An explicit multi-exponential model for semiconductor junctions with series and shunt resistances

Denise Lugo-Muñoz a, Juan Muci a, Adelmo Ortiz-Conde a, Francisco J. García-Sánchez a,⁎, Michelly de Souza b, Marcelo A. Pavanello b

a Solid-State Electronics Laboratory, Simón Bolívar University, Caracas 1080, Venezuela
b Department of Electrical Engineering, Centro Universitário da FEI, São Bernardo do Campo, SP, Brazil

ARTICLE INFO
Article history:
Received 21 April 2011
Received in revised form 25 June 2011
Accepted 27 June 2011
Available online 22 July 2011

ABSTRACT
An alternative explicit multi-exponential model is proposed to describe multiple, arbitrary ideality factor, conduction mechanisms in semiconductor junctions with parasitic series and shunt resistances. This Lambert function based model allows the terminal current to be expressed as an explicit analytical function of the applied terminal voltage, in contrast to the implicit-type conventional multi-exponential model. As a result this model inherently offers a higher computational efficiency than conventional models, making it better suited for repetitive simulation and parameter extraction applications. Its explicit nature also allows direct analytic differentiation and integration. The model's applicability has been assessed by parameter extraction and subsequent playback using synthetic and experimental diode forward I–V characteristics.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

The simplest approach to describe the electrical behavior of semiconductor junctions, be they bipolar or Schottky type, is by the ideal diode single-exponential Shockley equation. This well known equation may be readily modified into an implicit equation that includes the effects of possible parasitic loss mechanisms represented by series and parallel resistances added to the diode. Numerous analytic approximations have been proposed to overcome the computational inconvenience of its implicit nature [1,2]. However, the current and the voltage of the single-exponential modified junction equation may be explicitly solved without approximation making use of the Lambert functions [3]. Such solutions were first reported in 2000, for the case of only series resistance present [4], and for the more general case where in addition to the presence of parasitic series resistance there might be junction and/or peripheral shunt losses [5]. Those analytical solutions are widely used today in many applications, for example in different kinds of diodes [6–8] and solar cells [9–11].

When modeling real junctions however, a single-exponential equation is usually not enough to adequately represent the several conduction phenomena that frequently make relevant contributions to the total current of a particular junction. In such cases junctions need to be represented by lumped multi-diode equivalent circuits such as the one shown in Fig. 1. The junction is modeled in the presence of parasitic series and shunt resistances by the summation of multiple-exponential expressions, corresponding to each of the conduction mechanisms significant enough to justify its inclusion in the model, plus a resistive shunt leakage term. Accordingly, the total current has been traditionally described by the following conventional implicit equation:

I = ∑ k=1 N I pk exp [ (V − RkI/nkVth) − 1] + Gs(V − RsI),

where N is the number of different conduction mechanisms to be modeled, Ipk are the reverse current coefficients corresponding to each kth mechanism, nk are their respective “ideality” factors, Rs is the parasitic series resistance, Vth = kT/q is the thermal voltage, and Gs = 1/Rs is the shunt conductance.

An equation such as (1), with an appropriate number, N, of conduction mechanisms included, usually provides a sufficiently faithful representation of the junction’s I–V characteristics for most practical lumped modeling applications. Unfortunately, it has the serious shortcoming of not being explicitly solvable in general for either the terminal current or voltage. There are two very specific exceptions to this insolvability: (a) the case of the single exponential (N = 1) model already mentioned above whose explicit solutions are known [5], and (b) the case of a double-exponential (N = 2) model where the ideality factors are known fixed quantities, one equal to twice the other (n2 = 2n1), in which case it is possible to write a quadratic explicit solution for the terminal voltage [12,13].

The availability of analytically explicit device model equations constitutes a very desirable feature for circuit simulation...
applications, especially if the model is to be used repeatedly. Simulation times can be significantly reduced by the computational efficiency improvement afforded by avoiding the numerical iterations required by implicit equation models. Another advantage of models made of explicit equations is that they may be analytically differentiated and integrated directly, an undoubtedly advantageous quality for developing other model-derived functions, such as, for example, the dynamic resistance as a function of applied voltage. Not less important is the fact that in general any procedure to be used for model parameter extraction is easier to conceive and more efficiently implemented if it is based on explicit model equations.

Therefore, in order to circumvent the explicit insolvability of (1) and thus be able to benefit from the above mentioned advantages of an explicit model, we propose in what follows the use of an explicit multi-exponential alternative which might prove to be useful when modeling semiconductor junctions with multiple conduction types and series and shunt resistive-like losses.

2. Alternative model

The equivalent circuit of the proposed alternative model is present in Fig. 2. All current transport mechanisms considered by this model are additive and exhibit either exponential (rectifying) type or linear (shunt) type of dependence with respect to voltage, as indicated by the model equation. By solving each branch separately and adding the solutions, this model’s I–V characteristics may be expressed by the following explicit equation for the current:

$$I = \sum_{k=1}^{N} \left\{ \frac{n_k V_{th}}{R_{ka}} \exp \left( \frac{V + R_{ka} I_{th}}{n_k V_{th}} \right) - I_{th} \right\} + G_{Pa} V,$$

(2)

where $W_{0}$ is short hand notation for the principal branch of the Lambert function [3], $G_{Pa} = 1/R_{Pa}$ is the alternative outer shunt conductance and the rest of the parameters are defined as before. Notice that the single global series resistance, $R_{S}$, present in the conventional model, has been substituted in this alternative model by individual series resistances, $R_{ka}$, placed in each of the $k$ parallel current paths associated with the different conduction mechanisms.

The alternative model does not necessarily correspond in general to a physical current conduction phenomenology. Rather, it is proposed here primarily as a mathematical alternative to the conventional implicit current model, that can faithfully represent the current as an explicit function of the applied voltage. Although conventional and alternative models, represented by the equivalent circuits of Figs. 1 and 2, are not exactly bidirectionally equivalent, it is evident that there can be a broad range of parameter conditions for which the correspondence between both models can be excellent. For example, in practical devices, the shunt resistance $R_{Pa}$ is usually much greater than the series resistance $R_{S}$ of the conventional model ($R_{Pa} \gg R_{S}$). Therefore, in such cases, the shunt resistance $R_{Pa}$ of the alternative model is approximately equal to the shunt resistance $R_{S}$ of the conventional model ($R_{Pa} \approx R_{S}$). Also

$$1/R_{P} \approx \sum_{k=1}^{N} 1/R_{ka},$$

(3)

Care should be exercised however while using Eq. (2) for numerical calculations when the value of any of the individual series resistances $R_{ka}$ associated with a particular $k$th conduction mechanism is equal to zero. A value of $R_{ka} = 0$ may not be merely substituted into the corresponding $k$th summand. Instead, its limit as $R_{ka}$ goes to zero must be used, which is simply given by:

$$I_{k} = I_{th} \left[ \exp \left( \frac{V}{n_{k} V_{th}} \right) - 1 \right].$$

(4)

The choice of the number of branches to be included in the model must be made by the researcher. The decision criteria would generally depend on how many conduction phenomena are evident in the device’s measured I–V characteristics. The researcher may alternatively use fixed values for one or more ideality factors, thereby imposing on the model equation the type of phenomenology that is to be considered dominant or physically relevant for a particular device under given operating conditions.

3. Example of model correspondence

In order to validate the applicability of the alternative model, and since a two-diode equivalent circuit is frequently used to model junctions with two predominant conduction mechanisms, we generated synthetic I–V characteristics using the conventional implicit double-exponential equation given by:

$$I = I_{01} \left[ \exp \left( \frac{V - R_{d1}}{n_{1} V_{th}} \right) - 1 \right] + I_{02} \left[ \exp \left( \frac{V - R_{d2}}{n_{2} V_{th}} \right) - 1 \right] + G_{Pa}(V - R_{d}I),$$

(5)

which is just Eq. (1) for the particular case of $N = 2$.

Model parameters were then extracted from those synthetic I–V characteristics by directly fitting the logarithm of the alternative model double-exponential explicit equation:

$$I = \frac{n_{1a} V_{th}}{R_{S1a} W_{0}} \left\{ \frac{R_{ka} I_{th}}{n_{1a} V_{th}} \exp \left( \frac{V + R_{ka} I_{th}}{n_{1a} V_{th}} \right) \right\} + \frac{n_{2a} V_{th}}{R_{S2a} W_{0}} \left\{ \frac{R_{ka} I_{th}}{n_{2a} V_{th}} \exp \left( \frac{V + R_{ka} I_{th}}{n_{2a} V_{th}} \right) \right\} - (I_{01a} + I_{02a}) + G_{Pa} V,$$

(6)

to the logarithm of the synthetic I–V numeric data. The parameter values originally used to calculate the synthetic data with (5) are indicated on the left column of Table 1. The parameter values extracted from fitting (6) to the synthetically generated data are
As mentioned before, this alternative model’s explicit nature gives it a computational advantage over the conventional implicit model by avoiding numeric iteration. A quick test, run on a common mathematical software, consisting of calculating a thousand points of the present synthetic I–V characteristics example indicates that the computation time is reduced by a factor of about 4.5 when using the alternative model as compared to using the conventional model. Such significantly higher calculation efficiency of the alternative model strongly favors its use for repetitive circuit simulation purposes. Additionally, parameter extraction procedures can also be expedited by virtue of the alternative model’s explicit nature.

4. Modeling real devices

To further appraise the usefulness of the present alternative model, its double-exponential version \((N = 2)\) was again used to model the forward current of experimental lateral PIN diodes fabricated using a 150 nm technology from OKI Semiconductor on a 40 nm thick silicon layer over a 145 nm substrate oxide [15]. I–V characteristics were measured at forward voltages from 0.1 to 1.5 V, using voltage steps of 10 mV, at several temperatures of interest.

Fig. 4 presents the I–V characteristics of a 0.8 \(\mu\)m long 50 \(\mu\)m wide device measured at four temperatures from 300 to 390 K. The figure also includes the corresponding alternative model playbacks as calculated using (6) with the model parameter values extracted for each of the four temperatures. Model parameters were extracted, for both conventional and alternative double-exponential models, by globally fitting the logarithm of each model’s Eqs. (5) and (6) to the experimental data. The extracted parameter values are presented together in Table 2 for comparison.

As in the case of the conventional multi-exponential model, any traditional equation fitting procedure may be used for model parameter extraction. However, a faster convergence is to be expected when using this model, because of the explicit nature presented on the right column of that same table. Notice that, as expected: \(I_{0a1} \approx I_{01}, I_{0a2} \approx I_{02}, n_{a1} \approx n_1, n_{a2} \approx n_2\), the parallel combination of \(R_{S1}\) and \(R_{S2}\) is 29.85 \(\Omega \approx R_S\), in accordance with (3), and \(R_{S0} \approx R_P\) because \(R_P \gg R_S\).

It is worth mentioning here that Eq. (6) resembles the defining equation of a model proposed by Miranda et al. to express the leakage post-breakdown I–V characteristics of HfO2/TaN/TiN gate oxide stacks used in MOSFETs. In that unrelated model the most relevant post-breakdown conduction mechanisms that arise in the broken gate oxide are described by a parallel combination of two opposite-direction connected diodes with individual series resistances and a shunt leakage path [14].

The original synthetic data generated by the conventional implicit model (5) is presented in Fig. 3, together with the alternative model playback produced by the explicit double-exponential Eq. (6) using the extracted parameters. The three summands of (6) are also shown separately in Fig. 3 to illustrate their individual contributions to the total current.

The excellent correspondence between the two models in this synthetic example, with a calculated maximum absolute error of <0.25% in this case, attests to the suitability of the alternative explicit equation to faithfully reproduce the I–V characteristics of typical conventional junctions.

---

**Table 1**

<table>
<thead>
<tr>
<th>Parameter values used in the conventional model (5) to generate synthetic data</th>
<th>Extracted parameter values of alternative model (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_{0a1}) (A) 1.00 (\times) 10(^{-16}) &amp; (I_{0a1a}) (A) 1.15 (\times) 10(^{-16})</td>
<td></td>
</tr>
<tr>
<td>(I_{0a2}) (A) 3.00 (\times) 10(^{-15}) &amp; (I_{0a2a}) (A) 3.04 (\times) 10(^{-13})</td>
<td></td>
</tr>
<tr>
<td>(n_1) 1.000 &amp; (n_{1a}) 1.005</td>
<td></td>
</tr>
<tr>
<td>(n_2) 1.500 &amp; (n_{2a}) 1.503</td>
<td></td>
</tr>
<tr>
<td>(R_S) ((\Omega)) 30.00 &amp; (R_{S1a}) ((\Omega)) 33.72</td>
<td></td>
</tr>
<tr>
<td>(R_P) ((\Omega)) 1.00 (\times) 10(^{10}) &amp; (R_{P2a}) ((\Omega)) 259.99</td>
<td></td>
</tr>
</tbody>
</table>

---

**Fig. 3.** Synthetic I–V characteristics calculated from the conventional two-diode model (red circles) and the alternative model playback (black continuous line) with its three constituent branch currents (colored broken lines). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

**Fig. 4.** Measured (red symbols) and alternative model playbacks (black solid lines) forward I–V characteristics of an experimental lateral PIN diode at four temperatures. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
of its defining equation. Good initial parameter values that would expedite fitting may be easily found by prior observation of the measured $I-V$ characteristics.

It seems to be evident from Table 2 that the extracted conventional model parameter values are very similar to their corresponding alternative model values. In this particular device the resistive shunt loss is negligible ($R_{sh} \approx R_p$) compared to the other conduction mechanisms. Notice also that the parallel combination of $R_S$ and $R_{sh}$ is $\approx R_S$, as expected.

Fig. 5 shows the percent relative errors resulting from the use of each of the two models. These graphs are presented here only as a visual aid to facilitate the relative comparison of the errors’ magnitudes produced by both models. The specific shapes of error evolution with forward voltage that can be observed in the different plots of Fig. 5 should not be taken as necessarily representing a general behavior, since they are a consequence of the particular curve-fitting nonlinear optimization algorithm that we happen to use for model parameter extraction, which is not mentioned here because it is irrelevant for this present work.

The playback errors relative to the original measured data shown in Fig. 5 indicate that the alternative model produces a more accurate representation of this device’s forward conduction behavior at the four temperatures considered here. The maximum magnitude of the error produced by the alternative model is 6.5%, which is less than half of the 15.6% observed for the conventional model. Notice that above about 1 V, which is the region dominated by series resistance-limited linear-like conduction, the errors in both models decrease and become independent of temperature.

The superior description of these particular PIN diodes’ $I-V$ characteristics provided by the alternative double-exponential model suggests that the forward conduction mechanisms present in these devices are probably more adequately described by placing a separate resistance in series with each diode of the model’s two-diode equivalent circuit, than by the single series resistance of the conventional model.

Models somehow alike the alternative model mentioned here have been proposed to individually describe the different current conduction mechanisms present in poly and multicrystalline solar cells [16–19].

5. Discussion and conclusions

The presently described alternative approach constitutes a useful tool to model the various current transport mechanisms that usually coexist in real semiconductor junctions and which are in general each characterized by an arbitrary ideality factor. The distinctive quality of this approach is that the junction terminal current may be expressed as an explicit function of the applied terminal voltage. This feature of the presently proposed model represents a very valuable advantage over the conventional and traditionally used multi-diode model with series and shunt resistances, whose current–voltage equation is unavoidably implicit in general for arbitrary ideality factors [20].

The inherent explicit nature of the presently proposed alternative model allows a significantly higher numeric computational efficiency in simulation applications, facilitates curve fitting procedures for model parameter extraction [21], and allows for straightforward analytic differentiation and integration. All these are very attractive attributes for modeling a wide variety of junction types including solar cells [22].

The excellent suitability to faithfully describe junction behavior is validated by the close match found between test synthetic $I-V$ characteristics, generated by a conventional implicit two-exponential model equation, and the $I-V$ characteristics generated using the extracted model parameters of the present alternative model.

The applicability of the proposed alternative model was further tested on experimental PIN diode $I-V$ characteristics measured at several temperatures. Model parameter extraction was performing using global fitting of both the conventional and the present...
alternative model equations. Comparative error analysis of both models' playbacks relative to the original data indicates that when the presently proposed alternative explicit model is used for these experimental PIN diodes, the results obtained turn out to be more precise than those from the conventional traditional implicit model. The observed superior accuracy of the proposed alternative model to describe these PIN diodes suggests that perhaps this alternative model might also represent an even better phenomenological description of the conduction mechanisms prevalent in these particular devices. Further physical analysis of these specific devices in terms of the proposed model is currently underway, but it is beyond the interest and scope of the present paper.

Acknowledgments

The authors wish to express their gratitude to Prof. Denis Flan dre from the Electrical Engineering Department, ICTEAM Institute, Université Catholique de Louvain, Louvain-la-Neuve, Belgium, for providing the experimental samples. They also acknowledge the support received from CNPq PROSUL international cooperation (Brazil).

References