Multi-phase search optimisation algorithm for constrained optimal power flow problem

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Abstract: This paper proposes an enhanced solution for security constrained optimal power flow (SC-OPF) problem based on multi-phase search optimisation algorithm (MSOA). The objective is to minimise the generation costs by optimising the control variables, such as generator power, and satisfying system constraints. MSOA simulates the performance of humans’ intelligent search with memory, experience and uncertainty reasoning. The proposed algorithm is integrated with Lagrangian relaxation factors to deal with network constraints. The proposed technique is carried out on the IEEE 30-bus, 57-bus test systems and a real power system at West Delta Network as part of the Unified Egyptian Network. The space reduction strategy succeeded to decrease the search space in each generation causing fast convergence to the optimal solution. The obtained results are compared with particle swarm optimisation technique to prove the effectiveness of MSOA in solving SC-OPF problems in normal and emergency conditions.

Keywords: emergency conditions; genetic algorithm; optimal power flow; particle swarm optimisation; seeker optimisation algorithm; SOA.


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1 Introduction

Security constrained optimal power flow (SC-OPF) is a critical optimisation problem in power systems, which is characterised by the non-linear nature of the objectives and constraints. The SC-OPF problem aims to determine the lowest-cost and most efficient and reliable operation of power systems. This can be accomplished by the optimal setting of control variables in power systems such as dispatching the generation resources to supply the total load demand (Zhu, 2009).

Many conventional and modern meta-heuristics optimisation techniques have been applied for solving the SC-OPF problem. Comprehensive survey and discussions on the conventional techniques for solving SC-OPF problems such as linear programming, non-linear programming, quadratic programming, Newton-based techniques, sequential unconstrained minimisation technique, and interior point methods have been presented (Okamoto, 2011). Conventional techniques suffer from the problems of high dimensionality, sensitivity to the initial conditions, large simulation time and insecure convergence properties. The modern meta-heuristic algorithms are capable alternatives for solving many power system problems.

Examples of developed and successfully applied techniques for solving SC-OPF problems are genetic algorithm (Poland et al., 2012; Sheta et al., 2012; Muñuzuri et al., 2012; El-Sehiemy et al., 2013), artificial neural networks (Amirruddin and Zin, 2011), simulated annealing (Vanitha and Thanushkodi, 2012), evolutionary programming (Gaing and Ou, 2009; Niknam et al., 2011; Rajan, 2010), particle swarm (García-Gonzalo and Fernández-Martínez, 2012; Ali and Sabat, 2012; Abido, 2002; Maleh et al., 2013), ant colony (Panigrahi et al., 2007), differential evolution (Nomana and Iba, 2008; Rabiee et al., 2012; Duvvuru and Swarup, 2011) and bacterial foraging algorithm (Panigrahi and Pandi, 2008).

Recently, human thinking-based optimisation techniques, such as seeker optimisation algorithm (SOA) (Shaw et al., 2011; Dai et al., 2009a, 2009b, 2010), represented essential population-based heuristic search algorithms. The SOA is based on the concept of simulating the behaviour of humans’ intelligent search with their memory, experience, and uncertainty reasoning. SOA is applied in many fields for solving power system problems such as optimal load dispatch (Shaw et al., 2011), reactive power dispatch (Dai et al., 2009a, 2009b) and digital infinite impulse response ‘IIR’ filter design (Dai et al., 2010).

In this paper, suggested modifications are introduced to SOA algorithm to provide more convenient in solving the SC-OPF problem. Proper adjustments of committed generating units and system controlled variables to meet the power demand are achieved with preserving system constraints. Numerical results, carried out on the IEEE 30 bus network, the 57 bus network and West Delta Network (WDN), are compared with PSO results. Also, the SC-OPF problem is solved under power system emergency situations due to their importance to assess the system ability to face these unexpected conditions and to provide suitable preventive control actions.

2 Problem formulation

The SC-OPF problem can be expressed in the form of a constrained optimisation problem as follows:

\[
\text{Min } f(x) \quad (1)
\]

s.t.

\[
\begin{align*}
  g(x) &= 0 \quad (2) \\
  h(x) &\leq 0 \quad (3)
\end{align*}
\]

where \(f(x)\) is the objective function that can be generator-fuel costs, transmission line losses, etc., \(g(x)\) represents the equality constraints, \(h(x)\) represents the inequality constraints, and \(x\) is a vector of the controlled variables that can be the generator real power outputs, generator voltages, switchable reactive power, transformer tap setting, etc.

In this paper, the SC-OPF objective function is a non-linear equation representing the fuel cost of generators with quadratic functions depending on the generator real power outputs as follows:

\[
\text{min } F_i = \sum_{i=1}^{NG} f_i(PG_i) = \sum_{i=1}^{NG} a_i + b_i PG_i + c_i PG_i^2 \quad (4)
\]

where \(F_i\) is the non-linear objective function defining the total power generation cost of the system, \(a_i, b_i\) and \(c_i\) are the coefficients of the power generation cost function and \(NG\) is the number of generation buses.

The objective function given by equation (4) is subjected to the following system constraints.

2.1 Equality constraints

Two types of equality constraints are considered. The first is to simulate the total power balance for the network and the second represents the power balance at each bus.

2.1.1 Active/reactive power balance constraints

The generators real and reactive power outputs should be equal to the total load demand in addition to the transmission line losses. This constraint can be expressed as follows:

\[
\sum_{i=1}^{NG} PG_i = \sum_{j=1}^{NE} PD_j + P_L \quad (5)
\]
Multi-phase search optimisation algorithm for constrained optimal power flow problem  

\[ \sum_{j=1}^{NG} Q_{Gi} = \sum_{j=1}^{NL} Q_{Dj} + Q_{Li} \]  

where \( PG_i \) is the power generation at bus \( i \), \( QG_i \) is the reactive power generation at bus \( i \), \( PD_j \) is the load demand at load bus \( j \), \( QD_j \) is the reactive power demand at load bus \( j \), \( NL \) is the number of load buses, \( P_L \) is the total power losses in the system and \( Q_L \) is the total reactive power losses in the system.

The transmission power losses are equal to the sum of the total injected power at all system buses. The active/reactive injected power can be calculated as follows:

\[ P_i = V_i \sum_{j \in i} \left( G_{ij} \sin \theta_{ij} + B_{ij} \sin \theta_{ij} \right), \]

\[ P_L = \sum \left( P_i \right) \]

\[ Q_i = V_i \sum_{j \in i} \left( G_{ij} \sin \theta_{ij} - B_{ij} \sin \theta_{ij} \right), \]

\[ Q_L = \sum \left( Q_i \right) \]

where \( i \in n_i \) is a set of bus numbers except the swing bus, \( V_i \) is the voltage magnitude at bus \( i \), \( \theta_i \) is the voltage phase angle at bus \( i \), \( \theta_{ij} \) is the phase angle deference between buses \( i \) and \( j \), \( G_{ij} \) is the mutual conductance between buses \( i \) and \( j \), and \( B_{ij} \) is the mutual susceptance between buses \( i \) and \( j \).

### 2.1.2 Network constraints at each bus

The balance of active/reactive power defined by equations (5) and (6) does not ensure satisfying active and reactive power balance at each bus. Hence, additional network constraints must be added at each bus as follows:

\[ PG_i = PD_i + \sum_{k=1}^{nci} PF_{ik}, \quad \text{for } i = 1, 2 \ldots \text{NB} \]

\[ QG_i = QD_i + \sum_{k=1}^{nci} QF_{ik}, \quad \text{for } i = 1, 2 \ldots \text{NB} \]

where \( nci \) represents the number of lines connected to bus \( i \), \( PD_i \) is the load demand at load bus \( i \), \( QD_i \) is the reactive power demand at load bus \( i \) and \( PF_{ik} \) and \( QF_{ik} \) are respectively the active and reactive power flow in lines connected between buses \( i \) and \( k \).

### 2.2 Inequality constraints

#### 2.2.1 Generation constraints

The generation hard constraints include generator voltages, real power outputs, and reactive power outputs. These constraints are restricted by their physical lower and upper limits, where they can be simulated as:

\[ PG_i^{\min} \leq PG_i \leq PG_i^{\max} \]

\[ QG_i^{\min} \leq QG_i \leq QG_i^{\max} \]

\[ V_{Gi}^{\min} \leq V_{Gi} \leq V_{Gi}^{\max} \]

where \( PG_i^{\min} \) and \( PG_i^{\max} \) are the maximum and minimum power generation at bus \( i \) respectively, \( QG_i^{\min} \) and \( QG_i^{\max} \) are the maximum and minimum reactive power generation at bus \( i \) respectively and \( V_{Gi}^{\min} \) and \( V_{Gi}^{\max} \) are the maximum and minimum voltages at bus \( i \) respectively.

#### 2.2.2 Security constraints

To maintain system security, voltages at load buses and transmission line loadings should be kept within permissible limits and this can be expressed as:

\[ VL_i^{\min} \leq VL_i \leq VL_i^{\max}, \quad i = 1, \ldots, \text{NL} \]

\[ SL_i \leq SL_i^{\max}, \quad i = 1, \ldots, n_l \]

where \( VL \) is the load bus voltages, \( VL^{\max} \) and \( VL^{\min} \) are the maximum and minimum voltages at load bus \( i \) respectively, \( SL \) is transmission line loading in MVA and \( n_l \) is the number of system lines.

### 3 Enhanced SOA for SC-OPF

Each individual in the SOA population is called a seeker. The total population is randomly divided into \( n \)-subpopulations. These subpopulations search over several different domains of the search space. All seekers in the same subpopulation constitute a neighbourhood, which represents the social component for the social sharing of information (Shaw et al., 2011; Dai et al., 2009a, 2009b, 2010). In the SOA, a search direction \( d_i(t) \) and a step length \( \alpha_i(t) \) are computed separately for each \( i^{th} \) seeker and for each \( j^{th} \) variable at each step \( t \), where \( \alpha_i(t) \geq 0 \) and \( \alpha_i \in \{-1, 0, 1\} \). Here, \( i \) represents the population number and \( j \) represents the optimising variable number.

#### 3.1 Calculation of search direction

The search direction is the natural tendency of the swarms to reciprocate in a cooperative manner, while executing their needs and goals. Normally, there are two extreme types of cooperative behaviour prevailing in swarm dynamics. The first is egotistic and it is entirely pro-self and the second is altruistic and it is entirely pro-group. Every seeker, as a single sophisticated agent, is uniformly egotistic and should go towards its historical best position according to its own judgment. This attitude of the \( i^{th} \) seeker may be simulated by an empirical direction vector \( d_{i, ego}(t) \) as follows:
\[ d_{ego}(t) = \text{sign}\left( p_{g, best}(t) - x_i(t) \right) \]  

(16)

In altruistic behaviour, seekers try to communicate with each other, cooperate explicitly and adjust their behaviours in response to the other seekers in the same neighbourhood region for achieving the desired goal. Here, the seekers exhibit entirely pro-group behaviour. The population then exhibits a self-organised aggregation behaviour in which, the positive feedback takes the form of attraction towards a given signal source. These two optional altruistic directions are:

\[ d_{alt1}(t) = \text{sign}\left( p_{g, best}(t) - x_i(t) \right) \]  

(17)

\[ d_{alt2}(t) = \text{sign}\left( p_{l, best}(t) - x_i(t) \right) \]  

(18)

In equations (16) and (17), \( p_{g, best}(t) \) represents neighbours’ historical best position and \( p_{l, best}(t) \) stands for neighbours’ current best position. Moreover, seekers with the pro-activeness properties do not simply act in response to their environment but they are able to exhibit goal-directed behaviour. In addition, the future behaviour can be given as: empirical direction called ‘pro-activeness direction’ behaviour. Hence, each seeker is associated with an exhibit goal-directed behaviour according to its past seeker may be pro-active to change its search direction and predicted and guided by the past behaviour. As a result, the their environment but they are able to exhibit goal-directed

\[ d_{pro}(t) = \text{sign}\left( x_i(t_1) - x_i(t_2) \right) \]  

(19)

where \( t_1 \) and \( t_2 \in \{t, t-1, t-2\} \) and it is assumed that \( x_i(t_1) \) is better than \( x_i(t_2) \).

The Abovementioned four empirical directions, presented in equations (15) to (18), are considered to take a rational search direction decision. If the \( j^{th} \) variable of the \( i^{th} \) seeker goes towards the positive direction of the coordinate axis, the value of \( d_{j}(t) \) is taken as +1. On the other hand, if the \( j^{th} \) variable of the \( i^{th} \) seeker goes towards the negative direction of the coordinate axis, the value of \( d_{j}(t) \) is assumed as –1. The value of \( d_{j}(t) \) is assumed as 0 if the \( j^{th} \) variable of the \( i^{th} \) seeker stays at the current position. Every variable \( j \) of \( d_{j}(t) \) is selected by applying the proportional selection rule stated in the following equation:

\[ d_{j} = \begin{cases} 0 & \text{if } r_{j} \leq p_{j}^{(0)} \\ 1 & \text{if } p_{j}^{(0)} \leq r_{j} \leq p_{j}^{(0)} + p_{j}^{(1)} \\ -1 & \text{if } p_{j}^{(0)} + p_{j}^{(1)} < r_{j} \leq 1 \\ & \text{where } r_{j} \text{ is a uniform random number in } [0, 1], \\ & p_{j}^{(m)} (m \in \{1,0,-1\}) \text{ is the percentage of the numbers of ‘m’} \\ & \text{from the set } \{d_{ego}, d_{alt1}, d_{alt2}, d_{pro}\} \text{ on each variable } j \text{ of} \\ & \text{all the four empirical directions, i.e.,} \\ & p_{j}^{(m)} = \left\lfloor \frac{\text{the number of } m}{4} \right\rfloor \]  

(21)

3.2 Calculation of step length

The fitness function of SOA considers the fuzzy system to turn all the seekers into sequence numbers from 1 to \( S \) as the inputs to fuzzy reasoning. The membership function is used in the conditional part since the universe of discourse is a given set of numbers, that is, 1, 2, ..., \( S \), as:

\[ \mu_i = \mu_{max} \frac{S - I_i}{S - 1} (\mu_{max} - \mu_{min}) \]  

(22)

In equation (22), \( I_i \) is the sequence number of \( x_i(t) \) after sorting the fitness values; \( \mu_{max} \) is the maximum membership degree value that is equal to or a little less than 1.0. Here, the value of \( \mu_{max} \) is considered as 0.95. Thus, a minimum value of \( \mu_i, \mu_{min} = 0.0111 \), is selected. Moreover, the Bell membership function parameter, \( \delta \), is determined by:

\[ \delta = \omega \cdot \text{abs}\left( x_{best} - x_{rand} \right) \]  

(23)

where \( \text{abs} () \) operator refers to the absolute value of the input vector and the parameter \( \omega \) is used to decrease the step length with increasing time step so as to gradually improve the search precision. The value of \( \omega \) is linearly decreased from 0.9 to 0.1 during a run. The \( x_{best} \) and \( x_{rand} \) are the best seeker and a randomly selected seeker respectively, from the same subpopulation to which the \( i^{th} \) seeker belongs.

In the same subpopulation, \( x_{rand} \) is different from \( x_{best} \) and \( \delta \) is shared by all seekers. A uniformly random real number within the range \([\mu_{min}, 1]\) is returned by:

\[ \mu_j = \text{rand}\left( \mu_{min}, 1 \right) \]  

(24)

Equation (25) denotes the action part of fuzzy reasoning and gives the step length \( \alpha_j \) for every variable \( j \) as:

\[ \alpha_j = \delta \sqrt{-\ln(\mu_j)} \]  

(25)

3.3 Updating seekers’ position

In a population of size \( S \), for each seeker \( I \) (1 \( \leq i \leq S \)), the position update of each seeker \( j \) is given by:

\[ x_{j}(t+1) = x_{j}(t) - \alpha_j(t) \cdot d_{j}(t) \]  

(26)

3.4 Subpopulations learn from each other:

Each subpopulation searches to find the optimal solution using its specific information. It is possible for any subpopulation to trap into local optima yielding a premature convergence. For better performance, it is required for subpopulations to learn from each other about the optimum information they have acquired in their respective domain.
Thus, the position of the worst seeker of each subpopulation is combined with the best one in each of the other subpopulations using the following binomial crossover operator:

\[
X_{kj,worst} = \begin{cases} 
X_{lj,best} & \text{if } \text{rand}_j \leq 0.5 \\
X_{kj,j,worst} & \text{else}
\end{cases}
\]  

(27)

where \( \text{rand}_j \) is a uniform random real number within \([0, 1]\), \(X_{lj,best}\) is denoted as the \(j^{th}\) best position of the \(n^{th}\) individual in the \(l^{th}\) subpopulation and \(X_{kj,j,worst}\) is the \(j^{th}\) variable of the worst position in the \(k^{th}\) subpopulation. Here, \(n, k, l = 1, 2, ..., k - 1, k \neq 1, 2, \ldots, k - 1\).

3.5 Limit reduction strategy

According to the LRS strategy, the maximum and minimum limits of the controlled variables are modified in each iteration and hence, the search space is reduced. In SC-OPF problem, the controlled variable is the generator output power and hence, LRS is applied as follows:

\[
PG_l^{\text{max}}(t + 1) = PG_l^{\text{max}}(t) - \sigma \left( PG_l^{\text{max}}(t) - \text{g, best}(t) \right)
\]

(28)

\[
PG_l^{\text{min}}(t + 1) = PG_l^{\text{min}}(t) - \sigma \left( \text{g, best}(t) - PG_l^{\text{min}}(t) \right)
\]

(29)

where \(\sigma\) is a factor less than 0.1 randomly selected and adjusted according to the problem, where the most convenient value of \(\sigma\) for SC-OPF problem is 0.08.

3.6 Handling of constraints

The economic model of the non-linear fuel cost comprises many constraints. In case of tightly constrained problems, the infeasible solutions are known to cover the search space in the initial generations. Complete avoidance of infeasible solutions in this case leads to a high possibility of missing the area of global minimum. Also, moving the infeasible individuals to the nearest feasible area would be too complex and represents an extreme time-consuming process. Therefore, a penalty-function approach is applied to convert the constrained problem to an unconstrained one by augmenting additional cost terms with the main cost function. The additional terms assign non-linear costs for solutions that violate the constraints depending on their location relative to the feasible boundaries. The adequate choice of the penalty functions and their parameters is an essential factor in the optimisation process. A higher additional cost value has to be assigned to any infeasible solution to ensure the rejection of all individuals that violate the constraints. The evaluation function can be formulated as follows:

\[
\text{Min } F_l = \sum_{i=1}^{NG} f_i \left( PG_i \right) + \lambda \left[ \sum_{i=1}^{NG} PG_l + P_L - \sum_{j=1}^{NL} PD_j \right] + \mu \left[ S_l - S_l^{\text{max}} \right]
\]

(30)

From the previous discussion, the MSOA is adapted for solving the SC-OPF problem. The modifications are concentrated on reducing the search space of system variables for improving the convergence characteristics and proposing efficient handling method for equality and inequality constraints.

4 Implementation of MSOA algorithm

The main steps of the MSOA algorithm are summarised as follows:

Step 1 Initialisation

- entering data and reading cost curves of generators and Bs coefficients of the line power losses
- setting a counter
- identifying maximum population number of generator output parameter strings
- defining lower and upper limits of each generator output
- reading the SOA parameters
- setting termination criteria (i.e., max. iteration cycles).

Step 2 Initialising the positions of the seekers in the search space randomly and uniformly.

Step 3 Setting the iteration number, \( I = 0 \).

Step 4 Computing the evaluation function for initial positions and setting the personal historical best position of each seeker to its current position when achieving the initial historical best position among the population.

Step 5 Increasing the iteration number, \( I = I + 1 \).

Step 6 Selecting the neighbour of each seeker.

Step 7 Determining the search direction and step length for each seeker.

Step 8 Updating the position of each seeker.

Step 9 Satisfying the constrains of controlled variables.

Step 10 Alleviating the constrains violation by using Lagrangian relaxation factors.

Step 11 Computing the evaluation function for each seeker.

Step 12 Updating the historical best position among the population and historical best position of each seeker.
Step 13 Exchanging information between subpopulations, i.e., subpopulations learn from each other.

Step 14 Applying LRS.

Step 15 Repeating the procedures starting from step 5 (iteration cycles) until satisfying the stopping criterion.

Step 16 Determining the best generation schedule corresponding to the optimum objective function value.

5 Case studies

5.1 Test systems

The IEEE 30-bus and 57-bus test systems (El-Sehiemy et al., 2012; Power System Test Case Archive, 2012) are used to implement the proposed technique for SC-OPF using MSOA. The IEEE 30 bus test system has six generators, 19 fixed load and 41 branches as shown in Figure 1. The obtained results are compared with those obtained in a previous work using PSO techniques (Abido, 2002).

The second system is the IEEE 57-bus test system, which has seven generators, 43 fixed loads and 80 branches as shown in Figure 2. The third system is the WDN, which is a part of the Unified Egyptian Network (UEN) system. It has eight generators, 50 fixed loads and 108 branches as shown in Figure 3. Table 1 gives the parameters of the multi-phase search optimisation algorithm (MSOA) and the particle swarm optimisation algorithm.

Figure 1 Single line diagram of IEEE 30 bus-system
Figure 2  Single line diagram of IEEE 57 bus-system

Figure 3  Single line diagram of WDN bus system
Table 1  MSOA and PSO parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tool</th>
<th>MSOA</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total population size</td>
<td></td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>Generations total number</td>
<td></td>
<td>240</td>
<td>200</td>
</tr>
<tr>
<td>Inertia weight ((w))</td>
<td>0.9 &lt; w &lt; 1.1</td>
<td>0.3 &lt; w &lt; 0.8</td>
<td></td>
</tr>
<tr>
<td>Particle velocity limits</td>
<td>± 0.1 * Rang</td>
<td>± 0.08 * Rang</td>
<td></td>
</tr>
<tr>
<td>Number of subpopulations</td>
<td>3</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Membership function ((\mu))</td>
<td>0.011 &lt; \mu &lt; 0.3</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

5.2 Results and comments

The results of MSOA are compared with a modified version of PSO (MPSO) technique. The MPSO combines the original PSO technique with the LRS and the Lagrangian relaxation factors. Hybrid versions of SOA and PSO are developed to improve the solution quality using a two-stage procedure. In the first stage, the SC-OPF problem is solved using both MSOA and MPSO, while in the second stage the solution is enhanced and improved with Newton-Raphson ‘NR’ technique. The consideration of NR technique helps to efficiently assure satisfying the network and power balance constraints. The hybrid versions of SOA and PSO are abbreviated as HSOA and HPSO, respectively.

5.2.1 IEEE 30 bus test system

Table 2 compares the security constrained optimal power dispatch (SCOD) solutions obtained using MPSO, HPSO in addition to the modified versions MSOA and HSOA with the conventional PSO technique all applied to the IEEE 30 bus system. It is clear that, the MSOA algorithm has the lowest generation costs (798.19 $/hr) and power losses (7.88 MW) compared to other techniques for solving the SC-OPF problem. Figure 4 shows the convergence curves of MSOA and MPSO. MSOA reaches optimal solution faster than MPSO and hence, it is more convenient in online applications but the power losses are almost the same.

Figure 5 shows bus voltages under the optimal load dispatch. Although the line powers are within the acceptable limits and the active power balance at each bus is achieved, the voltages are lower than the minimum limit at some buses.

Table 2  SCOD solution using different optimisation algorithms

<table>
<thead>
<tr>
<th>Variables</th>
<th>(P_{\text{min}})</th>
<th>(P_{\text{max}})</th>
<th>PSO (Garcia-Gonzalo and Fernández-Martínez, 2012)</th>
<th>MPSO</th>
<th>HPSO</th>
<th>MSOA</th>
<th>HSOA</th>
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<tbody>
<tr>
<td>PG1</td>
<td>50</td>
<td>200</td>
<td>176.98</td>
<td>177.29</td>
<td>181.05</td>
<td>176.39</td>
<td>180.36</td>
</tr>
<tr>
<td>PG2</td>
<td>20</td>
<td>80</td>
<td>48.98</td>
<td>42.94</td>
<td>42.94</td>
<td>48.96</td>
<td>48.96</td>
</tr>
<tr>
<td>PG5</td>
<td>15</td>
<td>50</td>
<td>21.30</td>
<td>17.32</td>
<td>17.32</td>
<td>22.64</td>
<td>22.64</td>
</tr>
<tr>
<td>PG11</td>
<td>10</td>
<td>30</td>
<td>11.97</td>
<td>18.49</td>
<td>18.49</td>
<td>11.35</td>
<td>11.35</td>
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<tr>
<td>PG13</td>
<td>12</td>
<td>40</td>
<td>12</td>
<td>13.86</td>
<td>13.86</td>
<td>16.42</td>
<td>16.42</td>
</tr>
<tr>
<td>(\sum_{PG})</td>
<td></td>
<td></td>
<td>296.37</td>
<td>291.36</td>
<td>295.12</td>
<td>291.28</td>
<td>295.24</td>
</tr>
<tr>
<td>PL (MW)</td>
<td>12.97</td>
<td></td>
<td>12.97</td>
<td>7.96</td>
<td>11.72</td>
<td>7.88</td>
<td>11.84</td>
</tr>
<tr>
<td>Cost ($/h)</td>
<td>800.41</td>
<td></td>
<td>800.41</td>
<td>799.93</td>
<td>812.48</td>
<td>798.19</td>
<td>810.68</td>
</tr>
</tbody>
</table>

Note: Load = 283.4 MW

Figure 4  Convergence curves for MSOA versus MPSO for IEEE 30 bus system (see online version for colours)
Figure 5  Buses voltages of IEEE 30 bus system (see online version for colours)

![Figure 5](image)

Table 3  Comparison of various methods for IEEE 57-bus system

<table>
<thead>
<tr>
<th>Variable</th>
<th>$P_{\text{min}}$</th>
<th>$P_{\text{max}}$</th>
<th>MPSO</th>
<th>HPSO</th>
<th>MSOA</th>
<th>HSOA</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG$_1$</td>
<td>0</td>
<td>575</td>
<td>273.62</td>
<td>285.29</td>
<td>272.98</td>
<td>285.02</td>
</tr>
<tr>
<td>PG$_2$</td>
<td>0</td>
<td>100</td>
<td>100.81</td>
<td>100.81</td>
<td>100.89</td>
<td>100.89</td>
</tr>
<tr>
<td>PG$_3$</td>
<td>0</td>
<td>140</td>
<td>153.61</td>
<td>153.61</td>
<td>153.93</td>
<td>153.93</td>
</tr>
<tr>
<td>PG$_4$</td>
<td>0</td>
<td>100</td>
<td>100.81</td>
<td>100.81</td>
<td>100.89</td>
<td>100.89</td>
</tr>
<tr>
<td>PG$_5$</td>
<td>0</td>
<td>550</td>
<td>273.62</td>
<td>273.62</td>
<td>272.58</td>
<td>272.58</td>
</tr>
<tr>
<td>PG$_6$</td>
<td>0</td>
<td>100</td>
<td>100.81</td>
<td>100.81</td>
<td>101.59</td>
<td>101.59</td>
</tr>
<tr>
<td>PG$_7$</td>
<td>0</td>
<td>410</td>
<td>273.62</td>
<td>273.62</td>
<td>273.69</td>
<td>273.69</td>
</tr>
<tr>
<td>$\sum P_G$</td>
<td></td>
<td></td>
<td>1,276.9</td>
<td>1,288.57</td>
<td>1,276.5</td>
<td>1,288.6</td>
</tr>
<tr>
<td>PL (MW)</td>
<td></td>
<td></td>
<td>26.1</td>
<td>37.47</td>
<td>25.7</td>
<td>37.8</td>
</tr>
<tr>
<td>Cost ($/hr$)</td>
<td></td>
<td></td>
<td>3,171.3</td>
<td>3,240.1</td>
<td>3,167.5</td>
<td>3,236.1</td>
</tr>
</tbody>
</table>

Note: Load = 1,250.8 MW

Figure 6  Comparison between convergences of security constrained for IEEE 57 bus system (see online version for colours)

![Figure 6](image)

5.2.2 IEEE 57 bus test system

The comparison given in the previous section for the IEEE 30 bus system is repeated once again for the IEEE 57 bus test system and the results are summarised in Table 3. It is clear that, the MSOA algorithm has the lowest generation cost and power losses compared to all other techniques for solving the SC-OPF problem.

Figure 6 shows the convergence curves of MSOA and MPSO. Similar to the previous case, MSOA reaches optimal solution faster than MPSO. Thus it is more suitable for the online application although the power losses are...
almost the same. Figure 7 shows bus voltages under the optimal load dispatch. It is also noticeable in this case that some voltage-limit violations occur but the line powers are within limits and the active power balance at each bus is achieved.

5.2.3 WDN real system

Table 4 compares the SC-OPF solutions obtained using MPSO, HPSO, MSOA and HSOA applied to the WDN. Similar to the previous cases, the MSOA algorithm has the lowest generation cost and power losses compared to other techniques for solving the SC-OPF problem. Figure 8 shows the convergence curves of MSOA and MPSO. Similar to the previous case, MSOA is faster than MPSO. Figure 9 shows bus voltages under the optimal load dispatch. The voltage profile in this case has similar characteristics such as the two previous cases. Table 5 gives an evaluation of the SC-OPF process for IEEE 30-bus and 57-bus test systems.

**Figure 7** Buses voltages profile of IEEE 57 bus system (see online version for colours)

**Figure 8** Comparison between convergences of security constrained for WDN system (see online version for colours)

---

**Table 4** Comparison of various methods for WDN system

<table>
<thead>
<tr>
<th>Variable</th>
<th>MPSO</th>
<th>HPSO</th>
<th>MSOA</th>
<th>HSOA</th>
<th>$P_{\text{max}}$</th>
<th>$P_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG1</td>
<td>182.88</td>
<td>186.79</td>
<td>185.51</td>
<td>189.76</td>
<td>375</td>
<td>10</td>
</tr>
<tr>
<td>PG2</td>
<td>117.02</td>
<td>117.02</td>
<td>112.19</td>
<td>112.19</td>
<td>250</td>
<td>10</td>
</tr>
<tr>
<td>PG3</td>
<td>149.95</td>
<td>149.95</td>
<td>153.58</td>
<td>153.58</td>
<td>375</td>
<td>10</td>
</tr>
<tr>
<td>PG4</td>
<td>125.25</td>
<td>125.25</td>
<td>124.91</td>
<td>124.91</td>
<td>250</td>
<td>10</td>
</tr>
<tr>
<td>PG5</td>
<td>100.56</td>
<td>100.56</td>
<td>104.59</td>
<td>104.59</td>
<td>375</td>
<td>10</td>
</tr>
<tr>
<td>PG6</td>
<td>75.86</td>
<td>75.86</td>
<td>73.99</td>
<td>73.99</td>
<td>250</td>
<td>10</td>
</tr>
<tr>
<td>PG7</td>
<td>100.56</td>
<td>100.56</td>
<td>94.02</td>
<td>94.02</td>
<td>250</td>
<td>10</td>
</tr>
<tr>
<td>PG8</td>
<td>71.74</td>
<td>71.74</td>
<td>75.07</td>
<td>75.07</td>
<td>250</td>
<td>10</td>
</tr>
<tr>
<td>$\sum_{PG}$</td>
<td>923.81</td>
<td>927.72</td>
<td>923.86</td>
<td>928.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PL (MW)</td>
<td>34.06</td>
<td>37.97</td>
<td>34.18</td>
<td>38.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost ($/hr)</td>
<td>23977</td>
<td>24059</td>
<td>23958</td>
<td>24047</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Load = 889.75 MW
5.3 Emergency conditions

5.3.1 Increasing the load demand

Increasing the load demand is the most frequently repeated emergency in power systems and hence, the power system should be reliable and able to cover this load increase in addition to satisfying all system constraints. Table 6 shows the results of SC-OPF using MSOA for different percentages load increase over maximum load of the IEEE 30 bus system. Increasing the load by default increases system dispatch cost, losses and voltage deviations. However, the system is capable of feeding the load without exceeding generation limits or line-flow limits. Also, Table 6 shows the penalty effect of the $\lambda$ operator on the system costs. Lower values of $\lambda$ cause lower cost but increases the possibility of violating power flow limits. Thus, suitable selection of the $\lambda$ operator leads to insignificant increase in the cost but reduces the power losses and the voltage deviations.

5.3.2 Unexpected outage of a transmission line

The transmission line outage can be either expected, due to maintenance, or unexpected due to certain faults. There are restrictions on switching off some lines, e.g., to avoid isolating specific generators or loads from the network. This is the case in the IEEE 30-bus network, where disconnecting the line between buses 9 and 11 isolates the generator at buses 11. In addition, disconnecting the line between buses 12 and 13 isolates the generator at buses 13. On the other hand, disconnecting the line between buses 25 and 26 isolates the load at bus 26. In other words, some lines cannot be disconnected to prevent isolating important elements in the network regardless of their small effect on system losses, cost and power flow in the lines.

The results of the IEEE 30 bus system under emergency conditions represented by the outage of different transmission lines using MSOA and MPSO are shown in Table 7. Removing transmission lines leads to an increase in the power flow in other lines especially those having a direct connection with the removed line. MSOA technique shows less cost, losses and voltage deviation compared with MPSO. Also, MSOA results more reserve in power flow since MPSO results in power flow values close to the maximum limits of lines especially the critical lines listed in Table 7.

5.3.3 Effect of generation outage on the total cost

Sudden or expected outage of generation affects the SCOD and hence it directly affects the total generation cost. Generation outage can happen due to the depreciation of a specific plant and hence, it cannot deliver its full capacity. In addition, each plant consists of some generating units, where one or more may get out of service for maintenance or suddenly due to certain faults. Outage case study is performed on the IEEE 30-bus test system, where generation plants 1, 2 and 3 are disconnected. The study is performed using MSOA, where unit power reserve is not taken into consideration. Figures 10 to 12 show that the outage of generation in plant 1, plant 2, or plant 3 increases the total cost rate. However, there is a limit of generation outage that can be expressed as:
\[
\left( \sum_{i=1}^{NL} PG_{i} + (1 - \text{outage \%}) PG_{\text{max}} \right) \geq \left( \sum_{j=1}^{NL} PD_{j} + P_{L} \right) \]

where \( PG_{\text{max}} \) is the maximum generation of the plant where the outage is performed and % out is the percentage outage.

**Table 6** SC-OPF using MSOA for different percentages load increase over maximum load of IEEE 30 bus system

<table>
<thead>
<tr>
<th>%</th>
<th>5% Load increase</th>
<th>10% Load increase</th>
<th>15% Load increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda )</td>
<td>2.50 4.00 10.00</td>
<td>3.20 3.60 10.00</td>
<td>2.80 3.20 10.00</td>
</tr>
<tr>
<td>PG1</td>
<td>191.0 173.2 144.5</td>
<td>188.2 177.7 153.0</td>
<td>192.8 165.9 156.03</td>
</tr>
<tr>
<td>PG2</td>
<td>48.31 47.67 50.19</td>
<td>48.62 54.44 52.23</td>
<td>53.58 57.02 59.801</td>
</tr>
<tr>
<td>PG5</td>
<td>19.06 25.77 35.90</td>
<td>26.48 26.48 37.35</td>
<td>28.13 30.49 38.195</td>
</tr>
<tr>
<td>PG8</td>
<td>24.15 23.84 25.10</td>
<td>24.31 27.22 26.12</td>
<td>25.54 30.68 28.738</td>
</tr>
<tr>
<td>PG11</td>
<td>12.94 17.74 21.96</td>
<td>15.96 16.96 22.96</td>
<td>19.51 23.56 23.082</td>
</tr>
<tr>
<td>PF1</td>
<td>134.5 121.9 100.9 132.5 124.3 106.9</td>
<td>135.0 115.9 107.92</td>
<td></td>
</tr>
<tr>
<td>PF2</td>
<td>62.34 56.82 48.42 61.39 58.94 51.10</td>
<td>63.61 55.18 53.125</td>
<td></td>
</tr>
<tr>
<td>PF5</td>
<td>70.83 65.32 57.33 69.36 68.93 60.35</td>
<td>71.98 68.05 63.614</td>
<td></td>
</tr>
<tr>
<td>PF7</td>
<td>56.85 52.60 46.23 56.91 54.44 48.73</td>
<td>55.10 49.29 47.776</td>
<td></td>
</tr>
<tr>
<td>PL (MW)</td>
<td>13.15 11.29 8.81</td>
<td>13.00 12.27 9.80</td>
<td>14.02 11.71 10.752</td>
</tr>
<tr>
<td>VD</td>
<td>0.17 0.17 0.16</td>
<td>0.21 0.21 0.21</td>
<td>0.36 0.35 0.3541</td>
</tr>
<tr>
<td>Cost ($/hr)</td>
<td>863.9 865.2 887.2</td>
<td>918.9 918.8 942.1</td>
<td>974.5 984.8 999.01</td>
</tr>
</tbody>
</table>

Table 7 SC-OPF for different line outage for 30-bus system

<table>
<thead>
<tr>
<th>Line outage (no outage)</th>
<th>Base case</th>
<th>Line 3, bet. buses 3,4</th>
<th>Line 5, bet. buses 4,6</th>
<th>Line 6, bet. buses 6,7</th>
<th>Line 7, bet. buses 6,8</th>
<th>Max. limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>2.00</td>
<td>3.20 2.50</td>
<td>3.20 2.50</td>
<td>3.20 2.50</td>
<td>3.20 10.00</td>
<td>Max. limit</td>
</tr>
<tr>
<td>PG1</td>
<td>176.4 177.3</td>
<td>179.8 183.9</td>
<td>148.3 203.2</td>
<td>147 185.4</td>
<td>156.2 155.8</td>
<td>200 50</td>
</tr>
<tr>
<td>PG2</td>
<td>49 42.9 49.80</td>
<td>48.80 43.29</td>
<td>55.79 42.68</td>
<td>55.97 45.42</td>
<td>53.74 43.50</td>
<td>80 20</td>
</tr>
<tr>
<td>PG3</td>
<td>22.6 17.32</td>
<td>25.28 18.03</td>
<td>28.20 16.26</td>
<td>23.11 17.97</td>
<td>26.06 30.23</td>
<td>50 15</td>
</tr>
<tr>
<td>PG4</td>
<td>15.5 21.47</td>
<td>11.98 21.64</td>
<td>29.73 21.34</td>
<td>23.23 22.71</td>
<td>22.84 21.75</td>
<td>35 10</td>
</tr>
<tr>
<td>PG11</td>
<td>11.4 18.49</td>
<td>14.34 14.77</td>
<td>20.42 11.47</td>
<td>22.51 11.10</td>
<td>14.50 18.74</td>
<td>30 10</td>
</tr>
<tr>
<td>PF1</td>
<td>126.1 128.2</td>
<td>112.1 117.4</td>
<td>89.42 128.3</td>
<td>90.97 117.9</td>
<td>122.5 123.35</td>
<td>130 0</td>
</tr>
<tr>
<td>PF2</td>
<td>59.9 58.56</td>
<td>72.83 71.86</td>
<td>63.28 80.45</td>
<td>60.52 72.83</td>
<td>38.92 37.80</td>
<td>130 0</td>
</tr>
<tr>
<td>PF3</td>
<td>65.7 67.19</td>
<td>71.73 74.29</td>
<td>0.00 0.00</td>
<td>72.69 79.94</td>
<td>72.62 68.59</td>
<td>130 0</td>
</tr>
<tr>
<td>PF7</td>
<td>56.7 53.00</td>
<td>37.82 36.50</td>
<td>77.66 97.90</td>
<td>73.81 85.80</td>
<td>0.00 0.00</td>
<td>90 0</td>
</tr>
<tr>
<td>PL (MW)</td>
<td>7.88 7.96</td>
<td>12.53 12.69</td>
<td>16.72 24.52</td>
<td>10.27 13.57</td>
<td>11.44 10.82</td>
<td>90 0</td>
</tr>
<tr>
<td>VD</td>
<td>0.13 0.12</td>
<td>0.14 0.14</td>
<td>0.14 0.16</td>
<td>0.14 0.15</td>
<td>0.17 0.16</td>
<td>0.17 0.16</td>
</tr>
<tr>
<td>Cost ($/hr)</td>
<td>798 799.9</td>
<td>814.8 814.1</td>
<td>845.6 854.6</td>
<td>817.4 816.2</td>
<td>820.4 826.1</td>
<td>820.4 826.1</td>
</tr>
</tbody>
</table>

Note: Load = 283.4 MW
According to this condition, if plant 2 or plants 3 are totally out of service, there will be no problem indicating that the maximum permissible outage of plants 2 and 3 can reach 100% as shown in Figures 13 and 15. On the other hand, the maximum permissible outage of plant 1 in the IEEE 30 bus system is 75.3% as shown in Figure 11. If the percentage outage of plant 1 is increased over this limit, it can lead to load shedding since the system cannot feed the total load. Figures 13 to 15 shows that increasing the percentage outage of a specific plant decreases its sharing in the total load compared to the case of its full contribution.
6 Conclusions

This paper presents an approach based on MSOA technique to solve the SC-OPF problem with equality and inequality constraints under normal and emergency conditions. The proposed algorithm has been implemented on three test systems. The results obtained are compared with reference PSO and MPSO techniques presented in the literature. The results show that, MSOA provides lower generation cost for normal and emergency condition, while the power flows in all critical lines are kept within their permissible limits. The results ensured that the proposed MSOA is more accurate and represents a powerful tool to solve different power system problems in the online mode.
References


