ALIGNING THE FRAMES OF A NON STATIONARY IMAGING FOURIER TRANSFORM SPECTROMETER FOR SPECTRUM RETRIEVAL

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ABSTRACT

This paper presents an algorithm to align the frames obtained with a non-stationary imaging Fourier transform spectrometer (IFTS). These frames contain both relative motion and intensity variation due to the interference pattern of the IFTS called interferogram. Two motion estimation techniques are combined to register the frames with a sub-pixel precision. An approach based on mutual information is first used for pixel-precision registration with respect to a single reference frame. Sub-pixel precision is then achieved using a phase-based technique.

Simulation results show that our registration approach can successfully reconstruct the interferograms of the observed scene. Spectra obtained from these interferograms are undistorted and have better spectral resolution than spectra obtained with a registration algorithm using only phase-based motion estimation, or without motion compensation.

We also show that scene elements with different electromagnetic spectrum can be more easily identified in a spectral cube computed from registered interferogram frames.

Index Terms— IFTS, motion estimation, hyper-spectral

1. INTRODUCTION

Imaging Fourier transform spectrometers (IFTS) are deployed for remote sensing spectral imaging applications because of their ability to provide simultaneously both high spatial and spectral resolution images of a scene[1-2]. The spectral data obtained from a high resolution imaging FTS is often classified as “hyper spectral”. IFTS have proven their ability in many applications to provide rapid and non-destructive analysis, both spectrally and spatially [3-5]. In order to obtain high spectral accuracy, an IFTS has to be stationary for the complete acquisition time of the data cube, a condition that may not be fully met for some applications. Due to the relative motion between the instrument and the scene being scanned, the interferogram of a given pixel will be composed of data samples coming from different sub-areas of the scene, leading to corrupted spectra when Fourier transformed. Opto-mechanical means that keep the sensor pointing on the same ground area during the scan acquisition are not always capable of compensating for all the motion, and also add to the hardware complexity.

In this paper we define the problem and the effect of acquiring a data cube from a moving IFTS. We propose to perform motion compensation a posteriori using the data information. We combine two motion estimation algorithms to recover undistorted spectra. An approach based on mutual information is first used for pixel-precision registration with respect to a single reference frame. Sub-pixel precision is then achieved using a phase-based technique.

To test this algorithm, a scenario of IFTS working in the near infrared (NIR) is proposed, and the motion between the different frames is estimated. The motion estimation is then used for aligning the frames of the cube such that, along the optical path difference (OPD) axis of a given pixel, we find the interferogram samples from the same location of the scene. The method proposed enables either to recover the resolution of the spectrum computed for that scene area, or simply to retrieve a valid spectrum when the motion between frames is significant. This combined method is compared with the registration algorithm using only phase-based motion estimation, and gives better spectral resolution.

2. PROBLEM DESCRIPTION

A Fourier transform spectrometer (FTS) captures the electromagnetic spectrum of a radiative source by measuring its temporal coherence. A beam of light is split into two beams and reflected off two mirrors. One of the mirrors is moving to change the optical path difference between the two beams. The reflected beams are then brought together and the amplitude of the interference pattern is measured. An interferogram is measured by sampling the interference pattern for different optical path difference. The Fourier transform of this interferogram returns the spectrum of the input source [6].

Given a source with spectrum $A(\sigma)$, where $\sigma$ is the wave-number (reciprocal of wavelength), the intensity of the interferogram as a function of the optical path difference (OPD) $x$ between the two beams is given by

$$I_x(x) = 2 \int_0^\infty A(\sigma)(1 + \cos(2\pi \sigma x))d\sigma$$

(1)

where $I_x(x)$ is the intensity at a given OPD $x$. The Zero Path Difference (ZPD) is the location where the OPD is zero, and is the point where the interferogram is maximum.

In an imaging FTS (IFTS), the instrument captures an interferogram for each pixel of the detector, generating a 3D cube of data. If the scene and the instrument are both stationary, the
interference samples at a given pixel capture the spectrum of a scene element. A hyperspectral cube can then simply be generated by computing the Fourier transform of the data samples at each pixel.

However, for a data cube acquired with a relative motion between the instrument and the scene, the samples at one pixel are no longer capturing the interference pattern of the same spatial location. Let us consider we have a scene scanned with a moving IFTS. If we look at a given pixel in the cube, we will have interferogram samples from different spatial locations. When Fourier transformed, a distorted spectrum or with low spectral resolution will result. To retrieve the interferogram samples related to the same spatial location, we need to estimate the relative motion between the frames. The motion compensated cube is composed of the spatially registered frames.

3. MOTION ESTIMATION REGISTRATION

Motion compensation on the interferograms is difficult for two reasons. First, the intensity of the interferogram varies with the OPD, as shown by equation 1. Second, the intensity variation is not spatially uniform since we are looking at scene elements with different electromagnetic spectrum. Therefore, motion registration techniques based on constant intensity cannot be used to register the frames. Most motion registration techniques robust to intensity variation also assume, however, uniform spatial variation. In this paper, we combine two different registration algorithms to compute motion compensated cubes from which spectra information can be retrieved. We first describe these two algorithms, and then we explain how we combine them.

3.1 Mutual Information registration

Image registration based on mutual information (MI) has been shown to be robust against illumination variation. It relies on the degree of dependence of two random variables as determined by the distance between their joint distribution, and the distribution associated with the case of complete independence [7]. Let \( f_1(x, y) \) and \( f_2(x, y) \) be two frames in the hyper-spectral cube, where \((x, y)\) are the spatial coordinates for the pixels. Their mutual information \( I(f_1, f_2) \) is given by

\[
I(f_1, f_2) = H(f_1) + H(f_2) - H(f_1, f_2)
\]

where, \( H(f_1) \), \( H(f_2) \) and, \( H(f_1, f_2) \) are respectively their entropy and joint entropy, given by

\[
H(f_i) = -\sum_a P_{f_i}(a) \log P_{f_i}(a)
\]

\[
H(f_1, f_2) = -\sum_{a, b} P_{f_1, f_2}(a, b) \log P_{f_1, f_2}(a, b)
\]

where \( P_{f_1}(a) \) is the marginal probability mass function of \( f_1 \), and \( P_{f_1, f_2}(a, b) \) is their joint probability mass function.

The probability distributions in equations 3 and 4 are computed using histograms of image intensity values. The two-dimensional joint histogram \( h(a, b) \) is a function of the gray level intensities of images \( f_1 \) and \( f_2 \). The value \( h(a, b) \) supplies the number of corresponding pixel pairs having gray-level \( a \) in image \( f_1 \) and gray-level \( b \) in image \( f_2 \). Estimation for the joint probability distribution \( P_{f_1, f_2}(a, b) \) is obtained by normalizing the joint histogram of the image pair

\[
P_{f_1, f_2}(a, b) = \frac{h(a, b)}{\sum_{b} h(a, b)}
\]

The two marginal probability mass functions can then be obtained from

\[
P_{f_1}(a) = \sum_{b} P_{f_1, f_2}(a, b), \quad P_{f_2}(b) = \sum_{a} P_{f_1, f_2}(a, b)
\]

Given \( f_1 \) and \( f_2 \), we compute their MI \( I_{xy}(f_1, f_2) \). The image \( f_2(x + x_0, y + y_0) \) is registered with image \( f_1(x, y) \) for \((x_0, y_0)\) that maximize \( I_{xy}(f_1, f_2) \).

MI uses exhaustive search on the motion parameters to align the frames which is time-consuming. We therefore use this approach to register the frames with pixel-precision, and using as reference the ZPD frame of our data cube. This is critical as the intensity variation and spatial differences are larger around the ZPD frames. Sub-pixel registration is then performed using a phased-based approach.

3.2 Phase-based sub-pixel registration

Harold et al. [8] proposed a sub-pixel algorithm for image registration based on the relative phase between two images. The relative phase is a plane computed using 2D discrete Fourier Transforms, whose slope gives the 2D motion between the frames. If a motion is applied to the second frame to shift it by \((x_0, y_0)\) we can consider that \( f_2(x, y) = f_1(x - x_0, y + y_0) \). According to the Fourier shift property

\[
\hat{f}_2(u, v) = \hat{f}_1(u, v) \exp(-i(ux_0 + vy_0))
\]

where \( \hat{f}_1(u, v) \) and \( \hat{f}_2(u, v) \) are the Fourier transform of \( f_1(x, y) \) and \( f_2(x, y) \) respectively.

From the complex ratio

\[
\hat{C}(u, v) = \frac{\hat{f}_2}{\hat{f}_1} = \exp(-ix_0u - iy_0v)
\]

the linear phase difference between the 2 frames is given by

\[
\hat{\phi}(u, v) = \angle \hat{C}(u, v) = x_0u + y_0v
\]

The slope of the plane is computed using the least square estimate after masking out the spectral components that lie outside a chosen window around the center.

Note that if we have a uniform change of illumination between the two frames, the linear phase difference will not be affected. Moreover, working with phase provides a better approximation to the local velocity than do amplitude as reported by Fleet [9].
3.3 Mixed method

Both methods can work under illumination variation. However, MI is more robust to illumination variation that is not spatially uniform, which is the case in our scenario, mainly around the ZPD frames. The phase-based method is much faster than the MI with a higher accuracy. As the phase-based is a sub-pixel technique, we need to estimate the motion vectors between each two successive frames along the OPD and sum up the results. This introduces error accumulation. So, we register the frames using the MI with the ZPD as a reference for pixel precision, then use the phase-based algorithm for the sub-pixel registration. The accumulation of errors in the motion vectors is stopped since we estimate motion relatively to the ZPD frame.

4. EXPERIMENTAL DATA

4.1. Proposed scenario and motion pattern

To compute the performance of the different algorithms, we generate a hypercube with the following parameters. For the NIR range, we can assume that there is negligible emission from Earth's surface, so the radiation from the Sun that is transmitted through the column of the gas is reflected on Earth's surface towards the IFTS. The emission from the Sun in this spectral band is simulated from the spectral radiance of a blackbody (body transmits or absorbs all radiation) at temperature 6000K according to Planck's equation [10]. The IFTS is assumed to be placed on a plane scanning the column of gas at a height nearly 2 km above Earth's surface. The column of gas has an elliptical shape of CO2 for simulation purpose, with a maximum concentration of 0.0114 ppm at the center and decreasing gradually to the edges, and the working band for the sensors of the IFTS is between 6000-6500 cm⁻¹. The reflectance of Earth's surface is simulated from a NASA database [11] at this range and is normalized to 0.4 as a maximum reflection coefficient. The CO2 transmittance is obtained from Hitran spectroscopic database [12]. Figure 1 shows the setup for the mentioned scenario. The hyper-spectral cube we generate has 8000 frames of 64 x 64 pixels, with a 1.75 cm⁻¹ spectral resolution. The sampling rate or the minimum OPD step is set to twice the maximum wave-number (7000 cm⁻¹). For simplicity and to focus on the motion estimation performance, we assumed there is no off-axis effect (with off-axis, the spectral resolution varies spatially) or shear errors due to interferometer misalignment. The instrument is moving according to the motion pattern shown in Figure 1. This pattern is repeated every 1000 frames, while each path represented by an arrow lasts for 100 frames. At ZPD (frame 4001), the target is located at the lower right corner in the scene. On the right side of the figure, an example of the output frames is shown.

4.2. Results

In order to illustrate the effect of motion on the shape and resolution of the output interferogram and spectrum, we tracked the interferogram samples for a pixel located at the border of the CO2 column, and computed its spectrum. We compared them with the interferogram and spectrum coming from the same pixel but from a data cube with stationary CO2 column. The green curves in figures 3 and 4 are related to the non-stationary data, while the black ones are computed from the stationary data. The interferogram radiance (green curve) oscillates since it captures radiance from different spatial scene elements. Figure 2 shows the errors in the motion vectors for the phase-based and mixed methods. When using the mixed method, there is no error accumulation since the vectors are estimated relatively to the ZPD. To show the advantage of the mixed method over the phase-based one, interferograms, spectra, and SNR of a given pixel in their compensated cubes, are compared.
Figure 3 shows that the interferogram related to the phase-based method (blue curve) appears to oscillate around the ZPD, while the interferogram of the mixed method (red curve) coincides with that of the known one.

Figure 4 shows that the mixed method improves the resolution of the output spectrum, which matches the spectrum of the pixel assuming known motion vectors. Note that the difference between the spectrum computed using the known motion vectors and the ground truth (stationary) is caused by the interpolation errors. Figure 4 also shows a zoom at the wave-numbers where the CO2 absorbs. Computing the SNR for the spectra shows improved results for the mixed method over the phase-based method as shown in Figure 5. The SNR reference spectra were computed after aligning the frames using the known motion vectors.

After alignment of the frames and FFT of the interferograms, each frame corresponds to the spectrum intensity at a wave-number. Since the CO2 has high absorption around wave-numbers 6216, 6247, and 6366 cm\(^{-1}\), we show the corresponding frames from the spectral cube in Figure 6 with and without motion registration. The elliptical shape of the column is now apparent at those wave-numbers, although it was blurred before alignment. The location of the plume is related to the acquisition time of the ZPD frame since it was the reference frame used to align the other frames.

![SNR curves for the working pixel](image)

**Fig. 5.** SNR curves for the working pixel.

5. CONCLUSION AND FUTURE WORK

As a result of acquiring an hyper-spectral cube from a non stationary IFTS, the output spectra have degraded spectral resolution or are severely distorted. Two motion estimation techniques were combined to align the frames of the hyper-spectral cube: MI technique for aligning the frames on a pixel-based reference, followed by phase-based for the sub-pixel level. This technique was tested on a simulated IFTS working in the NIR band, to give output spectra with reduced errors as well as with a recognized increase in the SNR. Moreover, aligning the cube allows us to locate the target spatially and delimitate its shape.

![Frames at wave numbers 6216, 6247, and 6366 cm\(^{-1}\) before (up), and after (down) alignment.](image)

**Fig. 6.** Frames at wave numbers 6216, 6247, and 6366 cm\(^{-1}\) before (up), and after (down) alignment.

We have ignored the off-axis and shear effects characteristics to real IFTS instruments while simulating the cube. These factors can have a non trivial effect on the spectra and will be considered in the future work.

Determining the intensity values for aligned frames will require improved interpolation technique if the grid points do not coincide with the grid points of the reference frame. Using a suitable interpolation technique is important for recovering accurate spectra.

6. REFERENCES