Chirp zeta transform beamforming for three-dimensional acoustic imaging

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Abstract: This letter considers the possibility of the generation of three-dimensional acoustic images with a limited computational load by considering the extension of Chirp zeta transform beam forming to the case of planar array and near-field conditions. This extension, with a few innovative solutions, allows for a dramatic reduction in the number of on-line operations over traditional time-domain beamforming.

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1. Introduction
Despite the introduction of commercial equipment, the building of systems able to generate a real-time three-dimensional (3D) acoustic video of an investigated environment is still a challenge, mainly due to the cost of the needed planar array and the computational load of the associated signal processing. Wideband digital beamforming is the most applied signal processing method, for both underwater and medical applications.

Although the cost of planar arrays can be reduced by the use of the sparse array concept, some methods have been developed to implement the beamforming algorithm in the frequency domain to mitigate the computational load. Zero-padded fast Fourier transform (FFT) beamforming is the best known of these methods, being very efficient and producing exact results for narrowband signals. However, its extension to wideband signals is not straightforward and introduces some errors that reduce the method’s appeal.

Although less recognized, Chirp zeta transform (CZT) beamforming is a flexible frequency-domain method that can process wideband signals without any error, with a computational load equal to or lower than that of zero-padded FFT beamforming. However, its extension to planar arrays has not been attempted and its relative merits have not been assessed. Moreover, 3D imaging systems should be able to work in the near-field region, imposing a focused beamforming approach and this causes additional difficulties, especially with frequency-domain methods.

In this letter, we present an extension of wideband CZT beamforming to planar arrays and near-field conditions, introducing some innovative ideas. The near-field difficulties have been solved by the adoption of the Fresnel approximation, a useful definition of steering angles, and the setting of multiple focal regions. In addition, a technique to generate cubic resolution cells has also been introduced. The combination of such solutions can offer distinct computational advantages. Although the context assumed here refers to 3D acoustic imaging, mainly for underwater sonar systems, the proposed method can also be exploited in other contexts or environments.

2. CZT beamforming concept
Let us consider a linear array consisting of M sensors, with intersensor spacing d, and let us denote the steering angle with θ (measured from broadside) and the sound speed with c. If the time signals from the sensors are segmented into blocks of length K, then the discrete Fourier transform (DFT) coefficients, B(k, θ), of a corresponding segment of the beam signal steered in the direction θ, in the far-field region, are given by the following expression:
\[ B(k, \theta) = \sum_{m=0}^{M-1} w_m S_m(k) \exp\left(-j2\pi f_k \left( \frac{md \sin \theta}{c} \right) \right) \]  

(1)

where \( w_m \) is the apodizing weights applied to control the side-lobe level, \( S_m(k) \) is the DFT coefficients of the signal \( s_m(t) \) collected by the \( m \)-th sensor, and \( k \) is the frequency index, corresponding to the frequency \( f_k = k f_s / K \) (\( f_s \) being the sampling frequency). The product \( \frac{md}{c} \) represents the position, \( x_m \), of the \( m \)-th sensor along the array baseline.

According to Maranda, if the \( q \)-th beam of \( M_b \) beams is selected, the previous equation can be rewritten as follows:

\[ B(k, \theta_q) = W^{q^2/2} \sum_{m=0}^{M-1} w_m S_m(k) A^m W^{m^2/2} W^{(q-m)^2/2} \]  

(2)

where

\[ A = \exp\left(-j2\pi f_k \frac{d}{c} \sin \theta_t \right) \]  

(3a)

\[ W = \exp\left(j2\pi f_k \frac{d}{c} \Delta s \right) \]  

(3b)

\[ \Delta s = \frac{\sin \theta_f - \sin \theta_t}{M_b - 1} \]  

(3c)

where \( \theta_t \) and \( \theta_f \) are the initial and final steering angles, respectively, and the angle \( \theta_q \) is one of a set of \( M_b \) predefined steering angles equispaced in the sine domain. This beam spacing perfectly agrees with the related angular resolution.

It is worth noting that Eq. (2) expresses \( B(k, \theta_q) \) as a discrete convolution of the sequences:

\[ w_m S_m(k) A^m W^{m^2/2}, \quad W^{m^2/2}, \quad m \in [0, M-1] \]  

(4)

thereby allowing FFT methods for performing the so-called “fast convolution” to be used effectively. The computational advantage is duly discussed in Maranda.

3. Extension to planar array and near-field

Let us now consider a planar array placed on the plane \( z=0 \), centered on the coordinate origin, composed of \( M \times N \) sensors, with intersensor spacing \( d \) in both directions. The sensor identified by the indexes \( (m, n) \) is placed at position \((x_m, y_n)\) and generates the signal \( s_{m,n}(t) \). The steering direction is identified by two azimuth and elevation angles \( (\theta_a, \theta_e) \) that, different from conventional formalism, are defined as shown in Fig. 1. The choice of such angles is useful to easily extend CZT beamforming to a planar array. Although conventional formalism also makes it possible to express the main computation as a discrete convolution it is definitely less direct and intuitive.

Further, if near-field conditions are considered, the plane wave assumption that has been exploited in Eq. (1), allowing the use of the simple phase term \( x_m \sin \theta / c \), is no longer valid. To handle the curvature of the wave front, a focusing distance \( r_0 \) should be introduced in the phase term \( 1.4^2 \), which, in turn, becomes much more complex. To simplify such a phase term, the Fresnel approximation can be applied, obtaining the following term:
The validity region of the Fresnel approximation (i.e., the region where the approximation errors are negligible) is duly discussed in Ziomek. Here, we simply recall that, for a rectangular array centered on the coordinate origin, the focusing distance $r_0$ should satisfy the condition $r_0 > 0.678D$, $D$ being the diagonal size of the array.

Altogether, if the Fresnel approximation is adopted, the computation of the beam signal in the $(\alpha, \varepsilon)$ direction, focused at a distance $r_0$, requires the summing of the frequency bins as in Eq. (1), replacing the phase term $md \sin \theta/c$ with that in Eq. (5). In this way it is possible to obtain the following equation:

$$B(k, \theta_\alpha, \theta_\varepsilon) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} w_{m,n} \exp\left( j2 \pi f_k \left( \frac{x_m^2 + y_n^2}{2r_0c} \right) \right) S_{m,n}(k)$$

where $w_{m,n} = w_{a}^{\alpha/2} w_{e}^{\varepsilon/2}$, $S_{m,n}(k)$ is the frequency bin of the received signal at frequency $k$, and $x_m$ and $y_n$ are the coordinates of the $m$th and $n$th element of the array. The term $v_{m,n}$ is defined as follows:

$$v_{m,n} = \frac{x_m \sin \theta_\alpha + y_n \sin \theta_\varepsilon}{c} - \frac{x_m^2 + y_n^2}{2r_0c}$$
\[ v_{m,n} = w_{m,n} \exp \left\{ j2\pi f_k \left[ \left( m - \frac{M-1}{2} \right)^2 + \left( n - \frac{N-1}{2} \right)^2 \frac{d^2}{2r_0 c} \right] \right\} \]  

(8)

Actually, moving from Eq. (6) to Eq. (7), a phase term, linear with \( f_k \), which does not depend on the sensor indexes, has been neglected. This is reasonable as such a term simply introduces a known delay in the beam signal that can be easily fixed a posteriori.

In focused beamforming (independent of the time or frequency implementation), the depth of field (DOF)\(^1,6\) is defined as the range interval around the focusing distance \( r_0 \) inside which the performance, evaluated in term of angular resolution and amplitude gain, only marginally degrades. The extension of the DOF depends on the specific value of \( r_0 \): the smaller the focusing distance \( r_0 \), the smaller the DOF extension.\(^6\) Unfortunately, the range extension of the volume to be imaged typically exceeds that of the DOF. The problem is commonly solved by segmenting the received time signals into subsequent blocks that are processed using different focusing distances, which increase with time. In other words, the volume to be imaged is subdivided into multiple, adjacent focal regions. Each focal region is centered on a specific focusing distance and is contained inside the DOF corresponding to such a focusing distance. As in our method the time signals are already segmented into blocks of length \( K \), so it is easy to apply different focusing distances to each block. However, in setting the value of \( K \), it is essential that the spatial extension corresponding to the \( K \) time samples does not exceed the shortest of the extensions of the focal regions.

The general method and the specific solutions described above are used to extend the CZT beamforming to the planar array case and to allow correct focusing over an extended near-field volume. The accuracy of computing wideband beam signals by the proposed method is equal to that of the traditional delay-and-sum (D&S) beamforming,\(^1,4\) except for the errors introduced by the Fresnel approximation. However, as discussed and quantified in Trucco,\(^7\) such errors are really negligible inside the validity region\(^5\) of the approximation. Therefore, the described method represents a computationally convenient way to compute beam signals that, inside the Fresnel validity region, negligibly differ from those computed by D&S beamforming.

Below, two original solutions devoted to further reducing the computational burden will be briefly introduced:

1. It can generally be observed that it is computationally convenient to increase the length \( K \) of the signal blocks. This is in contrast to the need for short blocks close to the array, where the DOF extension is more limited. To overcome this problem the length of the signal blocks can vary, increasing in synchrony with the DOF extension.

2. The angular resolution is fixed and so the lateral resolution worsens with the distance. Instead, the range resolution depends on the bandwidth and does not generally vary with the distance. However, the generation of a cubic resolution cell is often welcome,\(^1\) and the worsening of the range resolution with the distance is perfectly acceptable. This makes it possible to save many operations by reducing the number of frequency bins considered (i.e., the bandwidth) with the distance. In other words, Eq. (7) can be computed for a number of indexes \( k \) that decreases block after block, according to the desired range resolution.

4. Results and conclusions

A comparison of the computational load of D&S beamforming\(^1,4\) and that of CZT beamforming is carried out below. For each beamforming type, only operations that must be performed on-line are considered, thus excluding all the operations that can be performed off-line and whose results can be stored inside a memory. Let us analyze a sonar system with the following design parameters: a square array of \( N \times N \) transducers (i.e., \( M = N \)), spaced \( d = 1.5 \) mm from each other; a carrier frequency \( f_c = 500 \) kHz, with a signal bandwidth of \( 250 \) kHz; a set of \( N_b \times N_k \) beams (i.e., \( M_b = N_b \)); a volume of interest ranging from 1 to 100 m of distance; a sampling frequency \( f_s = 1.5 \) MHz. The sound speed \( c \) is set at 1500 m/s. For the D&S beamforming, a
signal interpolation step has been considered, performed by a FIR filter with 100 stages. For the CZT beamforming, the block length $K$ ranges from 1024 to 4096, and the generation of cubic resolution cells is enabled.

Table 1 compares the number of on-line operations needed to create one image of the volume of interest. One can note that a computational advantage greater than two orders of magnitude is obtained for all the considered cases. As the proposed method applies the Fresnel approximation to the beamforming delays, the computed beam signals do not present any significant inaccuracy provided that they are pointed and focused inside the validity region of such an approximation.

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### References and links