ORIGINAL ARTICLE

Intelligent control for nonlinear inverted pendulum based on interval type-2 fuzzy PD controller

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Abstract The interval type-2 fuzzy logic controller (IT2-FLC) is able to model and minimize the numerical and linguistic uncertainties associated with the inputs and outputs of a fuzzy logic system (FLS). This paper proposes an interval type-2 fuzzy PD (IT2F-PD) controller for nonlinear inverted pendulum. The proposed controller uses the Mamdani interval type-2 fuzzy rule based, interval type-2 fuzzy sets (IT2-FSs) with triangular membership function, and the Wu–Mendel uncertainty bound method to approximate the type-reduced set. The proposed controller is able to minimize the effect of the structure uncertainties and the external disturbances for the inverted pendulum. The results of the proposed controller are compared with the type-1 fuzzy PD (T1F-PD) controller in order to investigate the effectiveness and the robustness of the proposed controller. The simulation results show that the performance of the proposed controller is significantly improved compared with the T1F-PD controller. Also, the results show good performance over a wide range of the structure uncertainties and the effect of the external disturbances.

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1. Introduction

The classical linear proportional-derivative (PD) controller and the proportional-integral-derivative (PID) controller cannot provide good enough performance in controlling highly complex, nonlinear, and uncertain processes [1,2]. Fuzzy control [3,4] has become, in the recent past, an alternative to conventional control algorithms to deal with complex processes and combine the advantages of classical controllers and human operator experience. The main advantage of a FLC is that it can be applied to systems that are nonlinear where their mathematical models are difficult to obtain. Another advantage is that the controller can be designed to apply heuristic rules that reflect the experiences of the human experts [5]. A suitable choice of control variables is important in fuzzy control design. Typically, the inputs to the fuzzy controllers are the error and the change of error. This choice is physically related to classical PID controllers. Usually, a fuzzy controller is either a PD- or a
PI-type depending on the output of fuzzy control rules; if the output is the control signal it is said to be a PD-type fuzzy controller and if the output is the change of control signal it is said to be a PI-type fuzzy controller [6].

Despite the significant improvement in these fuzzy controllers over their conventional counterparts, it should be noted that they are usually not effective if the system to be controlled has structure uncertainties because the ordinary fuzzy controllers (type-1 fuzzy controllers) have limited capabilities to directly handle data uncertainties [7]. There are five sources of the uncertainties in the type-1 fuzzy logic systems (T1-FLSs) [8,9]: (1) Uncertainties in the inputs to a FLS, which translate into uncertainties in the antecedents membership functions as the sensor measurements are affected by high noise levels from various sources. (2) Uncertainties in the control outputs, which translate into uncertainties in the consequents membership function of the FLS. (3) The meanings of the words that are used in the antecedents and consequents of rules can be uncertain (words mean different things to different people). (4) Uncertainties associated with the change in the operating conditions of the controller. Such uncertainties can translate into uncertainties in the antecedents and/or consequents’ membership functions. (5) The data that used to tune the parameters of a T1-FLS may also be noisy. All of these uncertainties translate into uncertainties about fuzzy set membership functions. T1-FLS is not able to directly model such uncertainties because their membership functions are totally crisp.

On the other hand, the type-2 fuzzy sets (T2-FSs) that introduced by Zadeh in 1975 are able to model such uncertainties because their membership functions are themselves fuzzy; there are very useful in circumstances where it is difficult to determine an exact membership function of a fuzzy set [10]. The concept of the T2-FSs is an extension of the concept of the ordinary fuzzy sets (type-1 fuzzy sets; T1-FSs). A T2-FS is characterized by a fuzzy membership function (i.e., the membership grade for each element of this set is a fuzzy set in [0, 1]), unlike a T1-FS where the membership grade is a crisp number in [0, 1] [10]. Therefore, a T2-FS provides additional degrees of freedom that make it possible to model and handle the uncertainties directly [11]. An IT2-FLC is a special case of a type-2 fuzzy logic system [9]. These are simpler to work with than general IT2-FLSs and distribute the uncertainty evenly among all admissible primary memberships [12]. The IT2-FLCs have been applied to various fields with great success [13–27].

The inverted pendulum on a cart system has the property of unstable, higher order, uncertain and highly coupled, which can be treated as a typical nonlinear control problem [28]. It provides an excellent experimental platform to test various control theories and techniques. Furthermore, an inverted pendulum system may also simulate many phenomena in the nature, such as walking robots, flying objects in space, and missile guidance [29]. The main problem in an inverted pendulum system is the uncertainty which is divided into two groups; parameter uncertainty and neglected linear, nonlinear and unmodelled dynamic uncertainty [30]. The parameter uncertainties can be caused by the parameters which are difficult or impossible to get a precise measure or that the parameters tend to vary as a function of time, temperature etc. For this pendulum system, the mean uncertain parameters are the Coulomb friction constants. With respect to neglected dynamics, the pendulum system does have unmodelled dynamics like bearings and track inclination. Further, the system includes also nonlinear elements dynamically. The nonlinear dynamic appears from the special the sine and cosine functions in the nonlinear model. Therefore, these uncertainties transmitted to the controller. So, the main objective of controlling the inverted pendulum is to minimize the effect of the system uncertainties. There are variety methods for an inverted pendulum control that are presented since now. The presented methods for an inverted pendulum control are divided generally into three groups. Classical methods such as PID controllers [31], modern methods such as an optimal control [32], and artificial intelligence methods such as neural networks and fuzzy control [33–37].

As reported in [38], a T1F-PD controller has been proposed for controlling an inverted pendulum system. Such a fuzzy PD structure is simple and can be theoretically analyzed. However, the main drawback of a T1F-PD controller is its limited capability to directly handle data uncertainties. Thus, the main objective of this paper is to develop an IT2F-PD controller taking into its consideration the advantages of the type-2 FLSs to overcome the uncertainty problems. The proposed IT2F-PD controller which contains two inputs (i.e., the error signal and the derivative of error signal) and one output (i.e., the control signal) has ability to minimize the effect of the structure uncertainties. The proposed controller is used for controlling the nonlinear inverted pendulum on a cart system to overcome the uncertainty problem and the effect of the external disturbances. The results are compared with other related controllers to test the robustness of the proposed controller to provide some improvements in performance over the related controllers under the effect of the system uncertainties and the external disturbances. The rest of this paper is organized as follows. In Section 2, the IT2F-PD controller is included. The description of the mathematical model of the nonlinear inverted pendulum is presented in Section 3. Section 4, presents the simulation results followed by the conclusions and the relevant references.

2. Interval type-2 fuzzy PD controller

The main objective of the controller design is to achieve better control performance in terms of stability and robustness for the system uncertainties and the environmental disturbances. Fig. 1, shows the structure of the IT2F-PD controller. It uses two input variables, i.e., \(e(k)\) and \(\Delta e(k)\), and one output variable, i.e., \(u(k)\). Two scaling factors \(G_e\) and \(G_{\Delta e}\) are employed to scale \(e(k)\) and \(\Delta e(k)\), respectively, as follows:

\[
E(k) = G_e e(k) = G_e (y(k) - y(k))
\]

\[
\Delta E(k) = G_{\Delta e} \Delta e(k) = G_{\Delta e} (e(k) - e(k-1))
\]

(1)

where \(y(k)\) is the system output reference signal, \(y(k)\) is the output of the system under control, and \(k\) is the sampling instance. The output variable \(u(k)\) and its scaling factor \(G_u\) are given by the following equation:

\[
u(k) = G_u U(k)
\]

(2)

where \(U(k)\) is the output of the IT2F-PD controller.

The IT2F-PD controller works as follows: The crisp input from the input variables are first fuzzified into input IT2-FSs. The input IT2-FSs then activate the inference engine and the rule base to produce output IT2-FSs. The IT2-FLS rules will remain the same as in a T1-FLS, but the antecedents...
and/or the consequences will be represented by IT2-FSs. The inference engine combines the fired rules and gives a mapping from input IT2-FSs to output IT2-FSs. The IT2 fuzzy outputs of the inference engine are then processed by the type reducer, which combines the output sets and performs a centroid calculation that leads to T1-FSs called the type-reduced sets. In this paper, the Wu–Mendel uncertainty bound method has been used to approximate the type-reduced set. After the type reduction process, the approximate type-reduced sets are defuzzified (by taking the average of the approximated type-reduced set) to obtain crisp outputs that are sent to the actuators.

2.1. Interval type-2 fuzzy set

An IT2-FS \( \tilde{A} \) is characterized as [8]:

\[
\tilde{A} = \bigcup_{x \in X} \bigcup_{x' \in \mathcal{J}_x} \left\{ (x, r) : r \in \mathcal{I}_x \subseteq [0, 1] \right\}
\]

where \( x \) is the primary variable and \( x \in X \); \( r \) is the secondary variable, \( r \in \mathcal{I} \) and it has domain \( \mathcal{I}_x \) at each \( x \in X \); \( \mathcal{J}_x \) is called the primary membership of \( x \) and is defined in Eq. (7); and, the secondary grades of \( A \) are all equal 1. \( \mathcal{I} \) denotes union over all admissible \( x \) and \( r \). \( \mathcal{J}_x \) is replaced by \( \sum \) when the universe of discourse is discrete. The union of all the primary memberships for a fuzzy set \( A \) is called the footprint of uncertainty (FOU) of \( A \) (see Fig. 2) i.e.,

\[
FOU(\tilde{A}) = \bigcup_{x \in X} \bigcup_{x' \in \mathcal{J}_x} \left\{ (x, r) : r \in \mathcal{I}_x \subseteq [0, 1] \right\}
\]

The upper membership function (UMF) and the lower membership function (LMF) of \( A \) are two type-1 MFs that bound the FOU. The UMF is associated with the upper bound of FOU \( A \) and is denoted \( \bar{\mu}_A(x) \), \( \forall x \in X \), and the LMF is associated with the lower bound of FOU \( A \) and is denoted \( \underline{\mu}_A(x) \), \( \forall x \in X \), i.e.,

\[
\bar{\mu}_A(x) = FOU(\tilde{A}) \quad \forall x \in X
\]

\[
\underline{\mu}_A(x) = FOU(\tilde{A}) \quad \forall x \in X
\]

When \( E(k) = x'_i \), the vertical line at \( x'_i \) intersects \( FOU(\tilde{A}) \) everywhere in the interval \([\underline{\mu}_A(x'_i), \bar{\mu}_A(x'_i)]\); and, when \( \Delta E(k) = x'_i \), the vertical line at \( x'_i \) intersects \( FOU(\tilde{B}) \) everywhere in the interval \([\underline{\mu}_B(x'_i), \bar{\mu}_B(x'_i)]\). Two firing levels are then computed, a lower firing level, \( f(x'_i) \), and an upper firing level \( \bar{f}(x'_i) \), where \( \bar{f}(x'_i) = \min \underline{\mu}_A(x'_i), \bar{\mu}_A(x'_i) \) and \( \bar{f}(x'_i) = \min \underline{\mu}_B(x'_i), \bar{\mu}_B(x'_i) \). The main observed comment from the result of the input and the antecedent operations is the firing interval \( F(x'_i) \), where \( F(x'_i) = [f(x'_i), \bar{f}(x'_i)] \). \( f(x'_i) \) is t-normed with LMF(\( \tilde{B} \)) and \( \bar{f}(x'_i) \) is t-normed with UMF(\( \tilde{B} \)).

2.2. Output processing

In this paper, the Wu–Mendel uncertainty bound method is used to calculate the output of the IT2F-PD controller. The steps of this method are calculated as follows [7]:

A. Computation of centroids of \( M \) consequent IT2-FSs:

\[
U_i^0(x) = \sum_{j=1}^{M} f U_j^i \quad U_i^M(x) = \sum_{j=1}^{M} f U_j^i
\]

B. Computation of boundary type-1 FLS centroids:

\[
U_i^0(x) = \sum_{j=1}^{M} f U_j^i \quad U_i^M(x) = \sum_{j=1}^{M} f U_j^i
\]
C. Computation of uncertainty bounds:

\[
\mathcal{U}(x) \subseteq U_l(x) \subseteq \bar{U}_l(x) \quad \mathcal{V}(x) \subseteq U_r(x) \subseteq \bar{U}_r(x)
\]

(10)

\[
\bar{U}_l(x) = \min\{U_l^{(0)}(x), U_l^{(1)}(x)\} \quad \bar{U}_r(x) = \max\{U_r^{(0)}(x), U_r^{(1)}(x)\}
\]

(11)

\[
L_l(x) = \bar{U}_l(x) + \left[\frac{\sum_{j=1}^{M} \hat{f}_l^i(j - f_l)}{\sum_{j=1}^{M} \hat{f}_l^i} \right]
\]

(12)

\[
L_r(x) = \bar{U}_r(x) + \left[\frac{\sum_{j=1}^{M} \hat{f}_r^i(j - f_r)}{\sum_{j=1}^{M} \hat{f}_r^i} \right]
\]

(13)

D. Computation of the approximate type-reduction sets:

\[
[U_l(x), U_r(x)] \approx [\hat{U}_l(x), \hat{U}_r(x)]
\]

\[
= \left[\frac{(L_l(x) + U_l(x))/2}{(U_r(x) + \bar{U}_r(x))/2}\right]
\]

(14)

E. Computation of approximate defuzzified output:

\[
U(x) \approx \hat{U}(x) = \frac{1}{2} \left[\hat{U}_l(x) + \hat{U}_r(x)\right]
\]

(15)

3. Nonlinear inverted pendulum system

3.1. Mathematical model

The inverted pendulum system defined here is shown in Fig. 3, which is formed from a cart, a pendulum and a rail for defining the position of the cart. The pendulum is hinged in the center of the top surface of the cart and can rotate around the pivot in the same vertical plane with the rail. The cart can move right or left on the rail freely. It is given that no friction exists in the system between the cart and the rail or between the cart and the pendulum [41].

The dynamic equations of the uncertain inverted pendulum system can be expressed as [41,42]:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
x_2 \\
f(x_1, x_2)
\end{bmatrix} + \Delta \begin{bmatrix}
x_1 \\
g_1(x_1)
\end{bmatrix} u
\]

(16)

\[
f(x_1, x_2) = \left[ g \sin(x_1) - \frac{m_p l_2^2 \sin(x_1) \cos(x_1)}{a} \right] / b,
\]

\[
g_1(x_1) = \cos(x_1)/ab
\]

(17)

(18)

where \(x_1\) is the angle of the pendulum, \(x_2 = \dot{x}_1\) is the angular velocity, and \(u\) is the control force in the unit (N) applied horizontally to the cart. The parameters, \(m_c\) and \(m_p\), are, respectively, the mass of the cart and the mass of the pendulum in the unit (kg), and \(g = 9.8\) m/s² is the gravity acceleration. The parameter \(l\) is the half length of the pendulum in meters. \(\Delta\) is the structural uncertainty of the inverted pendulum which indicates the parameter uncertainties which can be caused by parameters that are difficult or impossible to get a precise measure or that the parameters tend to vary as a function of time, temperature etc. For this pendulum system, the mean uncertain parameters are the Coulomb friction constants. Also, this uncertainty indicates to unmodelled dynamic uncertainty.

3.2. The proposed controller for the inverted pendulum

Fig. 4, shows the block diagram of the IT2F-PD controller for balancing an inverted pendulum on a cart. \(y_r\) denotes the desired angular position of the pendulum. The goal is to balance the pendulum in the upright position (i.e., \(y_r = 0\)) when it initially starts with nonzero angle of the vertical (i.e., \(x_1 \neq 0\)), \(u\) and \(d\) are the control signal (force) and the external disturbance, respectively. The proposed IT2F-PD controller for the inverted pendulum uses triangular membership functions. Figs. 5–7 show the membership function for the error signal \((\epsilon(k))\), the derivative of error signal \((\Delta \epsilon(k))\), and the control signal where the universe of discourse is divided into five overlapping IT2FSs values labeled NL (Negative Large), NS (Negative Small), Z (Zero), PS (Positive Small) and PL (Positive Large). The rule base is described in Table 1. The scaling factors \(G_\epsilon\), \(G_{\Delta \epsilon}\), and \(G_u\) are equal 1, 10, and 20 respectively.

4. Simulation results

In this section, an inverted pendulum-cart system has been simulated using the proposed IT2F-PD controller. In order to clear the improvement of the proposed controller, the T1F-PD controller also is implemented for comparison purposes using the same number of the membership functions, a number of rules, the same universe of discourse and the same scaling factors. The parameters of an inverted pendulum-cart system are given in Table 2. Three different tasks are considered.

4.1. Task 1: Normal case

Fig. 8, shows the response of the inverted pendulum system under the IT2F-PD and T1F-PD controllers with initial conditions \(x_1 = 0.3\) rad, and \(x_2 = 0\). Fig. 8a, shows the response of the pendulum angle which moves toward the desired position without a steady state error and without an overshoot for both the proposed IT2F-PD and the T1F-PD controllers. Fig. 8b and c shows the response of the angular velocity and the force applied to the cart of the pendulum, respectively. Fig. 8d,
The interval type-2 fuzzy PD controller for an inverted pendulum.

Membership functions of the error signal.

Membership functions of the derivative of error signal.

Membership functions of the control signal.
shows the phase plane trajectory of the inverted pendulum system which is moving from the initial conditions to the origin, indicating that the system is stable for both the T1F-PD and the proposed IT2F-PD controllers.

### 4.2. Task 2: Structure uncertainty

To simulate this type of uncertainty, we add the value of $\Delta A$ to the system states ($x_1$ and $x_2$) as shown above in Eq. (16). When this value is added, this means that the parameters of the inverted pendulum are changed. Fig. 9, shows the response of the inverted pendulum system using the IT2F-PD and the T1F-PD controllers for initial conditions $x_1 = 0.1$ rad, and $x_2 = 0$. The uncertainty value $\Delta A$ is defined as:

\[
\Delta A = \begin{bmatrix} 0.075 & 0.075 \\ 0.075 & 0.075 \end{bmatrix}
\]

This value is defined from time equal $1$ s. Fig. 9a, shows the response of the pendulum angle which is seemed to oscillate about the origin after adding the uncertainty value for the T1F-PD controller but, the response of the pendulum angle for the proposed IT2F-PD controller is good without an oscillation about the origin after adding the uncertainty value. Also, the angular velocity and the force are seemed to oscillate about the origin as shown in Fig. 9b and 9c respectively, for the T1F-PD controller. Fig. 9d, shows the phase plane trajectory which gives rise to a circular trajectory corresponding to an oscillatory system for the T1F-PD controller, but the system trajectory moves to the origin for the proposed IT2F-PD controller which indicates the system is stable. So, the proposed IT2F-PD controller has the superiority to respond the system uncertainties rather than the T1F-PD controller.

### 4.3. Task 3: external disturbances

The response of the inverted pendulum system using the IT2F-PD and the T1F-PD controllers after adding the angle disturbance value equal $0.06$ rad at time equal $3$ sec is shown in Fig. 10. The response of the pendulum angle for the proposed IT2F-PD controller is made significantly faster than the T1F-PD controller after adding the angle disturbance as shown in Fig. 10a. The phase plane trajectory of the inverted pendulum system leaves the origin and moves again to the origin after adding the angle disturbance which indicates that the system is stable for the T1F-PD controller and the proposed IT2F-PD controller. The response of the inverted pendulum system using the IT2F-PD and the T1F-PD controllers for the external disturbance value $d = 100$ N is shown in Fig. 11. The response of the pendulum angle for the proposed IT2F-PD controller has smaller settling time than the T1F-PD controller.

### Table 1: The rule base.

<table>
<thead>
<tr>
<th>Derivative of error signal</th>
<th>Error signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL</td>
<td>PL PL PL PS Z</td>
</tr>
<tr>
<td>NS</td>
<td>PL PL PS Z NS</td>
</tr>
<tr>
<td>Z</td>
<td>PL PS Z NS NL</td>
</tr>
<tr>
<td>PS</td>
<td>PS Z NS NL NL</td>
</tr>
<tr>
<td>PL</td>
<td>Z NS NL NL NL</td>
</tr>
</tbody>
</table>

### Table 2: The parameters of the inverted pendulum system.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter name</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_c$</td>
<td>The mass of the cart</td>
<td>0.5 kg</td>
</tr>
<tr>
<td>$m_p$</td>
<td>The mass of the pendulum</td>
<td>0.2 kg</td>
</tr>
<tr>
<td>$l$</td>
<td>The half length of the pendulum</td>
<td>0.5 m</td>
</tr>
<tr>
<td>$g$</td>
<td>The gravity acceleration</td>
<td>9.8 m/s$^2$</td>
</tr>
</tbody>
</table>

**Figure 8**  Response of the inverted pendulum system for normal case. (a) Pendulum angle. (b) Angular velocity. (c) Control signal. (d) Phase plane trajectory.
after adding the external disturbance at time equal 2 sec as shown in Fig. 11a. The phase plane trajectory of the system indicates the system is stable for both the proposed IT2F-PD and the T1F-PD controllers. Fig. 12, shows the response of the inverted pendulum system using the IT2F-PD and the T1F-PD controllers for external disturbance value, $d = 110$ N. The response of the pendulum angle for the proposed IT2F-PD controller is made significantly better than the T1F-PD controller as shown in Fig. 12a. After adding the external disturbance the phase plane trajectory leaves the origin and moves again to the origin for the proposed IT2F-PD controller which indicates the system is stable, but the trajectory moves away from the origin, implying an unstable system for the T1F-PD controller as shown in Fig. 12d. So, the proposed IT2F-PD controller is able to respond the effect of the external disturbance rather than the T1F-PD controller.

To show the visual indications of the control performance, an objective measure of an error performance was made using the integral of square of errors (ISE), the root mean square error (RMSE) and the mean average error (MAE) criteria.
ISE, the RMSE and the MAE are defined in Eqs. (19)–(21) respectively.

\[
ISE = \int_0^\infty |e(t)|^2 dt \tag{19}
\]

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (e(t))^2} \tag{20}
\]

\[
MAE = \int_0^\infty |e(t)| dt \tag{21}
\]

Tables 3–5 list the ISE, the RMSE and the MAE values respectively, for the proposed IT2F-PD controller, the T1F-PD controller which has been proposed previously [38] and the fuzzy sliding mode control (FSMC) which has been proposed in [36,37] for controlling the inverted pendulum system. It is clear that, all the values of the ISE, the RMSE and the MAE which obtained for the proposed IT2F-PD controller are lower than that obtained for the T1F-PD controller and the FSMC. So, the proposed IT2F-PD controller has the superiority to respond the system uncertainties and the effect of the external disturbances rather than the T1F-PD controller and the FSMC which

Figure 11  Response of the inverted pendulum system when the value of disturbance \(d = 100\) N. (a) Pendulum angle. (b) Angular velocity. (c) Control signal. (d) Phase plane trajectory.

Figure 12  Response of the inverted pendulum system when the value of disturbance \(d = 110\) N. (a) Pendulum angle. (b) Angular velocity. (c) Control signal. (d) Phase plane trajectory.

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have been proposed previously for controlling the inverted pendulum. Table 6, shows the computational time, the settling time and the maximum error for the proposed IT2F-PD controller, the T1F-PD controller and the FSMC. It is clear that, the proposed IT2F-PD controller has a large computation time but, the settling time and the maximum error for the proposed IT2F-PD controller are lower than the T1F-PD controller and FSMC.

5. Conclusions

In this paper, the IT2F-PD controller is proposed for controlling the uncertain inverted pendulum on a cart. It uses the Mamdani interval type-2 rule based, the interval type-2 fuzzy sets with triangular membership function, and the Wu-Mendel uncertainty bound method to approximate the type-reduced set. The proposed controller has been tested by using three simulation tasks including the normal case, the structure uncertainties, and the external disturbances. All the simulation results of the proposed IT2F-PD controller are compared with the results of the T1F-PD controller. In the normal case, the response of the inverted pendulum is good for both controllers. For the structured uncertainty case, the inverted pendulum system remains stable for the proposed controller. But, it becomes an oscillatory system for the T1F-PD controller. So, the proposed IT2F-PD controller can handle the structured uncertainties rather than the T1F-PD controller. For the external disturbances case, the proposed IT2F-PD controller has the ability to respond the effect of external disturbances rather than the T1F-PD controller. The test is carried using the three performance indices (the ISE, the RMSE and the MAE). All the values obtained for the proposed IT2F-PD controller are lower than that obtained for the T1F-PD controller and FSMC which have been designed previously for controlling the inverted pendulum system. The computation time for the proposed IT2F-PD controller is larger than the other controllers, but the settling time and the maximum error for the proposed controller are lower than other controllers. So, the proposed IT2F-PD controller has the superiority to respond the system uncertainties and the effect of the external disturbances for the inverted pendulum system rather than the T1F-PD controller and the FSMC.

The major contributions of this study are as follows: (1) the successful development of fuzzy PD controller to IT2F-PD controller. (2) The successful application of the proposed IT2F-PD controller for controlling the nonlinear inverted pendulum on a cart system. (3) The success of the proposed controller to minimize the effect of the system uncertainties and the external disturbances.

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