2 \). Finally, making use of the expression for TL in terms of the \( Z \) or \( S \) parameters obtained in Section-2, the TL of a side inlet and a side outlet elliptical chamber muffler is evaluated.

4. Results and Discussion

A reference configuration is first chosen and the TL performance of this configuration is evaluated using commercial vibro-acoustic analysis software based upon the 3-D Finite Element Analysis (FEA). Then, the results of the 3-D analytical method based upon the modal expansion and piston driven model are compared against the FEA results, thereby validating the present approach. In particular, we restrict our discussion in this work to ports situated on the side surfaces at canonical values of \( \eta = 0, \pi/2, \pi \) or \( 2\pi \) and thus study the effect of angular locations of the ports on the side surface. The effect of change of the axial length of the elliptical chamber muffler is also studied.

Fig. 1a is presented below in which the side view, top view and front view of an elliptical
chamber muffler is shown illustrating the angular location of the ports ($\eta_1, \eta_2$) along the side surface as well as the axial location ($l_1, l_2$), while Fig. 1b shows a 3-D view of the same for a particular case of axial and angular location of the side ports.

Figure 1. An elliptical chamber muffler with side inlet port (Port 1) and side outlet port (Port 2), a) Side, Top and Front View b) 3-D CAD model.

The speed of sound is taken to be $c_0 = 343.13$ m/s (i.e. at $20^\circ$ C) in all the ensuing computations of TL performance. The major axis and the minor axis of the elliptical chamber is fixed at $D_1 = 0.25$ m and $D_2 = 0.15$ m, respectively, while the diameters of the inlet and outlet ports are fixed at $d_1 = d_2 = d_0 = 0.04$ m. We first validate our 3-D analytical model with 3-D FEA results for two cases with the angular locations given by $\eta_1 = 0, \eta_2 = \pi/2$ and the axial locations and lengths of the chamber given by (1) $l_1 = 0.15$ m, $l_2 = 0.225$ m, $L = 0.3$ m, and (2) $l_1 = l_2 = 0.025$ m, $L = 0.05$ m. The results are shown in Figs. 2 and 3, respectively.

Figure 2. TL for the muffler configuration $\eta_1 = 0, \eta_2 = \pi/2$, $l_1 = 0.15$ m, $l_2 = 0.225$ m.

Figure 3. TL for the muffler configuration $\eta_1 = 0, \eta_2 = \pi/2$, $l_1 = l_2 = 0.025$ m.

It is indeed observed from Figs. 2 and 3 that the 3-D analytical approach and the 3-D FEA based predictions match very well throughout the frequency range of interest for elliptical chambers of longer ($L = 0.3$ m) and shorter axial length ($L = 0.05$ m), respectively, thereby validating the analytical procedure presented in this work. It is also seen from Fig. 2 that the 1-D axial plane wave matches the 3-D based predictions only for frequencies of up to than the cut-on frequency of the $(1,1)$ even mode (about 800 Hz in the present case). The elliptical chamber of short axial length shows a completely different attenuation pattern due to the dominance of the transverse modes. For such chambers, the axial plane wave theory would completely fail and therefore it is not shown in Fig. 3. Moreover, offsetting the inlet and outlet ports by angle of $\pi/2$ leads to a transverse resonance peak near about the cut-on frequency of the $(1,1)$ even mode. In fact, such resonance peaks are observed near about the cut-on frequencies of other higher order modes also.

Since the acoustic pressure field inside the elliptical chamber is obtained as a summation of infinite modes as shown in Eqs. (17), we truncate the same to a finite number of terms. In actual computations, the first six orders of the Even-Even ($m=0, 2, ..., 10$), Even-Odd, Odd-Even ($m=1, 3, ..., 11$) and Odd-Even ($m=2, 4, ..., 12$) series, and the first six roots for each order as well as first 10 axial modes, i.e. ($P = 0, 1, 2, ..., 10$) are considered in the Green’s solution which shows good convergence over the entire frequency range of interest. In fact, for axially long chamber, first several axial modes are considered to obtain a TL graph showing a convergent trend, however, for short
chamber mufflers, good convergence can be obtained by taking lesser number of axial modes. This point will be emphasised later in this work, when a 2-D model of short elliptical chamber is considered which is based only upon the transverse modes. Also, the plane wave assumption in the ports is indeed a good approximation as the diameters of the ports are small enough to support only planar wave propagation for the frequency range of interest of the present work. (The cut-on frequency of the \((1,0)\) mode in the port of \(d_0 = 0.04 \text{ m}\) for \(c_0 = 343.13 \text{ m/s}\) is evaluated at about 5027 Hz.)

\[
\begin{align*}
\eta_1 &= 0, \quad \eta_2 = \frac{\pi}{2}, \pi, 0 \\
\eta_1 &= \frac{\pi}{2}, \quad \eta_2 = \frac{\pi}{2}, 0, 3\frac{\pi}{2}
\end{align*}
\]

Figure 4: The effect of angular location of ports on TL for long chamber mufflers.

The effect of position of the angular ports on the side surface of the elliptical chamber is presented in Figs. 4a and 4b. The length of the chamber is held constant at \(L = 0.3 \text{ m}\), the axial location of the port 1 is at \(l_1 = 0.150 \text{ m}\) while the location of port 2 is \(l_2 = 0.225 \text{ m}\). In Fig. 4a, the angular location of the port 1 is fixed at \(\eta_1 = 0\), while the angular location of the port 2 is varied from \(\eta_2 = 0, \frac{\pi}{2} \text{ and } \pi\). The configuration in which port 2 is located at \(\eta_2 = \pi/2\) exhibits a broadband attenuation pattern. In fact, the offsetting of the ports by angle of \(\pi/2\) leads to a transverse resonance peak even in the case of an axially long chamber which greatly helps in raising the trough that would have occurred near about the cut-on frequency of the \((1,1)\) even mode if the ports were offset by an angle different from \(\pi/2\). The configuration in which the ports are offset by \(\eta = 0\) exhibits a transverse resonance peak earlier than the cut-on frequency of the \((1,1)\) even mode while the chamber with an offset angle of \(\eta = \pi\) exhibits a transverse resonance peak earlier than the cut-on frequency of the \((1,1)\) even mode.

Next, we fix the angular location of port 1 at \(\eta_1 = \pi/2\) and vary the angular location of port 2 as in the earlier case. The results are presented in Fig. 4b and similar conclusions can be made in this case also. If the relative angle between the ports is different from \(\pi/2\), a dip in TL graph is observed near about the cut-on frequency of the \((1,1)\) even mode resulting in a relatively better TL performance as compared with \(\eta_1 = 0, \eta_2 = 0\) in Fig. 4a.

The short axial length chambers are studied with \(l_1 = l_2 = 0.025 \text{ m}, \quad L = 0.05 \text{ m}\), the rest of the dimensions being unaltered. Fig. 5 shows the TL performance for the following cases:

1. \(\eta_1 = 0, \eta_2 = \pi/2\)
2. \(\eta_1 = 0, \eta_2 = \pi\)
3. \(\eta_1 = \pi/2, \eta_2 = 3\pi/2\)

It is observed from Fig. 6 shown on the next page that the configuration in Case 1 exhibits a transversal resonance peak not only at the cut-on frequency of the \((1,1)\) even mode but also at the cut-on frequencies of almost all the other higher order modes. This leads to the raising of dip in the TL spectrum leading to broadband attenuation pattern, while the configurations in Cases 2 and 3 behave like an expansion chamber with domes and troughs resembling that of a simple expansion chamber. However, the resonance peak present in the configuration of Case 3 can be due to the resonance along the direction of major axis due to even modes. The behaviour of all these short chambers are very different from that of a
chamber with longer axial length with the transverse modes completely dominating the axial modes. This suggests that if the summation over the index \(P\) is dropped in Eqs. (17) and (18), i.e., if only \(P=0\) is retained, it implies a constant pressure mode (no variation) along the axial direction.

Following this approach, only the 2-D transversal modes are considered for short elliptical chamber muffler. The validity of this approach is also tested in this work on configuration pertaining to Case 1 discussed in the foregoing paragraph. The TL graph considering only the transverse modes (a 2-D model) and the 3-D analytical model are compared in Fig. 6. The two TL graphs tally well up to a higher frequencies, thereby confirming our 2-D transverse model hypothesis.

5. Conclusions

A 3-D analytical approach based upon the uniform piston driven model and the modal expansion of the acoustic field in the elliptical chamber muffler with side inlet and side outlet is presented in this work. The TL predictions match well with the 3-D FEA results. The effect of the angular and axial location of ports is also examined. A useful guideline that has emerged from this work is that if one of the ports (say, the port \(i\)) is located at half the axial distance while the other port (port \(j\)) is located at three-fourths or one-fourth of the axial distance measured from the same direction, the angular location of the ports being either \(\eta_i=0, \eta_j=\pi/2\) or \(\eta_i=0, \eta_j=\pi/2\), then a broadband attenuation pattern is observed due to the resonances along the axial and the transverse directions tuning out the expansion chamber troughs. This is of great value for muffler designers.

References

12. P. M. Morse, and H.S. Feshbach, Methods of theoretical physics (Parts 1 and 2), New York, McGraw Hill.