Space-Time Correlated MIMO Mobile-to-Mobile Channels

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Abstract—A theoretical model is proposed for multiple-input multiple-output (MIMO) mobile-to-mobile (M-to-M) Rayleigh fading channels, such that the complex faded envelope does not depend on the distance between scatterers and antenna elements. From this model, a closed-form joint space-time correlation function is derived for 2-D non-isotropic scattering environment. Also, a space-frequency power density spectrum of the complex faded envelope is derived, assuming 2-D isotropic scattering environment. Finally, a statistical simulation model for MIMO M-to-M Rayleigh fading channels is proposed. The space-time correlation function of the simulation model is derived and verified by simulation, and an adaptive method for choosing the number of scatterers in simulations is proposed. The results show that the statistical simulation model is a good approximation of the theoretical model.

I. INTRODUCTION

Mobile ad-hoc wireless networks, intelligent transportation systems, and relay-based cellular networks all use mobile-to-mobile (M-to-M) communication channels, where both the transmitter ($T_x$) and the receiver ($R_x$) are in motion and equipped with low elevation antennas. M-to-M channels differ from conventional fixed-to-mobile (F-to-M) cellular radio channels, where the base-station is stationary, elevated, and relatively free of local scattering. Akki and Haber [1], [2] showed that the received envelope on M-to-M channels is Rayleigh faded under non line-of-sight (NLoS) conditions, but the statistical properties differ from F-to-M channels. They were the first to propose a reference model for single-input single-output (SISO) M-to-M Rayleigh fading channels. Methods for simulating SISO M-to-M channels have been proposed in [3]-[5]. Recently, Pätzold et al. proposed a theoretical model for narrow-band multiple-input multiple-output (MIMO) M-to-M channel [6], [7]. This reference model has disadvantage that the complex faded envelope depends on the distance between scatterers and antenna elements, usually unknown parameter in simulation trials. Furthermore, the space-time correlation function for 2-D isotropic scattering, derived in [6], does not depend on the distances between scatterers and antenna elements.

This paper proposes a theoretical model for MIMO M-to-M Rayleigh fading channels, such that the complex faded envelope does not depend on the distance between scatterers and antenna elements. This model is based on the “double-ring” geometrical model proposed in [8]. Our model differs from the model proposed in [8] because it includes mobility of both the transmitter and the receiver. From this theoretical model, we derive a closed-form joint space-time correlation function for 2-D non-isotropic scattering environment. We also show that, for 2-D isotropic scattering, our correlation function reduces to the one derived in [6]. Furthermore, we derive a space-frequency power density spectrum for an isotropic scattering environment.

The theoretical model assumes an infinite number of scatterers, which prevents practical implementation. Hence, we propose a statistical simulation model for 2-D isotropic scattering environment based on a statistical sum-of-sinusoids (SoS) model for M-to-M channels. SoS models approximate the underlying random processes by the superposition of a finite number of properly selected functions, and can be classified as either statistical or deterministic. Deterministic SoS models have sinusoids with fixed phases, amplitudes, and Doppler frequencies for all simulation trials. Statistical SoS models leave at least one of the parameter sets (amplitudes, phases, or Doppler frequencies) as random variables that vary with each simulation trial. The statistical properties of the statistical SoS models vary for each simulation trial, but converge to the desired properties when averaged over a large number of simulation trials. A joint space-time correlation function of the simulation model is derived and verified by simulation, and an adaptive method for choosing the number of scatterers in simulations is proposed. The results show that the statistical simulation model is a good approximation of the theoretical model.

The remainder of the paper is organized as follows. Section II describes the communication system and presents a theoretical model for MIMO M-to-M channels. Section III presents a derivation of the joint space-time correlation function for the theoretical model, as well as a derivation of the space-frequency power density spectrum. Section IV details the statistical SoS simulation model along with a derivation of the joint space-time correlation function. Section V compares the statistical simulation model to the theoretical model. Finally, Section VI provides some concluding remarks.

II. THEORETICAL MODEL FOR MIMO MOBILE-TO-MOBILE CHANNELS

A. System Model

This paper considers a narrow-band single-user MIMO communication system with $L_t$ transmit and $L_r$ receive omni-
rectangular antenna elements. It is assumed that both the transmitter (Tx) and the receiver (Rx) are in motion and equipped with low elevation antennas. The radio propagation environment is characterized by 2-D scattering with non-line-of-sight (NLoS) conditions between the transmitter and the receiver. The MIMO channel can be described by an \( L_x \times L_y \) matrix \( \mathbf{H}(t) = [h_{ij}(t)]_{L_x \times L_y} \) of complex faded envelopes.

### B. Theoretical Model

Fig. 1 shows a geometrical "double-ring" model for MIMO M-to-M channel with \( L_t = L_r = 2 \) antenna elements. This elementary \( 2 \times 2 \) antenna configuration can be used to construct other types of 2-D multielement antenna arrays, including linear, hexagonal, and circular multielement antenna arrays. The "double-ring" model has been proposed in [8], where it is used to model a F-to-M channel with local scattering around both the Tx and the Rx. Unlike the model proposed in [8], our work assumes mobility of both the transmitter and the receiver, which leads to different statistical properties of our theoretical model.

The "double-ring" model defines two rings of fixed scatterers, one around the Tx and another around the Rx, as shown in Fig. 1. Around the transmitter, \( M \) omnidirectional scatterers lie on a ring of radius \( R_t \), and the \( m^{th} \) transmit scatterer is denoted by \( S_t^{(m)} \). Similarly, around the receiver, \( N \) omnidirectional scatterers lie on a ring of radius \( R_r \) and the \( n^{th} \) receive scatterer is denoted by \( S_r^{(n)} \). The distance between the transmitter and the receiver is \( D \). Angles \( \gamma_t \) and \( \gamma_r \) describe the orientation of the Tx's antenna array and the Rx's antenna array, respectively, relative to the \( x \)-axis. Similarly, the Tx and the Rx are moving with speeds \( v_T \) and \( v_R \) in directions described by angles \( \gamma_T \) and \( \gamma_R \), respectively.

In the theoretical model, the number of local scatterers around the Tx and the Rx is infinite. Consequently, the received complex faded envelope can be approximated by

\[
h_{pq}(t) = \lim_{M,N \to \infty} \frac{1}{\sqrt{MN}} \sum_{m,n=1}^{M,N} e^{-j\frac{2\pi}{\lambda}(\epsilon_{pm} + \epsilon_{mn} + \epsilon_{nq}) + j2\pi(f_{t}^{(m)} + f_{r}^{(m)})t + j\phi_{mn}},
\]

where \( K = 2\pi/\lambda \), \( \lambda \) is the carrier wavelength, and frequencies \( f_{t}^{(m)} \) and \( f_{r}^{(n)} \) are equal to \( f_{\text{max}} \cos(\alpha_{T}^{(m)} - \gamma_T) \) and \( f_{\text{max}} \cos(\alpha_{R}^{(n)} - \gamma_R) \), respectively, where \( f_{\text{max}} = v_T/\lambda \) and \( f_{\text{max}} = v_R/\lambda \) are the maximum Doppler frequencies of the Tx and the Rx, respectively. The symbols \( \alpha_{T}^{(m)} \) and \( \alpha_{R}^{(n)} \) denote the angles of departures (AoD) and the angles of arrivals (AoA), respectively. Finally, the symbols \( \epsilon_{pm}, \epsilon_{mn}, \) and \( \epsilon_{nq} \) denote distances \( d_{p}^{(m)} - d_{r}^{(m)}, S_{t}^{(m)} - S_{r}^{(n)} \), and \( \sigma_{t}^{(m)} - A_{t}^{(n)} \), respectively, as shown in Fig. 1. It is assumed that \( \alpha_{T}^{(m)} \) and \( \alpha_{R}^{(n)} \) are uniformly distributed on the interval \([-\pi, \pi]\).

Distances \( \epsilon_{pm}, \epsilon_{mn}, \) and \( \epsilon_{nq} \) can be expressed as functions of random variables \( \alpha_{T}^{(m)} \) and \( \alpha_{R}^{(n)} \). From Fig. 1, assuming \( \max\{\delta_T, \delta_R\} \ll \max\{R_t, R_r\} \ll D \) and invoking the law of cosines, these distances are

\[
\begin{align*}
\epsilon_{pm} &= R_t - 0.5\delta_T \cos(\alpha_{T}^{(m)} - \theta_T), \\
\epsilon_{mn} &= R_r - 0.5\delta_R \cos(\alpha_{R}^{(n)} - \theta_R), \\
\epsilon_{nq} &= D - R_r \cos(\alpha_{R}^{(n)} - \theta_R) - \sin(\pi - \alpha_{R}^{(n)}) \approx D. 
\end{align*}
\]

Equations (2) and (3) can be generalized for any number of transmit and receive antennas as follows:

\[
\begin{align*}
\epsilon_{pm} &= R_t - \delta_T(0.5L_t + 0.5 - p) \cos(\alpha_{T}^{(m)} - \theta_T), \\
\epsilon_{nq} &= R_r - \delta_R(0.5L_r + 0.5 - q) \cos(\alpha_{R}^{(n)} - \theta_R).
\end{align*}
\]

where parameters \( p \) and \( q \) take values from the sets \( p \in \{1, \ldots, L_t\} \) and \( q \in \{1, \ldots, L_r\} \), respectively.

Substituting (4)–(6) into (1), the complex faded envelope of the link \( A_t^{(p)} - A_r^{(q)} \) becomes

\[
h_{pq}(t) = \lim_{M,N \to \infty} \frac{1}{\sqrt{MN}} \sum_{m,n=1}^{M,N} a_{p,m} b_{n,q} \exp\{j2\pi(f_{t}^{(m)} + f_{r}^{(n)})t + j\phi_{mn} + j\phi_{0}\},
\]

where \( \phi_{0} = -(2\pi/\lambda)(R_t + D + R_r) \) and

\[
a_{p,m} = \exp\{jK(0.5L_t + 0.5 - p) \delta_T \cos(\alpha_{T}^{(m)} - \theta_T)\},
\]

\[
b_{n,q} = \exp\{jK(0.5L_r + 0.5 - q) \delta_R \cos(\alpha_{R}^{(n)} - \theta_R)\}.
\]

### III. Space-Time Correlation Function and Power Density Spectrum

In this section, the joint space-time correlation function and the corresponding power density spectrum of the theoretical model presented in Section II are derived. The normalized space-time correlation function between two complex faded envelopes \( h_{pq}(t) \) and \( h_{pq',\tau}(t) \) is defined as [9]

\[
R_{pq,pq'}(\delta_T, \delta_R, \tau) = \frac{E[|h_{pq}(t)h_{pq',\tau}(t + \tau)|^2]}{E[|h_{pq}(t)|^2]E[|h_{pq',\tau}(t)|^2]}. \tag{10}
\]
where \( (\cdot)^* \) denotes complex conjugate operation, \( E[\cdot] \)
\( p, p' \in \{1, \ldots, L_v\} \), and \( q, q' \in \{1, \ldots, L_v\} \). Using (7) and (10), the space-time correlation function can be written as

\[
R_{pq,p'q'}(\delta_T, \delta_R, \tau) = \lim_{M,N \to \infty} \frac{1}{MN} \sum_{m,n=1}^{M,N} E[e^{-j2\pi(f_{pq}m + f_{p'q'n})}]
\]

\[
\alpha_{pq,p'q'} = \frac{1}{\delta_T + 0.5(L_v+1)} \int_0^\pi \int_0^\pi e^{-j2\pi(f_{pq}m + f_{p'q'n})} f(\alpha_T) f(\alpha_R) e^{j2\pi((p-p')\cos(\alpha_T-\gamma_T) + (q-q')\cos(\alpha_R-\gamma_R))} \sin(\delta_T - \delta_R) d\alpha_T d\alpha_R.
\]

The expression for the space-time correlation function is

\[
R_{pq,p'q'}(\delta_T, \delta_R, \tau) = \frac{1}{2\pi i0(\cdot)} \exp\{k \cos(\theta - \mu)\},
\]

where \( \theta \in [-\pi, \pi], I_0(\cdot) \) is the zeroth-order modified Bessel function of the first kind, \( \mu \in [-\pi, \pi] \) is the mean angle of scatterers’ distribution on the ring, and \( k \) controls the spread of scatterers around the mean. When \( k = 0 \), \( f(\theta) = 1/(2\pi) \) is a uniform distribution yielding 2-D isotropic scattering. As \( k \) increases, the scatterers become more clustered around angle \( \mu \) and the scattering becomes increasingly non-isotropic. By replacing \( f(\alpha_T) \) and \( f(\alpha_R) \) in (12) with (13) and by solving the integrals in (12) using [11, eq. 3.384-4], the closed-form expression for the space-time correlation function is

\[
R_{pq,p'q'}(\delta_T, \delta_R, \tau) = \frac{i_0(\sqrt{x^2 + y^2})}{i_0(kT) i_0(kR)},
\]

where parameters \( x, y, z, \) and \( w \) are

\[
x = j2\pi [(p' - p)\cos(\delta_T/\lambda) - f_{Max} \cos(\gamma_T)] + kR \cos(\mu_T),
\]

\[
y = j2\pi [(p' - p)\sin(\delta_T/\lambda) - f_{Max} \sin(\gamma_T)] + kR \sin(\mu_T),
\]

\[
z = j2\pi [(q' - q)\cos(\delta_R/\lambda) \sin(\delta_T/\lambda) - f_{Max} \cos(\gamma_R)] + kR \cos(\mu_R),
\]

\[
w = j2\pi [(q' - q)\sin(\delta_R/\lambda) \sin(\delta_T/\lambda) - f_{Max} \sin(\gamma_R)] + kR \sin(\mu_R).
\]

Many existing correlation functions are special cases of our MIMO M-to-M space-time correlation function in (14). The simplest special case of (14) is Clark’s temporal correlation function \( J_0(2\pi f_{RMax}) \) [12], obtained for \( \delta_R = 0 \) (isotropic scattering around \( R_0 \)), \( f_{RMax} = kT = 0 \) (stationary \( T_v \), no scattering), and \( \delta_T = \delta_R = 0 \) (single-antenna \( T_v \) and \( R_0 \)), where \( J_0(\cdot) \) is the first kind zeroth-order Bessel function. Expressions for other space-time correlation functions based on the “one-ring” model [8], [13], [14] can be similarly obtained. The temporal correlation function for \( M \)-to-\( M \) channels, assuming 2-D isotropic scattering, \( J_0(2\pi f_{RMax} \lambda) \) [1] is obtained for \( kT = kR = 0 \) and \( \delta_T = \delta_R = 0 \). Similarly, the spatial correlation function for \( M \)-to-\( M \) channel \( J_0(\cos(\pi f/\lambda)) J_0(2\pi (q' - q) \cos(\gamma_R/\lambda)) \) [15] is obtained for \( kT = kR = 0 \) and \( \tau = 0 \). Finally, the space-time correlation function for \( M \)-to-\( M \) channels, assuming 2-D isotropic scattering, \( J_0(\sqrt{x^2 + y^2}) J_0(\sqrt{z^2 + w^2}) \) [6] is obtained for \( kT = kR = 0 \).

The space-frequency power density spectrum (sf-psd) of the complex faded envelope is the Fourier transformation of the space-time correlation function. For the space-time correlation function in (14) and 2-D isotropic scattering, the corresponding sf-psd is

\[
S_{pq,p'q'}(\delta_T, \delta_R, f) = \int_{-\infty}^{\infty} J_0(\sqrt{x^2 + y^2}) J_0(\sqrt{z^2 + w^2}) e^{-2\pi f_T} dT.
\]

For \( \delta_T = \delta_R = 0 \) (SISO system), the sf-psd in (16) has the closed-form expression [1]:

\[
S_{pq,p'q'}(0, 0, f) = \frac{1}{\pi^2 \sqrt{Sf_{Max}}} K \left[ \frac{1 + \frac{f}{(1 + s) f_{RMax}}}{\pi f_{RMax} \sqrt{1 - \frac{f}{f_{RMax}}}} \right] \left( \frac{(p' - p)\cos(\gamma_T) \sin(\gamma_R)}{2\pi x^4 \sqrt{x^2 + y^2}} \right) \left( \frac{(q' - q)\sin(\gamma_T) \cos(\gamma_R)}{2\pi z^4 \sqrt{z^2 + w^2}} \right),
\]

where \( K[\cdot, \cdot] \) is the complete elliptic integral of the first kind and \( s = f_{RMax}/f_{Max} \).

To obtain the sf-psd for a MIMO system, integral in (16) can be evaluated numerically, but it is time consuming process. Hence, we derive a more general expression for the sf-psd in (16) that characterizes both, MIMO and SISO systems (i.e., \( \delta_T, \delta_R \neq 0 \) and \( \delta_T = \delta_R = 0 \)). By using [11, eq. 6.677-3] and solving the integral in (16), the sf-psd becomes

\[
S_{pq,p'q'}(\delta_T, \delta_R, f) = \left[ \cos \left( \frac{(p' - p)\cos(\gamma_T) \sin(\gamma_R)}{2\pi x^4 \sqrt{x^2 + y^2}} \right) \right] \left( \frac{\pi f_{RMax} \sqrt{1 - \frac{f}{(1 + s) f_{RMax}}}}{\pi f_{RMax} \sqrt{1 - \frac{f}{f_{RMax}}}} \right) \left( \frac{(q' - q)\sin(\gamma_T) \cos(\gamma_R)}{2\pi z^4 \sqrt{z^2 + w^2}} \right),
\]

where \( \odot \) denotes convolution, \( |f| \leq f_{RMax} + f_{Max} \), \( q_T = (p' - p)\cos(\gamma_T) \sin(\gamma_R) \), \( q_R = (q' - q)\sin(\gamma_T) \cos(\gamma_R) \), and \( s = f_{RMax}/f_{Max} \). Furthermore, in Fig. 2, we plot the sf-
The AoDs, \( \alpha \) and \( \gamma \), and the space-time correlation function of the statistical simulation model of the complex faded envelope obtained with \( N = M = 10 \) scatterers and \( N_{\text{stat}} = 50 \) simulation trials. Compared to the theoretical spatial correlation function, the maximum absolute error is less than \( 10^{-3} \) for \( \delta_T/\lambda, \delta_R/\lambda \in \{0, \ldots, 5\} \). Our simulations indicate that larger distances between antenna elements require larger number of scatterers to obtain adequate statistics. Instead of using many scatterers (e.g. 40) in all simulations, we adaptively select the number of scatterers depending on the distances between antenna elements. If we assume that the mean square error (MSE) of \( \leq 10^{-3} \) is required for the simulated space-time correlation function of the complex faded envelope (relative to the theoretical one), we need at least \( M = 14 + 2[\delta_T/0.3\lambda] \) and \( N = 14 + 2[\delta_R/0.3\lambda] \) scatterers. \(^1\) Fig. 5 shows that, for \( N = M = 18 \) scatterers, \( N_{\text{stat}} = 50 \) trials, and antenna distances \( \delta_T = \delta_R = 1/\lambda \), the space-time cross-correlation function (CCF) of the simulation model approaches the space-time CCF of the theoretical model. From Figs. 3-5, we can conclude that the statistical simulation model is a good approximation of the theoretical model.

\(^1\) Operation \([\cdot]\) denotes rounding up to the next integer.
in simulations is proposed. The results show that the statistical simulation model is a good approximation of the theoretical model.

DISCLAIMER

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APPENDIX A

DERIVATION OF THE SPACE-TIME CORRELATION FUNCTION FOR THE SIMULATION MODEL

The space-time correlation of the simulation model is

$$\tilde{R}_{pq,p,q'}(\delta_T, \delta_R, \tau) = \mathbb{E}[\hat{h}_{pq}(t)\hat{h}^{*}_{p,q'}(t+\tau)] = \frac{1}{4\pi^2 MN}$$ (22)

$$\sum_{m=1}^{M} \int_{-\pi}^{\pi} \exp \left\{ -j2\pi f_{R\max} \tau \cos \left( \frac{2\pi m}{M} + \frac{\psi - \pi}{2M} - \gamma_R \right) \right\} d\psi$$

From (22) we derive (21) by replacing the variables of integration, $\theta$ and $\psi$, with $\gamma_n = (2\pi n)/(N) + (\theta - \pi)/(N)$ and $\delta_n = (2\pi n)/(2M) + (\psi - \pi)/(2M)$, respectively and by using trigonometric equalities and [11, eq. 3.384-4].

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