A Novel Application of Multivariable $L_1$ Adaptive Control: from Design to Real-Time Implementation on an Underwater Vehicle

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Abstract—This paper presents the design and experimental implementation of the $L_1$ adaptive controller on a tethered underwater vehicle. This controller, well known for its fast adaptation and its robustness, is for the first time applied in the field of underwater vehicle control. This paper summarizes the implementation and experimental results obtained on a modified version of the AC-ROV underwater vehicle. Various scenarios are presented to illustrate the ability of the $L_1$ adaptive law not only to successfully control pitch and depth (even with strong modeling uncertainties), but also to be robust towards disturbances like waves or buoyancy changes.

I. INTRODUCTION

Underwater vehicles have gained an increased interest in the last decades given the multiple of operations they can perform in various fields. We are particularly interested in the category of tethered vehicles also called remotely operated vehicles (ROV). Different challenges in autonomous control of such systems arise from the inherent high nonlinearities and time varying behavior of the vehicle’s dynamics subjected to hydrodynamic effects and disturbances. Various approaches to solve this control problem can be found in the literature. Some $H_\infty$ control approaches were proposed and tested in simulations [1]. Various chattering free sliding mode schemes have been applied on such systems to cope with heavy uncertainties and were experimentally validated [2]. Intelligent control methods applying reinforcement learning or artificial intelligence can be found in [3][4] and [5] where simulation results are provided and an experimental study was reported in [6]. Adaptive controllers are seen to be very common in such applications such as in [7][8][9]. The use of an adaptive controller is motivated in particular by the presence of uncertainties in the model parameters and their likelihood to change. The salinity changes the buoyancy parameter; the addition of sensors or the manipulation of objects changes the mass parameters and the damping parameters are greatly affected by the encounter of algae or moving obstacles. For similar reasons, adaptive controllers have been used for system identification and are highly appreciated by the aircraft control community. Despite their success in many applications, adaptive controllers hold various drawbacks [10]. In [11] an extensive study has been made to show that a wide range of such controllers has been used with restrictive assumptions and concluded that adaptive controllers exhibit undesirable frequency characteristics. They also rely on the need of a persistancy in parameter excitation before convergence which may lead to a bad transient behavior [12]. Several attempts have been proposed to remedy the shortcomings of these controllers. A large adaptation gain leads to undesirable effects with the risk of parameter divergence. That’s why most of the methods suggest limiting this gain at the cost of a slower adaptation and convergence [13]. The recently developed $L_1$ adaptive controller eliminates the need of such a tradeoff since it relies on an architecture where robustness and adaptation are decoupled. It is based on the presence of a low pass filter in its feedback loop and is only limited by the hardware capabilities since the adaptation gain can be increased as much as needed to achieve a faster convergence given that the boundedness of the parameters is ensured by the projection operator present in the adaptation law. The proof of asymptotic stability is ensured by the small gain theorem [14]. This controller is able to revisit the failures of adaptive controllers by maintaining its performance and robustness in situations where other controllers cannot [15][16]. The $L_1$ adaptive controller has been validated through simulations and experiments mainly on aerial vehicles [17][18], but it was also seen in different applications such as the control of the acrobot [19] and the hysteresis in smart materials [20]. The main contribution of this paper is the experimental demonstration of a new application of this controller which is the depth and pitch control of an underwater vehicle. To the best knowledge of the authors, this method has never been applied yet to control underwater robots. This paper is organized as follows: in the second section we present the dynamic modeling of the system, the third section shows the theoretical aspect of this controller and its novel design of implementation on underwater vehicles. The fourth section presents the prototype and the experimental setup and in the fifth section we analyze the experimental results.

II. DYNAMIC MODELING OF THE SYSTEM

By considering the inertial generalized forces, the hydrodynamic effects, the gravity, and buoyancy contributions as well as the effects of the actuators (thrusters), the dynamic model of an underwater vehicle in matrix form, using the SNAME notation and the representation proposed by Fossen in [21], is written as:

$$\dot{\eta} = J(\eta)\nu$$  \hspace{1cm} (1)

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau + w_d$$  \hspace{1cm} (2)

where $\nu = [u, v, w, p, q, r]^T$, $\eta = [x, y, z, \phi, \theta, \psi]^T$ are vectors of velocities (in the body-fixed frame) and position/Euler angles (in the earth-fixed frame) respectively (Fig. 1).
We are studying in this paper the dynamics of the vehicle in frame and can be transformed to the earth frame by using actions on the system to fully control it. The presented \( \tau \) forces.

Coriolis-centripetal (including added mass), and damping respectively, while \( g \) is a vector of gravitational/buoyancy forces. \( \tau \) is the vector of control inputs and \( w_d \) the vector of external disturbances. In the case of our study, the vehicle used has a slow dynamics, and hence it will be moving at velocities low enough to make the Coriolis terms negligible (\( C(v) \approx 0 \)).

Equation (2) describes the dynamics of the system in 6 degrees of freedom taking into account the 3 translations and 3 orientations. The input vector \( \tau \in \mathbb{R}^6 \) considers 6 actions on the system to fully control it. The presented formulation of the robot’s dynamics is expressed in the body frame and can be transformed to the earth frame by using the kinematic transformations of the state variables and the model parameters as shown below:

\[
\dot{\eta} = J(\eta)v \\
\ddot{\eta} = J(\eta)\dot{v} + J(\dot{\eta})v \\
M^*(\eta) = J^{-T}(\eta)M J^{-1}(\eta) \\
D^*(v, \eta) = J^{-T}(\eta)D(v)J^{-1}(\eta) \\
g^*(\eta) = J^{-T}(\eta)g(\eta) \\
\tau^* = J^{-T}(\eta)\tau \\
w_d^* = J^{-T}(\eta)w_d
\]

Equation (2) can therefore be expressed in the earth frame as:

\[
M^*(\eta)\ddot{\eta} + D^*(v, \eta)\dot{\eta} + g^*(\eta) = \tau^* + w_d^* \tag{4}
\]

We are studying in this paper the dynamics of the vehicle in its translational motion along the \( z \) axis and its orientation with respect to the pitch angle. Therefore, we will get \( M^*(\eta), D^*(\eta) \in \mathbb{R}^{2 \times 2} \) and \( g^*, \tau^*, w_d^* \in \mathbb{R}^2 \).

\( \tau^* \) is the control input expressed in the earth frame in Newton and is given by:

\[
\tau^* = J^{-T}TKu
\]

where \( u \in \mathbb{R}^2 \) is the vector of control inputs in volts (as depicted on Fig.1, we have two vertical thrusters acting simultaneously on the two degrees of freedom of interest), \( K \) is the force coefficient in Newton.\( T \in \mathbb{R}^{2 \times 2} \) is the actuators’ configuration matrix, taking into account the position and orientation of the thrusters.

III. PROBLEM FORMULATION AND PROPOSED SOLUTION

Our objective is to achieve depth and pitch control of a highly nonlinear system with unknown and varying model parameters in presence of disturbances. For this purpose, a robust adaptive controller will be proposed. In this section, the state space representation extracted from the dynamical model (4) will be used for the design of the \( L_1 \) adaptive controller implemented for the first time on an underwater vehicle.

A. Problem Formulation

We consider the following class of systems as suggested in [14]:

\[
x_1(t) = x_2(t) \quad x_1(0) = x_{b1} \\
x_2(t) = A_2x_2(t) + f_2(t, x(t)) + B_2u(t) \quad x_2(0) = x_{b2} \tag{6}
\]

\( y(t) = Cx(t) \)

where \( x_1 \in \mathbb{R}^n \) and \( x_2 \in \mathbb{R}^m \) are the states of the system forming the complete state vector: \( x(t) = [x_1(t), x_2(t)]^T \). \( A_2 \) is a known \( n \times n \) matrix and \( B_2 \in \mathbb{R}^{m \times n} \) is a constant full rank matrix. \( u(t) \in \mathbb{R}^m \) is the control signal (\( m \leq n \)) and \( \omega \in \mathbb{R}^{m \times n} \) is the uncertainty on the input gain. \( C \in \mathbb{R}^{m \times m} \) is a known full rank constant matrix, \( y \in \mathbb{R}^m \) is the measured output and \( f_2 \) is an unknown nonlinear function. In matrix form, the system (6) becomes:

\[
\dot{x} = Ax + f + Bou
\]

with \( A = \begin{bmatrix} 0_{m \times n} & \mathbb{I}_n \\ 0_{n \times m} & A_2 \end{bmatrix} \), \( f = \begin{bmatrix} 0_{n \times 1} \\ f_2 \end{bmatrix} \) and \( B = \begin{bmatrix} 0_{n \times m} \\ B_2 \end{bmatrix} \)

Applying the same formalism as equation (6), the state space representation of the studied dynamics is extracted from the model (4) as:

\[
\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \\ \dot{\eta}_4 \\ \dot{\eta}_5 \\ \dot{\eta}_6 \end{bmatrix} = \begin{bmatrix} 0_{2 \times 2} & \mathbb{I}_2 \\ 0_{2 \times 2} & -\frac{c}{m} \mathbb{I}_2 \\ 0_{2 \times 2} & \frac{g}{m} \mathbb{I}_2 \\ 0_{2 \times 2} & -\frac{d}{m} \mathbb{I}_2 \\ 0_{2 \times 2} & \frac{d}{m} \mathbb{I}_2 \\ 0_{2 \times 2} & \frac{d}{m} \mathbb{I}_2 \end{bmatrix} \omega \tau^* \tag{8}
\]

where \( \eta_1 = [z, \theta] \) and \( \eta_2 = [\dot{z}, \dot{\theta}] \). \( \tau^* \) is expressed in Newton and \( \omega \) is the uncertainty on the input gain. In this case \( \omega \) is considered to be a diagonal matrix \( \in \mathbb{R}^{2 \times 2} \).

The output becomes:

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} z \\ \theta \end{bmatrix} \tag{9}
\]

B. Proposed Solution: \( L_1 \) Adaptive Controller

To control the system (8), an \( L_1 \) adaptive controller is implemented. The choice of this controller is motivated by its architecture based on the decoupling between adaptation and robustness. High adaptation gains can be chosen securing a fast convergence with a smooth transient response. This architecture described in [22] is shown in the following block diagram (Fig. 2). Block 1 is the studied system extracted from (6) with \( A_m \) being a known Hurwitz matrix determining the desired closed loop dynamics and calculated using a gain \( k_m: \ A_m = A - B k_m \) with \( A \) being the state matrix matrix and \( B \) the input matrix of the state space vector.
C. Implementation of the Controller on the Modified AC-ROV

The controller is designed using a state feedback control law. The closed loop filter is described in (8). The unknown linear function $f_2$ has been decomposed into a parameter space dependent part $\theta(t)||x(t)||_{\mathcal{L}_\infty}$, with $||x(t)||_{\mathcal{L}_\infty}$ being the infinity norm of the state $x$ at time $t$, and a nonlinear part $\sigma(t)$, which accounts for the external disturbances. The $\mathcal{L}_1$ architecture is shown in dotted lines in Fig. 2. It is composed of 3 main stages. The prediction stage (block 2), where the states of the system as well as the outputs are calculated at every iteration using the estimation of the parameters. The adaptation stage (block 3) uses the error between the measured states and the estimated ones to ensure their boundedness. The parameter vector $\hat{x}(t)$ is estimated by an alpha-beta observer. The last stage (blocks 4 and 5) pertains to the design and represents the uncertainties on the damping coefficients. The control law for our system becomes:

$$u(t) = \mathcal{Q} \hat{x}(t) + B_m(\omega(t)\omega(t)) + \mathcal{Q} \hat{\sigma}(t)$$

with $\mathcal{Q}$ being a positive feedback gain and $\hat{\sigma}$ a positive feedback gain

The control input is computed in the earth frame and should be transformed into the robot frame. We compute our input as:

$$u = J^T T^{-1} K^{-1}(u_c + u_m) \in \mathbb{R}^2,$$

with $u_c$ and $u_m$ explained in the previous section. The gains for depth control are chosen as $\Gamma_1 = 100000$ and $k_1 = 0.15$ being the adaptation gain and the feedback gain respectively. For the pitch angle, the gains are $\Gamma_2 = 1300$ and $k_2 = 1.2$. The filtering process was ensured by the choice of $D(s) = \frac{1}{\Delta s}$. It has to be mentioned that the gains must ensure the $\mathcal{L}_1$ norm condition, but have to be tuned empirically since there is no systematic method to chose them.

IV. EXPERIMENTAL SETUP

A. Modified AC-ROV experimental platform

The AC-ROV submarine (Fig. 1) is an underactuated vehicle, the propulsion system of which consists of six thrusters driven by DC motors and controlling five degrees of freedom. Motors 1, 2, 3 and 4 control simultaneously translations along $x$ and $y$ axes and rotation around the $z$ axis (yaw). Motors 5 and 6 control depth and pitch. Roll is left uncontrolled but remains naturally stable due to the positive metacentric distance. The robot weighs 3kg and has a rectangular shape with height 203mm, length 152mm and width 146mm. For measurements purposes, our prototype is equipped with various sensors. A 6 DOF IMU (Inertial Measurement Unit) measures roll, pitch, and yaw along with their respective velocities and a pressure sensor allows depth measurement. To pre-process and transmit the sensors’ data to the PC, a microcontroller board is used (Fig. 3). Once the control law has been computed by the control PC, the control input are transmitted to the power stage. Then, 6 PWM modulated signals are sent to the motors of the AC-ROV through the 40-meter long tether. Fig. 3 shows a schematic view summarizing the various components of the vehicle’s hardware and their interactions.

B. Conditions of the Experiments

The experiments have been performed in a $5m^3$ pool. The tether has been sufficiently unrolled to avoid inducing additional drag into the dynamics of the vehicle. The feedback gains computed for each of the control laws and used in nominal conditions, have been kept unchanged for the rest of the experiments despite some eventual changes in the model or its environment in order to evaluate the robustness of each controller. The noisy data of depth measured by the pressure sensor are filtered using a second order Butterworth filter. The information concerning the velocity in the $z$ direction is estimated by an alpha-beta observer. Fig. 4 displays the experimental test-bed used.

V. REAL TIME EXPERIMENTS

A. Experimental scenarios

The experiments presented in this section result from the application of the proposed controller detailed in section 3,
to the underwater vehicle testbed described in section 4. We will start by explaining the different performed scenarios and then we will analyze the obtained results presented through figures 5 to 7. The vehicle is regulated to reach a depth of 0.5m when starting from a static surface position. The pitch angle is controlled to follow a varying trajectory starting at 0° and changing to 15° at 35 seconds. The evolution of the control inputs, generated by the thrusters controlling the movements along the z axis and the angle around the y axis (pitch angle) are also plotted for each scenario as well as \( \theta \) and \( \sigma \) being respectively the parameters and the disturbances pertaining to each degree of freedom.

Three experimental scenarios were performed, namely:

**Scenario 1:** Control in Nominal Conditions.

The objective of this scenario is to control depth and pitch angle of the AC-ROV without any external disturbance. The gains for each controller have been tuned to accommodate this case and were kept unchanged for the rest of the experiments.

**Scenario 2:** Robustness towards Model Changes: Change in Buoyancy.

The model of the vehicle has been changed by the addition of a rectangular piece of polyester introducing a change of buoyancy of approximately 0.32N (which represents a +32% increase of the flottability). The objective of this scenario is to see whether the proposed controller is sufficiently robust to compensate this uncertainty and keep the performance of the controlled closed loop system. Such a disturbance may occur for instance when the robot navigates in environments with strong salinity changes (e.g. undersea fresh water spring) or when the payload of the robot is changed (e.g. additional sensors).

**Scenario 3:** Robustness towards Persistent External Disturbances: Waves.

Waves were generated manually by periodically disturbing the surface of the pool, which created waves of 15cm amplitude.

**B. Control in Nominal Conditions**

Fig. 5 displays the evolution of the controlled vehicle’s depth (0.5m) and pitch angle. The desired depth is reached smoothly in around 40 seconds. A similar response is observed for the pitch angle, except that the convergence time is longer (65 seconds). The small (\( \approx 3° \)) initial oscillations of \( \theta \) will be present in all the scenarios and they are caused by the differences in the starting torques (dry friction) of the thrusters 5 and 6. These latter reach steady state forces of \(-0.18N\) and \(-0.66N\) respectively. Their plots displayed in Fig. 5b converge to their final values with neither oscillation, nor overshoot, despite the lack of knowledge of our model parameters. The vectors \( \hat{\theta} \) and \( \hat{\sigma} \) initialized to 0 and depicted in Fig. 5c converge to the following steady state values: \( [\hat{\theta}_1 \, \hat{\sigma}_1]^T = [-8 \, -2.25]^T \) and \( [\hat{\theta}_2 \, \hat{\sigma}_2]^T = [-48 \, -5]^T \). It can be noticed that unlike nonlinear state feedback adaptive controller the \( L_1 \) controller ensures a fast convergence even without any a priori estimate of the unknown parameters.

**C. Robustness Towards Model Changes: Change in Buoyancy**

Like in the previous scenario, parameters are expected to adapt to their new values to compensate for this change in order for the controlled degrees of freedom to converge to their desired trajectories. The depth response (Fig. 6a) converges in 40 seconds as well, and it is seen to exhibit the same behavior observed in the nominal conditions (Fig. 5a).

A similar behavior is also seen with the pitch angle except that, for this degree of freedom, we observe an additional delay of 5 seconds in the convergence to the steady state value when compared to the nominal condition. Although the buoyancy change has hardly no effect on the responses, interesting changes can be observed in the control inputs and the parameters. The forces needed at steady state are \(-0.12N\) and \(-0.75N\), and are seen to be different than the nominal scenario since more force is now required to immerse the vehicle. The parameters converge to the following values: \( [\hat{\theta}_1 \, \hat{\sigma}_1]^T = [-8 \, -2]^T \) and \( [\hat{\theta}_2 \, \hat{\sigma}_2]^T = [-65 \, -6]^T \). It
was expected that vector $\hat{\theta}$ does not vary since it holds the parameters of damping that were kept unchanged with this modification, which only affected the buoyancy force present in vector $\hat{\sigma}$. This force has an important impact on the motion along the $z$ axis which is seen with the noticeable change in the parameter $\sigma_z$ (from -48 to -65). The main interest of this scenario is to highlight the fact that due to its very large adaptation gain, in this case the convergence time of the $L_1$ controller remains nearly constant even with a strong model change.

D. Robustness Towards Persistent Disturbances: Waves

The obtained results of this scenario are depicted in Fig. 7. Fig. 7a shows the system response of the robot in presence of waves. The depth is not seen to be affected by this persistent disturbance while varying oscillations of approximately 5° of amplitude around the regulated pitch angle are observed in the response of $\hat{\theta}$. This can be explained with the different dynamics of each degree of freedom. The translation around the $\theta$ axis is less sensitive to external disturbances than the pitch angle due to the robot’s inertia. Oscillations of 0.07N are also seen in the control input but they are more significant in $\hat{\theta}_y$ and $\hat{\sigma}_{\theta}$ which explains the maintained oscillations of the pitch response. $\sigma_z$ converged to the desired depth of 0.5m in 40 seconds with $\hat{\theta}_z = -14.5$ and $\hat{\sigma}_z = -60$. The parameters damping are changed with respect to their nominal conditions and this change is reflected in the steady state value of $\hat{\theta}_z$ that was $-8$ in the nominal case and became $-14.5$ in presence of waves. The induced disturbances along the $z$ axis caused by the waves are incorporated in $\hat{\sigma}_z$ that varied from $-48$ in the nominal conditions to $-60$ in this present scenario.

VI. CONCLUSION AND FUTURE WORK

This paper deals with the problem of depth and pitch control of an underwater vehicle. The proposed solution lies in the design and implementation of the $L_1$ controller, novel in the field of underwater robotics. This controller was tested in nominal case, as well as in presence of persistent disturbances (waves) and parameter change in order to highlight the robustness and adaptation features of this controller applied on a multi input multi output system with a coupled nonlinear dynamics. The $L_1$ controller was observed to converge smoothly to the desired trajectory in the two studied degrees of freedom and compensate for the external disturbances and the change in buoyancy. Our future work will include the control of the remaining degrees of freedom.

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Fig. 5: Control in nominal regime: plots of (a) the system outputs \( (z, \theta) \), (b) the control inputs, and (c) parameters \( \hat{\theta} \) and \( \hat{\sigma} \).

Fig. 6: Robustness towards model changes: change in buoyancy: plots of (a) the system outputs \( (z, \theta) \) are very similar to the nominal case. The change of buoyancy is observed through the plots of the control inputs (b) and the controlled parameters \( \hat{\theta} \) and \( \hat{\sigma} \) (c).

Fig. 7: Robustness towards persistant disturbances (waves): only the pitch angle was affected by the waves while the depth response has the same behavior as in the nominal case (a). The introduction of this external disturbance is reflected in the oscillations of the control inputs (b) and the controlled parameters (c).