MRI IMAGE COMPRESSION USING BIORTHOGONAL CDF WAVELET BASED ON LIFTING SCHEME AND SPIHT CODING

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Abstract. In the field of medical diagnostics, interested parties have resorted increasingly to medical imaging, it is well established that the accuracy and completeness of diagnosis are initially connected with the image quality, but the quality of the image is itself dependent on a number of factors including primarily the processing that an image must undergo to enhance its quality. We introduce in this paper an algorithm for medical image compression based on biorthogonal wavelet transform CDF 9/7 coupled with SPIHT coding algorithm, of which we applied the lifting structure to improve the drawbacks of wavelet transform. In order to enhance the compression by our algorithm, we have compared the results obtained with wavelet based filters bank. The results are very satisfactory regarding compression ratio, and the computation time and quality of the compressed image compared to those of traditional methods.

Keywords: Compression, MRI, Biorthogonal wavelets, CDF9/7, Lifting scheme, SPIHT.

1. Introduction

The massive use of numerical methods in medical imaging (MRI, X scanner, nuclear medicine, etc...) today generates increasingly important volumes of data. The problem becomes even more critical with the generalisation of 3D sequence. So it is necessary to use compressed images in order to limit the amount of data to be stored and transmitted.

Many compression schemes by transformation have been proposed, we can cite the standards JPEG images, MPEG 1 and 2 for compressing video. All of these standards are based on the discrete cosine transform (DCT).[1].

Over the past ten years, the wavelets (DWT), have had a huge success in the field of image processing, and have been used to solve many problems such as compression and restoration of images [2]. However, despite the success of wavelets in various fields of image processing such as encoding, weaknesses have been noted in its use in the detection and representation of the objects’ contours. The wavelets transform and other classical multi resolutions decompositions seem to form a restricted and limited class of opportunities for multi-scale representations of multidimensional signals.

To overcome this problem, new multi resolution decompositions better adapted to the representation of images. This is the case of decomposition by lifting scheme. This structure allows the construction decomposition transforms multi resolution nonlinear able to represent the dyadic wavelet transform.

In this work we propose the lifting structure algorithm for MRI image compression. For this reason, this paper is divided into three parts: the first of which is devoted to a representation of the lifting scheme, then we present the biorthogonal wavelet CDF 9/7.

In order to enhance the image compression by our algorithm, we compare the PSNR and MSSIM results obtained with the existing techniques namely the wavelets filter bank.

2. Wavelet Transforms

The wavelet transform (WT), in general, produces floating point coefficients. Although these coefficients can be used to reconstruct an original image perfectly in theory, the use of finite precision arithmetic and quantization results in a lossy scheme.

Recently, reversible integer WT’s (WT’s that transform integers to integers and allow perfect reconstruction of the original signal) have been introduced [3],[4]. In [5], Calderbank and al. introduced how to use the lifting scheme presented in [6], where Sweldens showed that the convolution based biorthogonal WT can be implemented in a lifting-based scheme as shown in figure(1) for reducing the computational complexity. Note that only the decomposition part of WT is depicted in figure(1) because the reconstruction process is just the reverse version of the one in figure(1). The lifting-based WT consists of splitting, lifting, and scaling modules and the WT is treated as a prediction-error decomposition. It provides a complete spatial interpretation of WT. In figure(1), let X denote the input signal and $X_{L1}$ and $X_{H1}$ be the decomposed output signals, where they are obtained through the following three modules of lifting-based 1DWT:

1. Splitting: In this module, the original signal X
is divided into two disjoint parts, i.e., $X_e(n)=X(2n)$ and $X_o(n)=X(2n+1)$ that denote all even-indexed and odd-indexed samples of $X$, respectively [7].

2. Lifting: In this module, the prediction operation $P$ is used to estimate $X_e(n)$ from $X_o(n)$ and results in an error signal $d(n)$ which represents the detail part of the original signal. Then we update $d(n)$ by applying

$$d(n) = X(n) - P(d(n-1), d(n-2)).$$

Figure 1. The lifting-based WT

3. Scaling: A normalization factor is applied to $d(n)$ and $s(n)$, respectively. In the even-indexed part $s(n)$ is multiplied by a normalization factor $K_e$ to produce the wavelet subband $X_{L1}$. Similarly in the odd-indexed part the error signal $d(n)$ is multiplied by $K_o$ to obtain the wavelet subband $X_{H1}$.

Note that the output results of $X_{H1}$ and $X_{L1}$ obtained by using the lifting-based WT are the same as those of using the convolution approach for the same input even they have completely different functional structures. Compared with the traditional convolution-based WT, the lifting-based scheme has several advantages. First, it makes optimal use of similarities between the highpass and lowpass filters, the computation complexity can be reduced by a factor of two. Second, it allows a full in-place calculation of the wavelet transform. In other words, no auxiliary memory is needed.

3. Biorthogonal Wavelets CDF 9/7

We are interested in this article by biorthogonal wavelet 9/7. These wavelets are part of the family of symmetric biorthogonal wavelet CDF. The low pass filters associated with wavelet 9/7 have $p=9$ coefficients in the analysis, $p=7$ coefficients to synthesize and are described in Table(1). The biorthogonal wavelets 9/7 are illustrated in figure(2), they have $N = 4$ null moments in analysis and $\tilde{N} = 4$ in synthesis. The wavelets 9/7 have a great number of null moments for a relatively short support. They are more symmetrical and very close to orthogonality. This is an important feature in coding which ensures that the reconstruction error is very close to the quantization error in terms of mean squared error. Antonini and Barlaud were the first [8] to show the superiority of Biorthogonal wavelet transform 9/7 for the decorrelation of natural images. It has been widely used in image coding [9],[10] and is used by the JPEG-2000 codec [11].

Table 1. a. The analysis filter coefficients. b. The synthesis filter coefficients.

<table>
<thead>
<tr>
<th>i</th>
<th>Low-pass filter</th>
<th>high-pass filter</th>
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<tbody>
<tr>
<td>0</td>
<td>0.6029490182363579</td>
<td>1.15087052457000</td>
</tr>
<tr>
<td>±1</td>
<td>0.266864118442875</td>
<td>0.591271763114250</td>
</tr>
<tr>
<td>±2</td>
<td>0.0782232665280900</td>
<td>0.057543526228500</td>
</tr>
<tr>
<td>±3</td>
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The Lifting scheme of the biorthogonal transform 9/7 is made up of four steps: two operators of predictions and two operators of update as shows it the figure(3) [12],[13].

Figure 2. CDF 9/7 wavelet $\psi$ and $\tilde{\psi}$ dual
The synthesis side of the filter bank simply inverts the scaling and reverses the sequence of the lifting steps. The following equation describes the 4-step lifting and scaling:

\[ \begin{align*}
Y(2n) &= (1/K) X Y(2n) \\
Y(2n+1) &= -K X Y(2n+1)
\end{align*} \]

where the values of the parameters are:

- \( K \) = 1.5861342, \( \alpha = 0.8829110762 \) and \( \delta = 0.4435068522 \).

The following equation describes the lifting steps:

\[ \begin{align*}
Y(2n) &= Y(2n) \\
Y(2n+1) &= Y(2n+1) + 1
\end{align*} \]

For lifting implementation, the CDF 9/7 wavelet filter pair can be factorized into a sequence of primal and dual lifting. The most efficient factorization of the polyphase matrix for the 9/7 filter is as follows:

\[ \begin{align*}
\begin{bmatrix}
Y(2n) \\
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Figure 4: Split, Predict and Update Steps of Forward and Update Steps. Figure 4 shows the synthesis side of the filter bank using lifting.

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decomposed into ten subbands. Then the method finds the maximum and the iteration number. Second step, the method puts the DWT coefficients into sorting pass that finds the significance coefficients in all coefficients and encodes the sign of these significance coefficients. Third step, the significance coefficients that be found in sorting pass are put into the refinement pass that use two bits to exact the reconstruct value for closing to real value. The front second and third steps are iterative, next iteration decreases the threshold ($T_n = T_{n-1}/2$) and the reconstructive value ($R_n = R_{n-1}/2$). Forth step, the encoding bits access entropy coding and then transmit [17]. The result is in the form of a bitstream.

All of the wavelet-based-image encoding algorithms improve the compression rate and the visual quality, but the wavelet-transform computation is a serious disadvantage of those algorithms.

Figure 6. Flowchart of SPIHT

5. Compression quality evaluation

The Peak Signal to Noise Ratio (PSNR) is the most commonly used as a measure of quality of reconstruction in image compression. The PSNR were identified using the following formulae:

$$MSE^2 = \frac{1}{M \times N} \sum_{i=1}^{m} \sum_{j=1}^{n} (l(i, j) - \hat{l}(i, j))^2$$

Mean Square Error (MSE) which requires two $m \times n$ grayscale images $I$ and $\hat{I}$ where one of the images is considered as a compression of the other is defined as:

The PSNR is defined as:

$$PSNR = 10 \log_{10} \left( \frac{\text{Dynamics of image}^2}{MSE} \right)$$

Usually an image is encoded on 8 bits. It is represented by 256 gray levels, which vary between 0 and 255, the extent or dynamics of the image is 255.

- The structural similarity index (SSIM)

The PSNR measurement gives a numerical value on the damage, but it does not describe its type. Moreover, as is often noted in [18],[19], it does not quite represent the quality perceived by human observers.

For medical imaging applications where images are degraded must eventually be examined by experts, traditional evaluation remains insufficient. For this reason, objective approaches are needed to assess the medical imaging quality.

We then evaluate a new paradigm to estimate the quality of medical images, specifically the ones compressed by wavelet transform, based on the assumption that the human visual system (HVS) is highly adapted to extract structural information. The similarity compares the brightness, contrast and structure between each pair of vectors, where the structural similarity index (SSIM) between two signals $x$ and $y$ is given by the following expression [20],[21]:

$$SSIM (x, y)=l(x, y)c(x, y)s(x, y)$$

Finally the quality measurement can provide a spatial map of the local image quality, which provides more information on the image quality degradation, which is useful in medical imaging applications.

For application, we require a single overall measurement of the whole image quality that is given by the following formula:

$$MSSIM (I, \hat{I})=\frac{1}{M} \sum_{i=1}^{M} \text{SSIM} (I_i, \hat{I}_i)$$

Where $I$ and $\hat{I}$ are respectively the reference and degraded images, $I$ and $\hat{I}$ are the contents of images at the $i$-th local window.

M: the total number of local windows in image. The MSSIM values exhibit greater consistency with the visual quality.

6. Results and discussion

We are interested in lossy compression methods based on 2D wavelet transforms because their properties are interesting. Indeed, the 2D wavelet transform combines good spatial and frequency locations. As we work on medical image the spatial location and frequency are important [22], [23].

In this article we have applied our algorithm to compress medical images, for this reason we have
chosen an axial slice of human brain size 512x512 (grayscale) encoded on 8 bits per pixel recorded by means of an MRI scanner (Figure 7.).

This image is taken from the GE Medical System database [24]. Our aim is to reduce the rates for which the image quality remains acceptable. In this work, estimates and judgments of the compressed image quality is given by the PSNR evaluation parameters and the MSSIM similarity Index. Figure (8) shown below illustrates the compressed image quality for different values of bit-rate (number of bits per pixel). According to the PSNR and MSSIM values, we note that from 0.5bpp, image reconstruction becomes almost perfect.

To show the performance of the proposed method, we will now make a comparison between these different types of transform (CDF 9 / 7 (Filter Bank); Gall5 / 3 (Lifting scheme) and CDF9 / 7 (Lifting scheme)) coupled with the SPIHT coding and CDF9 / 7 (Lifting scheme) combined with the EZW coding. For each application we vary the bitrate 0.125 to 2 and we calculate the PSNR and MSSIM. The results obtained are given in Table(2).
The comparison in terms of image quality for the four algorithms is given by the PSNR and MSSIM curves represented in figures 9 and 10.

Comparing the different values of PSNR and MSSIM, we show clearly the effectiveness in terms of compressed image quality by our algorithm.

This study was subsequently generalized to a set of MRI images of the GE Medical Systems database. The following figure (Figure 11.) presents the results obtained after application of different algorithms on another slice. These results are obtained with a the 0.75 bpp bite-rate.

We can say that compression degrades to a lesser extent the image structure for a low compression bit-rate. However, for high compression bit-rate, our algorithm better safeguards the various image structures.

### 7. Conclusion

The objective of this paper is undoubtedly the enhancement of medical images quality after the compression step. The latter is regarded as an essential tool to aid diagnosis (storage or transmission) in medical imaging. We used the Biorthogonal CDF9/7 wavelet compression based on lifting scheme, coupled with the SPIHT coding. After several applications, we found that this algorithm gives better results than the other compression techniques.

To develop our algorithm, we have applied this technique on different types of medical images (MRI). We have noticed that for 0.75 bpp bit-rate, the algorithm provides very important PSNR and MSSIM values for MRI images. Thus, we conclude that the results obtained are very satisfactory in terms of compression ratio and of compressed image quality.
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