Asymptotic Evaluation of Eigenvalue Distribution and Ergodic Capacity under Outdoor-Indoor MIMO Measured Channel

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I. INTRODUCTION

In wireless communications, multiple-input multiple-output (MIMO) techniques have recently been well-known as a new concept to achieve very high data rates. In particular, the advantage of the rich scattering radio environment causes MIMO systems to obtain higher benefits due to low correlated channel [1] and [2]. The characteristics of radio propagation channels between transmitter and receiver are very crucial to be understood in order to investigate performance limits of MIMO systems. It is well-known that the capacity increases linearly with the fixed power and bandwidth in independent and identically distributed (i.i.d.) Rayleigh fading channels, as the number of antennas at both the transmitter (TX) and the receiver (RX) get larger [1] and [2]. In practice, the fading correlation degrading the efficiency of MIMO systems exists between the signal transmitted or received at different antenna elements. Different analytic channel models pattern this correlated fading channel such as the popular separable Kronecker model [3], which is in good agreement for outdoor scenarios. More elaborate models for instance the Weichselberger model, which performs more excellent in indoor scenarios [4] and the virtual channel representation [5] have been proposed. Ref. [6] points out that the Weichselberger model is more applicable in outdoor-indoor scenarios. Several previous measurement campaigns have also studied radio channels in tunnels, corridors, or diffraction [7]-[9].

Many authors have analytically studied the asymptotic capacity limit, i.e., the capacity approaches a given value when the number of transmit and receive antennas increase without bound at the same rate. For instance, based on random matrix theory, [10] and [11] derived a closed form expression for the asymptotic capacity per antenna in the uncorrelated channel. The case of spatially correlated fading channels are considered in [12] and [13]. However, the analysis of the asymptotic capacity based on a real channel measurements has been only rarely considered. Although there have been a great number of MIMO channel measurement campaigns investigated, very few papers analyze outdoor-indoor environment.

In this paper, we firstly investigate the most practical cases of outdoor-indoor, i.e., the fading correlation between the array elements at the receiver (RX) is low due to the existence of local scatterers around it, the channel in the room and the corridors are investigated. We show that unlike in [6], the one-sided Kronecker model is reasonably suitable to be applied for our measured channels. Subsequently, the analytic results of the asymptotic distribution of the channel eigenvalues and the closed-form solutions of the ergodic channel capacity based on the one-sided Kronecker model are reviewed and analyzed with the measurement data.

The organization of this paper is as follows. In Section II, the measurements set up and the description of the measurement environment are described. The preliminary signal analysis and channel modelling are done in Section III. We define the system model and MIMO channel capacity in Section IV. The asymptotic behavior of eigenvalue distribution and ergodic channel capacity are analyzed in Section V. The conclusions of this study are finally summarized in Section VI.

II. MEASUREMENTS: SETUP AND DESCRIPTION OF MEASUREMENT ENVIRONMENT

The measurement campaign was conducted with Propsound T.M., channel sounder [14] performed at a center frequency of 5.25 GHz and a signal bandwidth of 100 MHz. As illustrated in Fig. 1, the transmit antenna was a 16-elements dual polarized uniform linear patch array with element spacing $\lambda/2$. Only the...
9-elements dual polarized antenna (the ring below) from the 25-elements dual polarized omni-directional receive antenna was used covering the entire 360-degree view in azimuth. However, the same polarization is the only case for analyzing. The 100 Mchip/s sounding signal consisting of a 1023-chip M-sequence was applied. Fig. 2 illustrates the floorplan of the measurement place, where is the building with 30 cm thick concrete walls. As can be seen, the transmitter (TX) was located outside the building at the height of 12 m and the receiver (RX) was moving with the speed of 15 cm/s in the building as depicted on the floorplan. The example measurement routes were in the office room (route no.1), which are surrounded by tables and electronic equipments, and corridors (route no.2 and no.3).

![Fig. 1. A 16-elements dual polarized uniform linear patch array at the transmitter and a 9-chosen elements (the ring below) dual polarized omni-directional antenna at the receiver.](image1)

![Fig. 2. The floorplan of the building and the measurement routes.](image2)

III. SIGNAL ANALYSIS AND CHANNEL MODEL

We preliminarily analyze the channels from the measurement campaign and model the channels for further analysis in the next sections.

First we consider the spatial correlation of the channel matrix $H$ of the largest case system, i.e., $8 \times 16$ MIMO. Like in [15], the magnitude of spatial TX and RX correlation matrices ($16 \times 16 \text{R}_{TX}$ and $8 \times 8 \text{R}_{RX}$) are plotted in Figs. 3.a and 3.b. The magnitude of spatial MIMO fully correlation ($128 \times 128 \text{R}_{MIMO}$) calculated directly from the measurement in the room $E[\text{vec}(H)\text{vec}(H)^H]$ and from Kronecker product $\text{R}_{TX} \otimes \text{R}_{RX}$ are shown in Figs. 4.a and 4.b. As one can see in Fig. 5, the difference between the MIMO spatial correlation calculated directly from the measured and the one from the Kronecker product is small. Therefore, the spatial correlation matrix of the RX can be assumed to be independent from the correlation of the TX. In addition, the RX correlation is so small that it can be disregarded. The results for the channels in the corridors behave the same way (not shown in the paper). Consequently, the one-sided TX correlation model can be applied for the channels both in the room (route no.1) and the corridors (route no.2 and 3). The probability density function (PDF) of the envelop of channel coefficients for each route are shown in Figs. 6 and 7, respectively. The mean values of the ordered channels eigenvalues for the measured channels and the simulated channels are presented comparing with those for the independent and identically distributed (i.i.d.) channel in Fig. 8. The difference between the means of the largest and second largest channel eigenvalues are 4 dB, that is quite small. In addition, those largest and second largest of the

![Fig. 3. The magnitude of spatial correlation matrices at a.) TX and b.) RX of the channel in the room.](image3)

![Fig. 4. The magnitude of spatial MIMO correlation matrices of the channel in the room, a.) from the measurement and b.) from the Kronecker product.](image4)
simulated channels eigenvalues are reasonably fit to those of the measured channels. Both histograms in Figs. 6 and 7 as well as the plots in Fig. 8 confirm the goodness of fit of the applied one-sided Kronecker model.

IV. SYSTEM MODEL AND MIMO CHANNEL CAPACITY

In a MIMO system where the transmitter (TX) employs $N_T$ antennas and the receiver (RX) has $N_R$ antennas, the $N_R$-dimensional output signal $\mathbf{y}$ can be expressed as a column vector

$$\mathbf{y} = \mathbf{H} \mathbf{x} + \mathbf{n},$$  

(1)

where $\mathbf{x}$ denotes the $N_T$-dimensional transmit signal vector, $\mathbf{n}$ is the $N_R$-dimensional circularly-symmetric complex additive white Gaussian noise (AWGN) vector with zero mean and covariance $E[\mathbf{n} \mathbf{n}^H] = \sigma^2 \mathbf{I}_N$ observed at the RX, as well as $\mathbf{H}$ is the $N_R \times N_T$ channel matrix with complex entries including phase noise due to the phase locked loops in the time division multiplexing (TDM) switched channel sounder [16] and [17]. Nonetheless, the channels from the measurements have high rank as reported in Fig. 8. Moreover, the signal-to-noise ratio (SNR) of the measured signal is in the range of 10-15 dB. Thus, the impact of phase noise on capacity can be neglected in this study [16] and [17]. Therefore, the impact of phase noise on capacity can be neglected in this study.

In the following, we assume that our analysis is restricted to frequency flat channel matrix $\mathbf{H}$. We also assume that we are in the situation where the channel is perfect known only at the RX and is unknown to the TX. The uniform distribution of the power is the most reasonable power allocation scheme to use. The general capacity expression of a random MIMO channel in bits per second per hertz (bit/s/Hz) can be expressed as

$$C = \log_2[\det(\mathbf{I}_{N_{\min}} + \frac{\rho}{N_T} \mathbf{W})],$$  

(2)

where $\mathbf{I}_{N_{\min}}$ is the $N_{\min} \times N_{\min}$ identity matrix, $\rho$ is the average signal-to-noise (SNR) ratio at the RX branch, $N_{\min} = \min(N_R, N_T)$ and the $(N_{\min} \times N_{\min})$ matrix $\mathbf{W}$ defined as:

$$\mathbf{W} = \begin{cases} \mathbf{H}^H \mathbf{H} & \text{if } N_R > N_T, \\ \mathbf{H} \mathbf{H}^H & \text{if } N_R \leq N_T, \end{cases}$$  

(3)
where \((\cdot)^H\) denotes transpose conjugate. By using eigenvalue decomposition, the capacity (2) can be expressed as

\[
C = \sum_{i=1}^{N_{\text{min}}} \log_2 \left( 1 + \frac{\rho}{\beta} \lambda_i \right),
\]

where \(\{\lambda_i\}_{i=1}^{N_{\text{min}}}\) denotes the eigenvalues of \(N_R^{-1}W\) and \(\beta = N_T/N_R\).

V. ASYMPTOTIC ANALYSIS

In this section, the analytical expressions for the ergodic capacity of a MIMO channel with fading correlation are reviewed. The ergodic capacity is defined as the statistical expectation of the capacity over time and expressed as

\[
E[C] = E \left[ \sum_{i=1}^{N_{\text{min}}} \log_2 \left( 1 + \frac{\rho}{\beta} \lambda_i \right) \right].
\]

(5)

Because of the lack of an analytical closed-form expression of this capacity, the asymptotic behavior of capacity is analyzed, e.g. when the number of transmit and receive antennas increase without bound at the same rate \((N_T, N_R \to \infty)\). The eigenvalues of \(W\) are nonnegative real number as \(W\) is Hermitian. Let \(f_W(x)\) denote the empirical probability density function of the eigenvalues of \(W\) and it is given by

\[
f_W(x) = \frac{dF_W(x)}{dx} = \frac{1}{N_{\text{min}}} \sum_{i=1}^{N_R} \delta(x - \lambda_i),
\]

(6)

where \(F_W(x)\) denotes the empirical cumulative distribution function (cdf) of the eigenvalues and is defined as

\[
F_W(x) = \frac{\left\{ \lambda_i \mid \lambda_i \leq x \right\}}{N_{\text{min}}}.
\]

(7)

The ergodic capacity can be repeated by Lebesgue integral over \(dF_W(x)\) and given by [13]

\[
E[C] = N_{\text{min}} \int_{-\infty}^{\infty} \log_2 \left( 1 + \frac{\rho}{\beta} x \right) f_W(x) dx.
\]

(8)

As can be seen, the ergodic capacity depends on the empirical distribution of the eigenvalues. Therefore, the asymptotic properties of the ergodic capacity depends on how the empirical distribution of the eigenvalues converges to a limiting distribution. When the entries of \(H\) are independent and identically distributed (i.i.d.) circularly symmetric Gaussian-distributed with zero mean and unit variance, the empirical distribution function of the eigenvalues of \(N_R^{-1}W\) almost surely converges to the so-called Marchenko-Pastur distribution [10] and [11].

The integral in (8) is written in the closed form, as derived in [13]. In realistic channels, the effect of fading correlation between the entries of \(H\) at the TX or the RX or both of them must be taken into account. We investigate in this paper the most practical case of outdoor-indoor, i.e., the fading correlation between the array elements at the receiver (RX) is low enough to be ignored due to the existence of local scatterers around it. There is approximately just the fading correlation at the TX.

A. Fading correlation at the TX

The channel matrix in (1) is modelled by the one-sided Kronecker model [13]

\[
H = GR^{1/2}_{TX},
\]

(9)

where \(G\) is an \(N_R \times N_T\) matrix with i.i.d. circularly symmetric complex Gaussian-distributed with zero mean and unit variance, \(R_{TX}\) is an \(N_T \times N_T\) fading correlation matrix observed on the TX side defined by \(R_{TX} = E[H^H H]\) (see the square and circle plots in Fig. 9), and \(R^{1/2}_{TX}\) stands for the Hermitian positive definite square root of \(R_{TX}\).

Fig. 9. The magnitude of correlation coefficient vs the antenna element spacing (\(\lambda/2\)) in the room.

The matrix \(N_R^{-1}W\) when \(N_R \leq N_T\) in (3) is now expressed as \(W = \frac{1}{N_T} GR^{1/2}_{TX} R^{1/2}_{TX} G^H\). With the fact that \(W = \frac{1}{N_T} GR^{1/2}_{TX} R^{1/2}_{TX} G^H\) has the same eigenvalues as \(R^{1/2}_{TX} R_{TX} R^{1/2}_{TX} G^H G = R_{TX} V\), with \(V = \frac{1}{N_T} G^H G\), but the size is different. Hence the empirical probability density function of \(W\) can be expressed in term of the empirical probability density function of \(R_{TX} V\) as

\[
f^\infty_W(x) = \beta f^\infty_{R_{TX} V}(x) + (1 - \beta) \delta_0(x),
\]

(10)

where \(\delta_0(x)\) is the Dirac’s delta function centered at \(x = 0\). Based on free probability theory stated that these matrices are asymptotically free, therefore the empirical probability density function \(f^\infty_{R_{TX} V}(x)\) is the free multiplicative convolution of the empirical probability density function of \(R_{TX}\) and \(V\). This is achieved by the means of the so-called S-transform, since the S-transform of \(f^\infty_{R_{TX} V}(x)\) is equal to the product of S-transform of \(f^\infty_{R_{TX}}(x)\) and \(f^\infty_V(x)\).

\[
S_W(z) = S_V(z) S_{R_{TX}}(z).
\]

(11)

The derivation in [13] shows that the S-transform of \(f^\infty_V(x)\) is equal to \(S_V(z) = 1/1 + z\). The S-transform of the final density is

\[
S_W(z) = \frac{1}{1 + \beta z} S_{R_{TX}}(z).
\]

(12)

Then we calculate \(S_{R_{TX}}(z)\) from \(f^\infty_{R_{TX}}(x)\) derived with specific form of a tilted semicircular law and the assumption of the
exponentially decaying correlation model \( \{ R_{TX} \}_{i,j} = \rho^{i-j} \) (see the solid and dash curves in Fig. 9). \( R_{TX} \) for the channel in the corridors is almost identical to that in the room, i.e. \( \rho \) at TX \( \approx 0.8 \) and \( \rho \) at RX \( \approx 0.2 \), which are applied for further calculations. Thus \( f_{R_{TX}}^{\infty} \) is given by [13]

\[
f_{R_{TX}}^{\infty}(x) = \frac{1}{2\pi \mu L^2} \sqrt{\left( \frac{x}{\sigma_1} - 1 \right) \left( 1 - \frac{x}{\sigma_2} \right)} 1_{[\sigma_1, \sigma_2]}(x),
\]

(13) where \( \mu = \frac{\sqrt{\rho}}{1 - \rho}, \sigma_1 = \frac{1 - \rho}{1 + \rho}, \sigma_2 = \frac{1 + \rho}{1 - \rho} \) and

\[
1_I(x) = \begin{cases} 
1, & x \in I \\
0, & \text{otherwise}.
\end{cases}
\]

First we compute the Stieltjes transform of \( f_{R_{TX}}^{\infty} \)

\[
m_{R_{TX}}(z) = \int_{-\infty}^{\infty} \frac{1}{z - x} f_{R_{TX}}^{\infty}(x) dx
\]

(14) and

\[
\psi_{R_{TX}}(z) = z^{-1} m_{R_{TX}}(z^{-1}) - 1.
\]

(15)

Then we obtain the S-transform \( S_{R_{TX}}(z) \) by

\[
S_{R_{TX}}(z) = \frac{z + 1}{z} \psi_{R_{TX}}^{-1}(z) = 1 - \mu z.
\]

(16)

Inserting (16) in (12) and take the inverse Stieltjes transform [13], then we get

\[
f_{W}^{\infty}(x) = \max \left\{ 0, 1 - \frac{1}{\beta} \right\} \delta_0(x/\beta) + \frac{1}{2\pi} \sqrt{\frac{\beta}{\mu}} \left( 1 + \frac{1}{\beta} \right) - 1_{[\xi_1, \xi_2]}(x).
\]

(17)

Replacing \( f_{W}(x) \) with \( f_{W}^{\infty}(x) \) in (8), we finally obtain a closed form for the asymptotic capacity per antenna for the transmit correlation case integrating \( E[C] \) with respect to \( \rho \) and forcing \( E[C]_{|\rho=0} = 0 \) as

\[
E[C] = \beta \log_2 \frac{\beta}{\mu} w \left( \frac{\beta}{\beta'}, \mu \right) + \frac{\beta}{\mu} \log_2 \left| 1 - \frac{1}{\beta} \right| u \left( \frac{\beta}{\beta'}, \mu \right) - \left( 1 - \beta \right) \log_2 \left| u \left( \frac{\beta}{\beta'}, \mu \right) \right|,
\]

(18)

where \( w, v, \) and \( u \) are given by

\[
u(\rho, \beta, \mu) = \frac{\beta + (1 - \beta) \rho + 2 \mu \rho^2 - \sqrt{\gamma}}{2 \beta (\rho - \mu)} - 2 \rho \left[ 1 + (1 + \beta) \mu + \beta \rho \right]^2,
\]

\[
u(\rho, \beta, \mu) = \frac{\beta + (1 - \beta) \rho + 2 \mu \rho^2 - \sqrt{\gamma}}{2 \beta (\rho - \mu)} - 2 \rho \left[ 1 + (1 + \beta) \mu + \beta \rho \right]^2,
\]

\[
u(\rho, \beta, \mu) = \frac{\beta + (1 + \beta) \rho + \sqrt{\gamma}}{2 \beta (\rho - \mu)} - 2 \rho \left[ 1 + (1 + \beta) \mu + \beta \rho \right]^2,
\]

\[and\]

\[
y = [\beta + (1 + \beta) \rho]^2 - 4 \beta (\rho - \beta) \mu.
\]

(19)

Fig. 10 illustrates the PDFs of the empirical channel eigenvalues of the measured and simulated channels when \( N_{RX} = N_{TX} = 8 \) compared to the asymptotic PDFs for the i.i.d channel \( f_{W}^{\infty}(x) \) and the TX correlation channel \( f_{W}(x) \). The asymptotic PDF \( f_{W}^{\infty}(x) \) seems to be a good approximation to those of the measured and simulated channels. Fig. 11 presents the comparison between the ergodic capacity per receive antenna \( C/N_R \) versus signal-to-noise ratio (SNR) for \( N_{RX} = N_{TX} = 8 \) and the asymptotic capacity per receive antenna \( C^{\infty}/N_R \) at \( \beta = 1 \). It seems that the asymptotic capacity is a reasonable approximation to those of the measured and simulated channels. The same kind of plots are shown in Fig. 12 for \( N_{RX} = 8, N_{TX} = 16 \) comparing with the asymptotic capacity per receive antenna \( C^{\infty}/N_R \) at \( \beta = 2 \).

VI. CONCLUSIONS

We investigated the MIMO channels in the outdoor-indoor environments. The fading correlation between the array elements at the receiver (RX) was low due to the existence of local scatterers around it. Moreover, the preliminary results
showed that the correlations at the TX and the RX were independent of each other. Therefore, the one-sided Kronecker model is reasonably suitable to be applied for our measured channels. Subsequently, the analytic equations of the asymptotic distribution of the channel eigenvalues and the ergodic channel capacity based on the one-sided Kronecker model were reviewed and analyzed with the measurement data. We showed that these closed-form solutions were also valid for measured channels and realistic number of antenna elements.

REFERENCES