**Optimal Random Access for a Cognitive Radio Terminal with Energy Harvesting Capability**

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**Abstract**—We consider a cognitive radio scenario with an energy harvesting secondary user (SU) attempting to access a primary channel randomly. We assume multipacket reception (MPR) capability and investigate a system in which the SU may or may not exploit the primary feedback messages. The access probabilities are obtained to maximize the secondary throughput under the constraints of primary queue stability and such that the primary queueing delay is kept below a specified value in order to guarantee a certain quality of service (QoS) for the primary user (PU). We investigate the impact of the energy queue arrivals, MPR capability, and the primary queueing delay constraint on the maximum secondary throughput.

**Index Terms**—Cognitive radio, energy harvesting, queueing delay, throughput, multipacket reception.

I. INTRODUCTION

In many practical situations, the secondary user (SU) is a battery-powered device and, hence, is energy-constrained. Consequently, the SU must optimize its spectrum access decisions to efficiently utilize the energy at its disposal.

Data transmission by an energy harvesting transmitter with a rechargeable battery has thus got a lot of attention [1]–[3]. Throughput maximization via energy allocation over a finite horizon was investigated in [1]. The authors in [2] investigated the maximum stable throughput region of a system composed of one primary user (PU) with a rechargeable battery and an SU plugged to a reliable power supply. The author of [3] investigated the optimal cognitive sensing and access policies for an SU with an energy queue based on Markov-decision process (MDP).

In the present paper, we study a cognitive setting with an energy harvesting cognitive radio user that exploits primary automatic repeat request (ARQ) feedback for channel accessing. There have been several proposals to employ the feedback from the primary receiver (PR) to the primary transmitter in order to optimize the secondary transmission strategies. In [4], the authors proposed a cognitive access methodology that exploits the feedback channel in two-way primary communication links for better spectrum utility and protection against interference. The SUs dynamically control access and power based on primary acknowledgment/negative acknowledgment (ACK/NACK) messages. The authors of [5] applied partially observable MDPs to devise an optimal secondary channel access strategy based on the ACK/NACK messages from the PR and the prior knowledge of the PU idle-busy probability distribution.

In this letter, we investigate the optimal mean service rate, $\mu_s$, of an SU randomly accessing a primary channel and possibly leveraging the primary feedback messages if they are available. We assume multipacket reception (MPR) capability added to the physical layer of the receiving nodes. We investigate the impact of the energy queue arrivals, the MPR capability strength, and the tolerable primary queueing delay on the maximum $\mu_s$.

II. SYSTEM MODEL

The network consists of one PU and one energy harvesting SU. The channel is slotted in time and a slot duration equals the packet transmission time. The PU and the SU have infinite buffers $Q_p$ and $Q_s$, respectively, for storing fixed-length data packets. The arrivals at queue $Q_j$ are independent and identically distributed (i.i.d.) Bernoulli random variables from slot to slot with mean $\lambda_j$, where $j$ reads ‘$p$’ for the PU and $j$ reads ‘$s$’ for the SU [2]. Arrival processes are independent from terminal to another. The SU has an additional energy queue, $Q_e$, to store the energy harvested from the environment. The arrivals at the energy queue are also Bernoulli with mean $\lambda_e$ and are independent from arrivals at the other queues [2], [3]. The Bernoulli arrival model is simple but it still can capture the random and sporadic nature of packet arrival [3]. More importantly, in the analysis of discrete-time queues, Bernoulli arrivals see time averages (BASTA). This is the BASTA property equivalent to the Poisson arrivals see time averages (PASTA) property in continuous-time systems [7].

We adopt a general MPR channel model similar to the one specified in [2], [6]. This means that concurrently transmitted packets can survive if the received signal-to-interference-plus-noise ratio (SINR) exceeds the threshold required for successful decoding at the receiver. This is in contrast with collision models where interference, regardless of its magnitude, causes packets to be lost. In our model, the SU accesses the channel whenever it has a packet to send, while the SU employs a random access-based approach. That is, the PU sends the packet at the head of its queue, whereas the SU, when it has energy in its energy queue, randomly accesses the channel without employing any spectrum sensing scheme. Note that the SU can utilize the channel simultaneously with the PU to take advantage of the MPR capability. The PU receives an ACK/NACK feedback at the end of the time slot to indicate the status of packet decodability at the PR. The SU can overhear

1If in practice these buffers are large enough compared to packet size, this is a reasonable approximation (see [2], [6] for a similar assumption).
We characterize the success of packet reception by the probability $P_r$, which denotes the probability of correct reception of the packet transmitted by user $j$ to its respective receiver. When there is concurrent transmission, a superscript ‘$c$’ is appended to the symbol. It is always the case that $P_j^{(c)} \leq P_j$ due to interference.

Parameters, such as $\lambda_p$ and $P_p$, can be estimated by the SU as follows. The SU remains silent for a number of time slots, $N$, observing the primary ACK/NACK feedback. The mean primary service rate, denoted by $\mu_p$, is the probability of the primary channel not being in outage, $P_p$, and can be estimated by observing the ratio of ACKs to total number of transmissions or slots during which the PU is active. The ratio of total transmissions to $N$ gives the probability of the queue being nonempty, which is equal to the ratio $\lambda/N$. Thus,

$$\mu_p^{(\text{est})} = \frac{P_p^{(\text{est})}}{N} = \frac{A}{M} \lambda_p^{(\text{est})} = \frac{M}{N} \frac{\mu}{N}$$ (1)

where $A$ is the total number of ACKs during the observation period and $M$ is the total number of transmissions.\(^3\)

III. QUEUEING ANALYSIS OF THE PRIMARY QUEUE

A. The Case of the SU not Employing Primary Feedback

We start with the case of the SU accessing the primary channel without making use of the primary feedback. The SU attempts to access the channel when both its data and energy queues, $Q_d$ and $Q_e$, are nonempty with some access probability $p_t$. Concurrent transmission occurs when the nonempty queue SU accesses the channel while $Q_p$ is nonempty. An energy packet is consumed from the energy queue if the SU decides to access the channel while $Q_e \not= 0$. The queues mean service rates of $Q_s$, $Q_e$, and $Q_p$ are given by

$$\mu_s = p_t \Pr\{Q_p = 0, Q_e = 0\} P_s^{(c)} + P_s^{(c)} P_{s}\Pr\{Q_p = 0, Q_e = 0\}$$

$$\mu_e = p_t \Pr\{Q_s = 0\}$$

$$\mu_p = (1 - \xi) P_p + \xi P_p^{(c)}, \quad \xi = p_t \Pr\{Q_s = 0, Q_e = 0\}$$ (2)

Since the queues are interacting, we make the following approximations to decouple queue interaction and render the problem tractable. We assume that $\mu_s = 1$ as in [2]. This makes the probability of $Q_e$ being nonempty equal to $\mu_e$. Note that this assumption underestimates $\mu_e$ by increasing the probability that $Q_e$ is empty. We also assume that the SU sends dummy packets when $Q_s$ is empty (as in [2], [6] and the references therein). Based on these approximations, the mean service rates become:

$$\mu_s = \lambda_e p_t \Pr\{Q_p = 0\} P_s^{(c)} + P_s^{(c)} P_{s}\Pr\{Q_p = 0\}$$

$$\mu_e = \frac{1 - \lambda_e p_t}{\lambda_e p_t + \lambda_e p_t} P_p$$ (3)

Solving the state balance equations of the Markov chain modeling the primary queue, it is straightforward to show that

Fig. 1. Markov chain of the PU for the feedback-based access scheme. State self-transitions are not depicted for visual clarity. Note that $\pi = 1 - x$.

The mean primary queueing delay, $D_p$, is $D_p = \frac{1 - \lambda}{\mu_p + \lambda_p}$. For the optimal secondary random access, we solve the following constrained optimization problem. We maximize $\mu_e$ under the constraints that the primary queue is stable and that the primary packet delay is smaller than or equal to a specified value $D_p$. The value of $D_p$ is application-dependent and is related to the required quality of service (QoS) for the PU. The optimization problem can be stated as

$$\max_{0 \leq p_t \leq 1} \lambda_e p_t \left(1 - \frac{\lambda_p}{\mu_p} \right) \left(\frac{P_p^{(c)}}{P_p} \frac{\lambda_p}{\mu_p}\right) \left(\frac{P_p^{(c)}}{P_p}\right), \text{s.t. } \lambda_p \leq \mu_p, \quad D_p \leq \frac{1}{P_p^{(c)}}$$ (4)

The problem is convex and can be solved using the Lagrangian formulation. Let $a = \frac{P_p}{\mu_p}$, $b = \lambda_e \frac{P_p^{(c)}}{\mu_p}$, $c = \lambda_p \frac{P_p^{(c)}}{\mu_p}$, and $d = \frac{1}{P_p^{(c)}}$. The delay constraint subsumes the stability constraint, $\lambda_p \leq \mu_p$, and $p_t$ is upperbounded by $U$

$$U = \min \left\{1, \frac{a - \left(1 - \frac{\lambda_p}{\mu_p} + \lambda_p\right)}{b} \right\}$$ (5)

The second term in $U$ must be nonnegative for the problem to be feasible. The optimal access probability is thus given by

$$p^*_t = \min \left\{U, \max \left\{a - \sqrt{ac/d}, 0\right\} \right\}$$ (6)

B. Primary Feedback-based Access

We provide here the primary queueing analysis assuming the SU operates under the approximations in the previous subsection and leverages the primary ARQ feedback. The SU has two access probabilities: $p_t$ if an ACK or "nothing" was overheard at the end of the previous time slot, and $p_r$ if a NACK was overheard. We assume that the PU retransmits an erroneously received packet by the PR until it is received correctly. The Markov chain describing the PU’s queue is shown in Fig. 1. The probability of the queue having $k$ packets and transmitting for the first time is $\pi_k$, where $F$ in Fig. 1 denotes first transmission. The probability of the queue having $k$ packets and retransmitting is $\epsilon_k$, where $R$ in Fig. 1 denotes retransmission. Define $\alpha_p$ as the probability of successful transmission of the PU’s packet in case of first transmission and $\Gamma_p$ as the probability of successful transmission of the

\(^2\)We assume that the feedback packet is short relative to the slot duration and is always received correctly due to the use of a strong channel code.

\(^3\)The accuracy of estimation depends on $N$. Therefore, we assume that $N$ is high enough for error-free estimations.
PU’s packet in case of retransmission. It can be shown that both probabilities are given by
\[
\alpha_p = (1 - \lambda_p)\,\overline{P}_p + \lambda_p\left(p_t\overline{P}_p^c + (1 - p_t)\overline{P}_p\right)
\]
\[
\Gamma_p = (1 - \lambda_p)\,\overline{P}_p + \lambda_p\left(p_r\overline{P}_p^c + (1 - p_r)\overline{P}_p\right)
\]  
(7)

Solving the state balance equations, we can obtain the state probabilities which are provided in Table I. The probability \(\pi_0\) is obtained using the normalization condition \(\sum_{k=0}^{\infty} (\pi_k + \epsilon_k) = 1\). It should be noticed that \(\lambda_p < \eta\), where \(\eta\) is defined in Table I, is a condition for the sum \(\sum_{k=0}^{\infty} (\pi_k + \epsilon_k)\) to exist. This condition ensures the existence of a stationary distribution for the Markov chain and guarantees the stability of the primary queue. The mean service rate of the SU is given by

\[
\mu_s = \lambda_c \left[\pi_0 p_t \overline{P}_s + \left(\sum_{k=1}^{\infty} \pi_k \right) p_t \overline{P}_s ^c + \left(\sum_{k=1}^{\infty} \epsilon_k \right) p_r \overline{P}_s ^c \right]
\]
(8)

where the probability summations are given in Table I. Applying Little’s law, the mean delay of \(Q_p\) is \(D_p\). \(\lambda_p = \sum_{k=1}^{\infty} k \left(\pi_k + \epsilon_k\right)\). After some mathematical manipulations,

\[
D_p = \frac{(\alpha_p - \eta)(\eta - \lambda_p)^2 + \lambda_p^2 \alpha_p \eta}{(\eta - \lambda_p)\lambda_p \eta \Gamma_p}
\]
(9)

For a fixed \(\lambda_p\), the maximum service rate for the SU is given by solving the following optimization problem:

\[
\max_{p_t, p_r} \mu_s \quad \text{s.t.} \quad 0 \leq p_t, p_r \leq 1, \quad \lambda_p \leq \eta, \quad D_p \leq \overline{D}_p
\]
(10)

The optimization problem is non-convex. It can be solved via a one-dimensional grid search for the optimal value of \(p_r\). That is, fixing \(p_r\) makes the optimization problem convex. Afterwards, we solve a family of convex optimization problems parameterized by \(p_r\). The optimal \(p_r\) is taken as that which yields the highest value of the objective function. Let \(\alpha_p = a - b p_r\), \(\Gamma_p = a - b\), where \(a\) and \(b\) are defined in the previous subsection, and \(\delta_j = \overline{P}_j^c / \overline{P}_j\) which represents the ability of the receiver of user \(j\) to detect packets correctly in the presence of concurrent transmissions. We obtain the following optimization problem for a given \(p_r\):

\[
\max_{0 \leq p_t \leq 1} \left(\frac{a - \lambda_p b p_r - \lambda_p p_t \Gamma_p + \lambda_p b p_r \delta_s}{\Gamma_p} + \lambda_p \delta_s\right) p_t - \lambda_p b p_t^2 \Gamma_p \leq 0
\]

s.t.

\[
(\eta - \Gamma_p) (\eta - \lambda_p)^2 + \lambda_p (h - \eta) + \overline{D}_p (\eta - \lambda_p) (\eta - 1) \Gamma_p \lambda_p \leq 0
\]
(11)

where \(h = \lambda_p + \overline{D}_p \Gamma_p\) and \(\overline{D}_p \geq 1\). It can be shown that problem (11) is convex. The solution can be obtained efficiently using any convex programming technique.\(^5\)

IV. NUMERICAL RESULTS, SIMULATIONS AND CONCLUSIONS

In this section, we present some numerical results. In Fig. 2, we provide the optimal access probabilities, \((p_r^*, p_t^*)\), which achieve the optimal \(\mu_s\) for different \(\overline{D}_p\) and \(\lambda_c\) when the SU leverages primary feedback. The access probabilities are nonincreasing with increases in \(\lambda_p\) for both systems, with and without feedback. It is worth noting from the figure that the optimal access probabilities, \((p_r^*, p_t^*)\), increase with the decrease of \(\lambda_c\) and with the increase of \(\overline{D}_p\). That is, for small \(\lambda_c\) or for high \(\overline{D}_p\), the SU will be more likely to access each time slot. Ditto for the optimal access probability in the system without employing feedback. Fig. 3 shows the impact of the energy arrival rate. The increase of \(\lambda_c\) boosts the optimal \(\mu_s\). Note that the maximum \(\mu_s\) for \(\lambda_c = [0.8, 1]\), for the system employing feedback leveraging, and \(\lambda_c = [0.4, 1]\), for the without feedback system, coincide due to the fact that even if the SU has enough energy it cannot increase its service rate in order to avoid violating PU’s QoS. Note also that the case of \(\lambda_c = 1\) represents an SU plugged to a reliable power supply.

The benefit of the MPR capability is seen in Fig. 4. An enhancement in MPR capability is modeled by increasing the MPR capability is modeled by increasing the error in estimating parameter \(\alpha\) on the optimal access probabilities, \(\lambda_p\), when the estimated \(\lambda_p\) is lower than the actual \(\lambda_p\), and the positive errors in estimating \(\overline{P}_p\), the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
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<tbody>
<tr>
<td>(\eta)</td>
<td>(\lambda_p \alpha_p + \overline{P}_p \Gamma_p)</td>
<td>(\pi_0)</td>
<td>(\pi - \lambda_p / \overline{P}_p)</td>
</tr>
<tr>
<td>(\epsilon_0)</td>
<td>0</td>
<td>(\psi)</td>
<td>(\lambda_p \eta)</td>
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<td>(\pi_0)</td>
<td>(\lambda_p \alpha_p + \overline{P}_p \Gamma_p)</td>
<td>(\epsilon_1)</td>
<td>(\pi - \lambda_p \overline{P}_p / \eta)</td>
</tr>
<tr>
<td>(\pi_k, k \geq 2)</td>
<td>(\lambda_p \alpha_p \overline{P}_p / \eta)</td>
<td>(\epsilon_k, k \geq 2)</td>
<td>(\lambda_p \alpha_p \overline{P}_p / \eta)</td>
</tr>
<tr>
<td>(\sum_{k=1}^{\infty} \pi_k)</td>
<td>(\lambda_p \alpha_p \overline{P}_p / \eta)</td>
<td>(\sum_{k=1}^{\infty} \pi_k)</td>
<td>(\lambda_p \alpha_p \overline{P}_p / \eta)</td>
</tr>
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Feedback, \(p_r^*\)

![Feedback, \(p_r^*\)](image)

Feedback, \(p_t^*\)

![Feedback, \(p_t^*\)](image)

Fig. 2. Access probabilities for the parameters: \(\overline{P}_p = 0.7, \delta_s = 0.8, \delta_p = 0.4, \overline{P}_s = 0.8, \) and \(\overline{D}_p = 2\) time slots.

5Note that the optimization problems (4) and (11) are solved at the SU.
access probabilities are computed under a smaller primary arrival rate and under a higher probability of correct primary packet reception and may exceed the value they should have if \( \lambda_p \) and/or \( \overline{P}_p \) are/is known perfectly. Operating with such increased access probabilities may cause an actual violation of the PU’s QoS. We therefore suggest the following measure to provide protection for the PU against possible service disruption induced by parameter estimation errors. The SU will always multiply its estimate of \( \lambda_p \) by a factor \( \kappa_e \geq 1 \), and will multiply each estimated value of \( \overline{P}_p \) by a factor \( \kappa_o \leq 1 \). These factors depend on the accuracy of the estimators. This approach makes sense in a cognitive context where protecting the PU should take precedence over secondary rate concerns. Fig. 6 presents the impact of positive and negative estimation errors on \( \mu_s \). The parameters used to generate the figure are: \( \kappa_e = 1.25 \), \( \kappa_o = 0.9 \), \( \overline{P}_p = 2 \), \( \lambda_e = 0.5 \), \( \overline{P}_p = 0.7 \), \( \overline{P}_s = 0.8 \), \( \delta_s = 0.8 \) and \( \delta_p = 0.4 \). For both subfigures, the estimation errors cause the degradation of \( \mu_s \) while preserving the PU’s QoS. Note that if we do not use the factors \( \kappa_e \) and \( \kappa_o \), the negative estimation errors of \( \lambda_p \) and the positive estimation errors of \( \overline{P}_p \) would have caused an increase in \( \mu_s \) over the perfect estimation case, but also the PU’s QoS would have been violated for some \( \lambda_p \) and/or \( \overline{P}_p \) values.

Fig. 3. Impact of varying the energy arrival rate, \( \lambda_e \) packets per time slot, on the maximum \( \mu_s \) for the parameters: \( \overline{P}_p = 0.7 \), \( \overline{P}_s = 0.8 \), \( \delta_s = 0.4 \), \( \delta_p = 0.3 \), and \( \overline{D}_p = 2 \) time slots.

Fig. 4. Optimal secondary mean service rate for the parameters: \( \lambda_e = 0.8 \) packets per time slot, \( \overline{P}_p = 0.7 \), \( \overline{P}_s = 0.8 \), and \( \overline{D}_p = 2 \) time slots.

Fig. 5. The effect of varying the delay constraint, \( \overline{D}_p \), on the optimal \( \mu_s \) for the parameters: \( \lambda_e = 0.7 \) packets per time slot, \( \overline{P}_p = 0.7 \), \( \overline{P}_s = 0.8 \), \( \delta_s = 0.4 \) and \( \delta_p = 0.3 \).

Fig. 6. Impact of estimation errors on \( \mu_s \) for the system employing feedback leveraging. The left subfigure represents the impact of \( \overline{P}_p \) estimation error while having perfect \( \lambda_p \) whereas the right subfigure represents the error in estimating \( \lambda_p \) while \( \overline{P}_p \) is estimated perfectly. The parameters used to generate the figure are: \( \kappa_e = 1.25 \), \( \kappa_o = 0.9 \), \( \overline{D}_p = 2 \), \( \lambda_e = 0.5 \) packets per time slot, \( \overline{P}_p = 0.7 \), \( \overline{P}_s = 0.8 \), \( \delta_s = 0.8 \) and \( \delta_p = 0.4 \).

REFERENCES