Using Homo-Separation of Variables for Solving Systems of Nonlinear Fractional Partial Differential Equations

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Abstract: In this research work, a new method proposed and coined by the authors as the Homo-Separation of Variables method is utilized to solve systems of linear and nonlinear fractional partial differential equations (FPDEs). The new method is a combination of two well-established mathematical methods; namely the homotopy perturbation method (HPM) and the separation of variables method. When compared to existing analytical and numerical methods, the method resulted from our approach shows that it is capable of simplifying the target problem at hand and reducing the computational load that is required to solve it, considerably. The efficiency and usefulness of this new general-purpose method is verified by several examples, where different systems of linear and nonlinear FPDEs are solved.

Keywords: Homo-separation of variables, Riemann–Liouville, Mittag-leffler, Relaxation-oscillation

1 Introduction

In the recent years, it has turned out that many phenomena in various technical and scientific fields can be described very successfully by using fractional calculus. In particular, fractional calculus can be employed to solve many problems within the biomedical research field and get better results. Such a practical application of fractional order models is to improve the behavior and efficiency of bioelectrodes. The importance of this application is based on the fact that bioelectrodes are usually needed to be used for all types of biopotential recording and signal measurement purposes such as Electroencephalography (EEG), Electrocardiography (ECG) and Electromyography (EMG) [18, 19, 15]. Another promising biomedical application field is proposed by Arafa et al. [3] where a fractional-order model of HIV-1 infection of CD4+ T-cells is introduced. Other examples of applications of fractional calculus in life since and technology can be found in botanics [13], biology [6], rheology [7] and elastography [5].

Numerous analytical methods have been presented in the literature to solve FPDEs, such as the fractional Greens function method [20], the Fourier transform method [19], the Sumudu transform method [9], the Laplace transform method and the Mellin transform method [24]. Some numerical methods have also widely been used to solve systems of FPDEs, such as the variational iteration method [21], the Adomian decomposition method [19], the homotopy perturbation method [1] and the homotopy analysis method [8]. Some of these methods use specific transformations and others give the solution as a series which converges to the exact solution.

In addition, some numerical methods use a combination of utilizing specific transformations and obtaining series which converge to the exact solutions. An example of such a method is the iterative Laplace transform method which is a combination of the Laplace transform method and an iterative method [12]. Another such a combination is the homotopy perturbation transformation method, which is constructed by combining two powerful methods; namely the Laplace transform method and the homotopy perturbation method [16]. A third example is the Sumudu decomposition method, which is a combination of the Sumudu transform method and Adomian decomposition method [17]. And a fourth such an approach is combining the Sumudu transformation method with the homotopy perturbation method, which gives a new method called the homotopy perturbation Sumudu transform method [26].

Recently, the homotopy perturbation method and the Adomian decomposition method are frequently used for solving nonlinear FPDE problems.

According to the combinational methods mentioned above, it is possible to notice that HPM have strong potential to be combined with other method to produce a more efficient approach. The main reason is that HPM is an efficient method for solving PDEs and ODEs (ordinary differential equations) with integer or fractional order. Recently, Karbalaie et al.