A Nonlinear PID Stabilizer With Spherical Projection for Humanoids: From Concept to Real-time Experiments

David Galdeano¹, Ahmed Chemori¹, Sébastien Krut¹ and Philippe Fraisse¹

Abstract—This paper deals with a stabilizer for a hybrid kinematic/dynamic control scheme for humanoid robots. The proposed solution is based on a nonlinear PID regulation of the ZMP, coupled with a spherical projection in the CoM’s control space. The result of such a stabilizer is a dynamically stable motion, even with large variations in the inclination of the ground. The effectiveness of this new stabilizer has been demonstrated through real-time experiments on the humanoid robot HOAP-3. The produced motion is smooth and dynamically stable.

I. INTRODUCTION

The goal of developing humanoid robots is to make them working in dull, dirty and dangerous environments where human workers could be in danger. However, up to now, most of humanoid robots are still confined to research laboratories, since they are not reliable enough in real operating conditions.

One of the main features that should be improved to produce reliable humanoid robots is the dynamic stability of movements. Their fall can be dangerous for humans standing in a close proximity of the robots. The robot ability to move in its environment, even if the environment is degraded, is needed to bring robots to consumers application.

Even with an optimal joint trajectory design, the stability of the motion could not be guaranteed if a perturbation occurs. In order to produce stable motions, the robot movement should be adapted online to the external environment using the sensors feedback information. This reactive adaptation allows the robot to evolve in a real world environment. The stability indicators are mathematical tools to quantify the degree of stability stability. Mainly two indicators of stability can be used:

• the Center of Mass (CoM),
• and the Zero Moment Point (ZMP).

The COM is the mean location of all masses of the robot’s links. This indicator is often used for static stability [1]. The ZMP [2] is the point of junction between the resultant vertical reaction force and the ground. It is the most used indicator for dynamic stability [3], [4].

The ZMP indicator can be used in the design of a control law to improves the dynamic stability. The robot is stable if the control is able to keep the ZMP inside the support polygon during robot’s motions.

In the literature, various methods have been proposed to ensure the stability of the robot motions based on the ZMP stability indicator.

One of the most used methods is called the Inverted Pendulum Model (IPM) [5], [6] that considers the robot as a single pointwise mass and massless legs. This method simplifies the dynamics of the robot by using the relationship between the CoM and the ZMP. The robot’s dynamics is approximated by the one of an inverted pendulum with a pointwise mass linked through a telescopic leg to a spherical ground/leg joint. Since then, different extensions have been proposed such as the Linear Inverted Pendulum Model (LIPM) [7], [8]. The preview control of the ZMP [9] has improved the LIPM by reducing the error induced by the simplification of the dynamics of the humanoid robot. The previous three methods are very efficient to perform walking motions, but the model equations are based on the assumption that the ground is flat, therefore the range of admissible motions is limited.

A control law can be design to allows a compliant interaction with external forces. This technique have been implemented on several robots to allows the generation of stables motions by absorbing the external perturbations [10], [11]. However, this technique requires a torque level control [12] to be efficient which is not possible on all humanoids robots.

Some other proposed techniques are based on a sensory feedback system [13], [14]. A stabilizer can be used to control the torque of the ankle joint [15], [16], [17], the angular momentum of the hip [18], [19], the trajectory of the CoM [20], [21] or the decision to take a step [22], [23] in order to ensure the motion stability. The above approaches are very reactive and can adapts to large disturbances. However, they are difficult to be combined with frameworks using the rest of the body of the robot in other useful tasks.

In this paper, a nonlinear ZMP based stabilizer is proposed. This stabilizer is sensory feedback based and has been developed within the framework of an hybrid dynamic/kinematic whole-body control strategy. The whole control architecture with a simpler stabilizer has been proposed in a previous works of the authors [24]. The basic idea of this stabilizer is to use a nonlinear PID controller to regulate the ZMP error and to project this regulation in a CoM tracking objective using a spherical projection. The nonlinear PID controller allows a fast tracking with a favorable damping, improving the stability of the controlled
robot. The spherical projection of the ZMP error compensation allows to adapt the robot posture to large variations in the inclination of the ground.

The proposed stabilizer allows to increase the rejection of external disturbances applied to the humanoid robot.

In the literature, most of the proposed stabilizers relies on precomputed trajectories for the desired ZMP trajectory. With the proposed stabilizer, the regulation of the center of pressure (CoP) is used to produce dynamically stable motions.

The stabilizer has been implemented on HOAP-3, a small robot with position control, noisy sensors data and low computational power. The control scheme can be easily deployed on other robots if they are equipped with joint encoders and CoP sensors on both feet.

This paper is organized as follows: in next section, our humanoid robot demonstrator HOAP-3 is introduced. Section III is devoted to the proposed nonlinear ZMP based stabilizer. The ZMP regulation is mapped in the CoM’s control space using a spherical projection. In this section, its basic principle is presented and discussed. Real-time experimental results are introduced in section IV, with a presentation and discussion of the obtained results. The paper ends with some concluding remarks and an overview on future work.

II. DESCRIPTION OF OUR DEMONSTRATOR

The proposed control scheme within this work will be implemented on HOAP-3 humanoid robot (Cf. Fig. 1), manufactured by Fujitsu Automation. HOAP stands for "Humanoid for Open Architecture Platform".

The robot is equipped with an incremental encoder per joint, a three-axis acceleration sensor, a three-axis gyro sensor, two CDD cameras and four force sensors per feet.

A control PC with RT-Linux real-time operating system allows the control of the robot with a sampling time of 1 msec.

This robot is a versatile testbed for whole-body motion control. Our stabilizer algorithm is implemented in a C language code for real-time experiments.

III. MAIN CONTRIBUTION: A ZMP ERROR PROJECTION BASED NONLINEAR STABILIZER

A. General overview of the control scheme

In a previous work of the authors, a hybrid kinematic/dynamic whole-body control framework has been proposed [24]. This framework has been experimentally validated for squat-like motions.

The framework consist in a kinematic control including four objectives, namely (i) the robot’s feet relative-pose, (ii) the CoM, (iii) the body orientation and (iv) joints’ limits avoidance as illustrated in Fig. 2. The kinematic control is based on the task formalism [25], [26].

![Task formalism](image)

Fig. 2: Block diagram of the control framework proposed in [24] with the control of (i) the robot’s feet relative-pose, (ii) the CoM with a ZMP regulation, (iii) the body orientation and (iv) joints’ limits avoidance.

A ZMP based dynamic feedback has been considered to produce a dynamically stable motions. This stabilizer uses the ZMP error to modify the CoM trajectory.

In this work, we aim at improving this stabilizer through two main contributions:

- A nonlinear feedback control of the ZMP error to improve the rapidity and stability of the stabilizer,
- a sphere projection of this regulation to produce human-like motion even with large variations in the inclination of the ground.

B. Nonlinear ZMP regulation controller

Nonlinear PD (NPD) controllers have been proposed in robotics field as an improvement of the classic linear PD controllers [27], [28], [29], [30].
The nonlinear ZMP regulation control is used to produce a reactive control with a better damping [30]. This allows a faster response without introducing instabilities.

Contrary to classical linear PD controller, the NPD controller uses time-varying gains depending on the tracking errors instead of fixed gains. The general expression of the NPD can be expressed as:

\[ u(t) = k_p(\cdot)e(t) + k_d(\cdot)\dot{e}(t) \]  

where \( k_p(\cdot) \) and \( k_d(\cdot) \) are the time-varying proportional and derivative gains, \( e(t) \) is the system error and \( \dot{e}(t) \) its first time derivative (velocity error).

The time-varying proportional and derivative gains may depend on the system state, its inputs or other variables. In our case, we consider a proportional gain depending on the position error and a derivative gain depending on the velocity error.

The nonlinear proportional gain, illustrated in Fig. 3, is expressed by:

\[ k_p(e) = \begin{cases} 
    k_{p0} |e|^{\alpha_1 - 1}, & |e| > \delta_1, \\
    k_{p0} \delta_1^{\alpha_1 - 1}, & |e| \leq \delta_1.
\end{cases} \]  

where \( k_{p0} \) is the maximum proportional gain, \( \alpha_1 \) is the non-linearity tuning parameter and \( \delta_1 \) its threshold of activation.

\[ \text{Fig. 3: Typical evolution of the nonlinear proportional gain } k_p \text{ versus position error } e \text{ with } \alpha_1 = 0.75 \text{ and } \delta_1 = 1. \]

The nonlinear derivative gain, illustrated in Fig. 4, is given by:

\[ k_d(\dot{e}) = \begin{cases} 
    k_{d0} |\dot{e}|^{\alpha_2 - 1}, & |\dot{e}| > \delta_2, \\
    k_{d0} \delta_2^{\alpha_2 - 1}, & |\dot{e}| \leq \delta_2.
\end{cases} \]  

where \( k_{d0} \) is the maximum derivative gain, \( \alpha_2 \) is the non-linearity tuning parameter and \( \delta_2 \) its threshold of activation.

\[ \text{Fig. 4: Typical evolution of the nonlinear derivative gain } k_d \text{ versus velocity error } \dot{e} \text{ with } \alpha_2 = 1.25 \text{ and } \delta_2 = 1. \]

A NPD controller is not designed to compensate static errors since it does not contain an integral action. Consequently an integral action should be considered to compensate the ZMP static errors. The ZMP error compensation can then be defined as:

\[ u_{ZMP} = k_p(\varepsilon_{ZMP})\varepsilon_{ZMP} + k_d(\frac{d\varepsilon_{ZMP}}{dt})\frac{d\varepsilon_{ZMP}}{dt} + k_i \int \varepsilon_{ZMP} \]  

where \( u_{ZMP} \) is the ZMP error compensation, \( k_i \) is the integral gain and \( \varepsilon_{ZMP} \) the ZMP tracking error.

The ZMP tracking error (as illustrated on Fig. 5), \( \varepsilon_{ZMP} \in \mathbb{R}^2 \times 1 \), is computed from the center of pressure (CoP) measurement as follows:

\[ \varepsilon_{ZMP} = \alpha \cdot dCoP_{left} + (1 - \alpha) \cdot dCoP_{right} \]  

where \( dCoP_{left} \) and \( dCoP_{right} \) are illustrated in Fig. 5 and computed as follows:

\[ dCoP_{left} = CoP_{left, desired} - CoP_{left, measured} \]  

\[ dCoP_{right} = CoP_{right, desired} - CoP_{right, measured} \]

where \( CoP_{left, desired} \in \mathbb{R}^2 \times 1 \) and \( CoP_{right, desired} \in \mathbb{R}^2 \times 1 \) are the desired CoP positions respectively under the right and left feet. Their values are chosen to be at the center of each foot \( \{x,CoP_{left,desired} = 0 ; y,CoP_{left,desired} = 0\} \). \( CoP_{left, measured} \in \mathbb{R}^2 \times 1 \) and \( CoP_{right, measured} \in \mathbb{R}^2 \times 1 \) are the measured (thanks to the force sensors) CoP positions, under the right and left foot respectively.

The weight \( \alpha \) of the distribution in eq. (5) is computed from the amplitudes of contact forces measured under the feet:

\[ \alpha = \frac{ACoP_{left}}{ACoP_{left} + ACoP_{right}} \]  

where \( ACoP_{left} \) and \( ACoP_{right} \) are the amplitudes of forces measured respectively on the right and left foot of the robot.
The ZMP tracking error, $\varepsilon_{ZMP}$, is a weighted distribution of ZMP errors on both feet. The weights of this distribution are proportional to the amplitude of the ground contact forces on both feet. This distribution allows a balanced regulation during the double support phase. However, during the single support phase, the error of the force sensor on the swing foot is zero and consequently neglected by the weighted distribution while the ZMP error regulation is focused on the foot of support.

In eq. (5), the ZMP error compensation is not computed using a ZMP reference trajectory in global coordinates but rather a local coordinates CoP reference trajectory since the desired CoP is at the center of each foot.

C. Spherical projection of the ZMP regulation

In the control scheme proposed in [24], the ZMP error compensation was directly added to the CoM objective. Since the CoM trajectory was modified in the transverse plane, this concept was limited to small ZMP errors and flat ground. However, for the case of large ZMP errors or the case of an inclined ground, the ZMP error compensation should be carefully managed to avoid structural singularities produced by stretching the desired CoM out of reach.

The proposed solution is then to project the ZMP error compensation on a virtual sphere (as illustrated in Fig. 6). This sphere is defined by its center positioned on the ground, at the ZMP position, and its radius is equal to the initial CoM height as illustrated on Fig. 6.

The spherical projection equation of the ZMP error compensation $\varepsilon_{SP} = [\varepsilon_{SPX} \varepsilon_{SPY} \varepsilon_{SPZ}]^T$, with $u_{ZMP} = [u_{ZMPX} u_{ZMPY}]^T$ from eq. (4), can be expressed as:

$$
\varepsilon_{SPX} = h_{CoM} \sin \left( \beta \cdot \frac{u_{ZMPX}}{h_{CoM}} \right), \\
\varepsilon_{SPY} = h_{CoM} \sin \left( \gamma \cdot \frac{u_{ZMPY}}{h_{CoM}} \right), \\
\varepsilon_{SPZ} = \sqrt{h_{CoM}^2 - u_{ZMPX}^2 - u_{ZMPY}^2},
$$

where $h_{CoM}$ denotes the initial CoM height, $\beta$ and $\gamma$ are adjustable parameter.

The spherical projection parameters $\beta$ and $\gamma$ allows to tune the shape of the spherical projection as illustrated on Fig. 7.

The projection of the ZMP error compensation is then added to the CoM tracking as follows:

$$
\varepsilon_{CoM\&ZMP} = \varepsilon_{CoM} + \varepsilon_{SP}
$$

Fig. 5: Illustration of the tracking errors on the ZMP position.

Fig. 6: Illustration of the sphere projection space w.r.t. the norm of $u_{ZMP}$ ($\beta = 1$ and $\gamma = 1$). Footprints are displayed in gray.

Fig. 7: Sphere projection space w.r.t. the norm of $u_{ZMP}$ ($\beta = 0.5$ and $\gamma = 0.75$). Footprints are displayed in gray.

In order to produce an human-like reaction to slope variation of the ground and to avoid auto-collision of the legs...
of the robot, the torso orientation must be managed. Since the upper-body orientation is not applied on the CoM tracking but on the torso orientation tracking, this does not modify the stability of the robot. The desired torso orientation becomes:

\[
T_{\text{torso \_ori \_d}}(x) = \frac{\arctan2(u_{ZMP}(y), h_{CoM})}{\pi},
\]

\[
T_{\text{torso \_ori \_d}}(y) = \frac{\arctan2(u_{ZMP}(x), h_{CoM})}{\pi},
\]

where \( T_{\text{torso \_ori \_d}}(x) \) and \( T_{\text{torso \_ori \_d}}(y) \) are the desired torso orientations. The desired torso orientation is managed in a different regulation objective than the stability objective.

The Fig. 8 illustrate the CoM trajectory in the frontal plane if a disturbance is applied on the ZMP on \( y \) axis.

This stabilizer improves the stability locally by modifying the reference position of the CoM. The quick reaction of the ZMP-based stabilizer to CoP errors allows to converge to stable motions if quick and small perturbations occurs. Large and slow disturbances are taken into account using the integral part and the spherical projection to allows a stable posture on inclined ground.

It does not guarantees a global stability, which needs the decision of taking a step to keep stability in case of quick and large disturbances.

IV. REAL-TIME EXPERIMENTAL VALIDATION

The proposed stabilizer presented in section III has been implemented with the control scheme of Fig. 2 on the HOAP-3 humanoid robot presented in section II.

Three experimental scenarios have been performed to show the efficiency of the proposed stabilizer. For the two first scenarios, the robot’s reference trajectories of the relative feet pose are chosen to be constant, which means that the feet should remain stationary. The CoM position is set to the initial CoM position and kept constant, which means that the robot should not move excepted for ZMP stabilization.

In the first scenario, the slope of the ground has been varied to demonstrate the adaptation of the proposed stabilizer to an inclined ground.

In the second scenario, the robot is pulled with a constant force to demonstrate the reaction of the proposed stabilizer to external disturbances.

The third scenario demonstrates a task of dynamic walking on uneven ground with an unexpected variation in the slope of the ground.

In the sequel, the three scenarios will be detailed.

A. Scenario 1: Online adaptation toward slope variation of the ground

The objective of this scenario is to demonstrate the adaptation of the proposed stabilizer against slope variation of the ground. The experimental test platform consist of a wooden board (as shown in Fig. 9) which is lifted from one side, therefore creating a rotation around the opposite edge of the board. The spherical projection tuning parameter are set to \( \beta = 1 \) and \( \gamma = 1 \) for this scenario.

![Fig. 8: Frontal view of the sphere projection with \( u_{ZMPX} = 0 \) and \( u_{ZMPX} \) varying in \([-h_{CoM} \frac{\pi}{2}; h_{CoM} \frac{\pi}{2}] \) domain. Robot’s structure and footprints are displayed in gray. Spherical projection tuning parameter are set to \( \beta = 1 \) and \( \gamma = 1 \) for this scenario.](image)

![Fig. 9: Illustration of the real-time adaptation against ground’s inclination.](image)
displacement. The CoM trajectory is modify to keep balance. Along the $z$ axis, the trajectory is also kept constant at the height of the initial CoM position.

In Fig. 11, the evolution of the measured ZMP and CoM positions are plotted with respect to the footprints of the robot. The trajectory of the desired CoM is constant. The observed variation on the CoM position is due to the adaptation of the stabilizer to the ZMP displacement to keep balance.

The robot’s body adapts to ground’s inclination variation. The combination of the CoM position adjustment and the hip rotation allow a smooth motion. The posture of the robot during large ZMP disturbances on Fig. 9(b) is looking natural, the torso is not inclined like the ground.

**B. Scenario 2: Robustness towards external disturbances**

The objective of this scenario is to demonstrate the robustness of the proposed stabilizer towards external disturbances.

The experimental test platform consists in hooking a lightweight rope to the shoulder of the robot. Then, the rope is pulled using a pulley system with a constant counterweight. This system allows, under the gravity, to apply a constant external force (due to the counterweight) on the robot body as illustrated in Fig. 12.

The spherical projection tuning parameter are $\beta = 1$ and $\gamma = 1$ for this scenario.

In Fig. 13, the evolution of the CoM position expressed in the right foot’s reference coordinate is displayed. Along the $x$ axis, the trajectory is constant since no perturbation on this axis has been introduced. Along the $y$ axis, the trajectory of the CoM should be regulated around its desired value (constant). Indeed, the observed variation is due to the adaptation of the stabilizer to the ZMP displacement. The CoM trajectory is modify to keep balance. The trajectory converges to stable positions whether the weight is added or removed. Along the $z$ axis, the trajectory is kept constant at the height of the initial CoM position.

In Fig. 14, the evolution of the measured ZMP and CoM positions are displayed with respect to the footprints of the biped robot.

According to this result, one can conclude that the proposed stabilizer complied with the task-based controller is robust towards external forces disturbing the ZMP position. The stabilizer modifies automatically the desired CoM to fairly distribute the ground’s reaction forces, this produces a smooth motion to resist to the introduced external perturbation.
C. Scenario 3: Walking on an uneven ground

The objective of this scenario is to show the robustness of the control scheme against ground’s slope variation while walking.

The robot is walking on a flat ground followed by a five degrees inclined plane as illustrated in Fig. 15. A simple B-splines based pattern generator under Matlab\textsuperscript{1} software is used to produce the desired CoM position and the feet relative pose trajectories.

This pattern generator was used to produce the feet and the CoM trajectories in operational space. The objectives’ trajectories have been designed to produce a walk of six steps. The produced feet and CoM trajectories are then expressed in the right foot reference coordinates in order to be used as reference trajectories.

The spherical projection tuning parameter are $\beta = 1$ and $\gamma = 1$ for this scenario.

In Fig. 16, the evolution of the CoM position expressed in the right foot reference coordinates is displayed. On the $x$ axis, the trajectory is similar to the classical trajectory given by an inversed pendulum model, except that the ZMP regulation shifts the CoM forward (increase on $x$ axis) to maintain the stability of the robot when walking on the inclined ground. On the $y$ axis, the trajectory is similar to the classical trajectory given by an inversed pendulum model. On the $z$ axis, the trajectory is at the height of the CoM; however, it is is lowered when the reference foot is lifted above the ground, to keep the same CoM height.

\textsuperscript{1}Matlab is a registered trademark of The Mathworks, Inc.

V. CONCLUSION AND FUTURE WORK

This paper deals with a new efficient stabilizer for humanoid robots. The proposed solution is based on a nonlinear PID based regulation, coupled with a spherical projection of the ZMP regulation error in the CoM jacobian control.
The main advantage of this stabilizer lies in the enhancement of the stability of the robot under the control of a previously proposed control framework for whole-body motions control [24].

The proposed stabilizer is based on a reactive behavior and uses the center of the feet as reference’s trajectories for the CoP to compute a ZMP error. Therefore, the obtained motion is more natural than the one obtained by classical control methods such as the linear inverted pendulum in case of disturbance of the stability. The obtained results are very promising.

In future work, we aim at extending this work for more complex scenarios including carrying objects or in human-robot interactions.

REFERENCES


