Monte Carlo simulations. As shown, the estimate of the proposed algorithm without a local search was sufficiently close to the peak of the 3-D MUSIC spectrum.

Table 1: Mean and variance of distance between peak of 3-D MUSIC spectrum and estimate of proposed algorithm without local search

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range (½R)</td>
<td>Mean</td>
<td>0.04399</td>
<td>0.03635</td>
<td>0.03374</td>
</tr>
<tr>
<td></td>
<td>Var.</td>
<td>0.00107</td>
<td>0.00883</td>
<td>0.00858</td>
</tr>
<tr>
<td>Elevation angle (Deg.)</td>
<td>Mean</td>
<td>0.03874</td>
<td>0.03328</td>
<td>0.02960</td>
</tr>
<tr>
<td></td>
<td>Var.</td>
<td>0.00996</td>
<td>0.00808</td>
<td>0.00844</td>
</tr>
<tr>
<td>Azimuth angle (Deg.)</td>
<td>Mean</td>
<td>0.02577</td>
<td>0.02247</td>
<td>0.01658</td>
</tr>
<tr>
<td></td>
<td>Var.</td>
<td>0.00956</td>
<td>0.00804</td>
<td>0.00829</td>
</tr>
</tbody>
</table>

Table 2 lists the number of floating-point operations required to set the initial points for the local search with respect to the number of grids in the range using MATLAB function 'flops'. The Quasi-Newton method was used to find the peaks of the 2-D MUSIC spectrum required to initialise the proposed algorithm. For both the 2-D MUSIC and 3-D MUSIC, the grid size of the incident angles was smaller than the 3-D MUSIC. Furthermore, since most of the computations involved in the proposed algorithm are due to searching for the peaks of the 2-D MUSIC spectrum, an increase in the number of grids in the range did not cause much of an increase in the number of computations required by the proposed algorithm.

Table 2: Number of floating-point operations required to set initial points for local search

<table>
<thead>
<tr>
<th>Number of grids</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D MUSIC</td>
<td>93 708 033</td>
<td>93 733 493</td>
<td>93 784 738</td>
<td>93 887 264</td>
</tr>
<tr>
<td>Proposed</td>
<td>823 058 689</td>
<td>1 645 699 148</td>
<td>3 290 951 896</td>
<td>6 581 350 220</td>
</tr>
<tr>
<td>3-D MUSIC</td>
<td>93 266 396 (Independent of range grid)</td>
<td>93 733 493</td>
<td>93 784 738</td>
<td>93 887 264</td>
</tr>
</tbody>
</table>

Conclusion: An algorithm is proposed for localising 3-D near-field sources using a UCA, whereby the algebraic relations between the incident angles under the far-field assumption and the actual near-field location are used as paths to follow to the peak of the 3-D MUSIC spectrum. Simulation results demonstrate that the proposed algorithm is computationally efficient.

Acknowledgments: This work was supported in part by the Underwater Acoustic Research Center, SNU, Korea, and in part by the BK21 project.

References

Matched filtering for CMA-based blind channel estimation

B. Baykal and A.G. Constantinides

A novel blind matched filter receiver (MFR) is presented. The development depends on rearrangement of the optimum MFR and the use of the constant modulus algorithm (CMA) to estimate the matched filter. Using the blind MFR and CMA together, a novel blind channel estimation technique is presented.

Introduction: In this Letter we show how a matched filter receiver (MFR), i.e. the optimal receiver in the presence of inter-symbol-interference (ISI) and additive white Gaussian noise (AWGN) which requires explicit knowledge of the communications channel, can be turned into a blind MFR. At a first glance, it would seem impossible to use an MFR blindly without calling for a blind channel estimate obtained from rather complicated singular value decomposition or higher-order statistics based algorithms [1]. However, if the matched filter (MF) itself is made the sole unknown in the receiver it can be estimated effectively using much simpler approaches such as the constant modulus algorithm (CMA). The significant benefit that accrues from the estimate of the matched filter impulse response is that if the MF is estimated, then since the MF is the time-reversed and conjugated channel impulse response (CIR), we also obtain the CIR directly. This approach is fundamentally different from the existing methods [2, 3] in that matched filtering has not been considered yet in a blind framework. The main contributions of this work are: to show how an MFR can be used in a blind framework, particularly in blind channel estimation; and, to use the CMA as a direct CIR estimator. Standard matrix-vector notation is used with frequency domain quantities denoted as capital letters, k denotes discrete-time index and * sign refers to estimated quantities.

Fig. 1 Baseband model of time-invariant communications channel and its MFR where MLSE, DFE and LE are shown

Blind matched filter configuration: Referring to the symbol-rate sampled configuration in Fig. 1, the received signal can be expressed as

\[ r_k = \frac{g_k}{G(z)} h_d k + n_k \quad (1) \]

where \( L \) is the channel memory and \( h = [h_0, \ldots, h_L]^T \) is the symbol-spaced time-invariant channel impulse response (CIR). An MFR is an optimal receiver that minimises the bit error probability in the presence of ISI and AWGN. It consists of an MF, whitening filter (WF) \( w(z) \), and maximum-likelihood sequence estimation (MLSE). The MLSE is applied to the combined MF + WF filtered channel output, \( y_k \), using the so-called transformed channel impulse response (TCIR) \( f(z) \) to compute path metrics when the switch SW in Fig. 1 is in position 1. A suboptimal MFR configuration, namely a decision feedback equaliser (DFE) can be developed when switch SW in Fig. 1 is in position 2, by choosing the feedback filter (FBF) \( g(z) \) such that \( G(z) = f(z) - 1 \).
moved to the position 'LE' in Fig. 1, the linear equaliser (LE) configuration is obtained. In all these configurations, WF, which is anticausal and infinite impulse response (IIR), is approximated as a causal finite impulse response (FIR) filter. Usually the length of the WF is chosen such that $N_f > 5L$. Note that explicit knowledge of the channel is required to be able to use the MFR configurations. With the following rearrangement of the filters, the system can be turned into a blind MFR. Indeed, WF and FBF are moved to the front end and they process the channel output as shown in Fig. 2a for blind DFE and Fig. 2b for blind LE. Then the MF, the length of which is $N_m - L + 1$, is applied to the processed signal. The WF and FBF can be computed without explicit knowledge of the channel as:

$$
\begin{bmatrix}
\psi_{-N_c,-N_d} & \cdots & \psi_{-N_c,0} & \psi_{-N_c,1} & \cdots & \psi_{0,-N_d} & \psi_{0,0} & \cdots & \psi_{0,1} & \cdots & \psi_{N_c,-N_d} & \psi_{N_c,0} & \cdots & \psi_{N_c,1} & \cdots & \psi_{N_c,N_d}
\end{bmatrix}
\begin{bmatrix}
\rho_{-N_c} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\rho_0 \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\rho_{N_c}
\end{bmatrix}
= 
\begin{bmatrix}
\rho_{-N_c} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\rho_0 \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\rho_{N_c}
\end{bmatrix}
$$

(2)

where $N_c$ is the WF length, $\psi_{i,j} = \sum_{n=0}^{N_c} \rho_n r_{-n-i} r_{-n-j}$, $i = -N_c, \ldots, 0$ and $\rho_i = E(|r|^2) - \sigma_i^2$, $\sigma_i^2 = E(|h|^2)$, and $f_j = \sum_{i=0}^{N_c} w_i r_{-n-i}$, $i = 0, \ldots, L$.

The WF computed in (2) is anticausal and hence should be delayed to obtain a causal WF response. The length of the TCF is $N_f = L + 1$. The FBF $g$ is such that the first tap of $g$, i.e. $g_0$ is removed, i.e. $G(z) = F(z)$. Then the length of $g$ is $N_m = L$. The channel correlation $\rho_i$ can be estimated in the receiver using sample averaging as follows, with particular attention on $\rho_0$:

$$
\hat{\rho}_i = \frac{1}{N_c} \sum_{n=0}^{N_c} x_{n-i} r_{-n-i} \quad i \neq 0
$$

(4)

$$
\hat{\rho}_0 = \frac{1}{N_c} \sum_{n=0}^{N_c} x_{n} r_{-n} - \sigma_i^2
$$

(5)

where the AWGN power $\sigma_i^2$ estimation must be performed, for instance using eigenvalue decomposition of the autocorrelation matrix of the fractionally spaced channel output and selecting the minimum eigenvalue. Note that autocorrelation estimation and AWGN power estimation are fairly well-established in the literature [1]. Time dependence of estimated quantities is not shown in (4) and (5) as the sample averaging is made using a fixed time instant $k_0$. This is feasible in stationary channels. However if the channel is time-varying, estimated quantities will be dependent on time and should be updated as the channel conditions vary, implying that the WF and FBF, which are time-invariant in stationary channels, should also be updated. Hence, when the autocorrelation estimates are updated in (4) and (5), then WF and FBF should be recalculated. The MF can be estimated using any blind equalisation technique. The CMA turns out to be a simple alternative to a quasi-minimum mean square estimation (MMSE) solution [6].

Blind channel estimation technique using CMA: The CMA converges to a quasi-minimum mean square estimation (MMSE) solution [6] due to AWGN which necessitates further processing on the MF to obtain the channel estimate. The quasi-MMSE solution that CMA provides estimation $\hat{h}$ of the channel output $h$ is computed in each run of the algorithm where $\hat{h}$ is obtained as in (7) and $\phi = 0, 0.5$. Monte Carlo trials are performed. The CIR against SNR plot is shown in Fig. 3. Channel autocorrelation estimates are computed using the first 2000 samples of the channel output, i.e. $N_c = 2000$ in (4) and (5). AWGN power is computed as selecting the minimum eigenvalue of the autocorrelation matrix of the fractionally spaced version of the channel output $x_k$. CMA is let run over 10,000 samples. A satisfactory performance is obtained from the proposed method, it can be observed that the method in (7) provides estimation error reduction as much as 20 dB. Algorithms that are based on optimisation of convex functions of higher order statistics, e.g. cumu-
Stochastic resonance and data processing inequality

M.D. McDonnell, N.G. Stocks, C.E.M. Pearce and D. Abbott

The data processing inequality of information theory proves that no more information can be obtained out of a set of data than was there to begin with. However, many papers in the field of stochastic resonance report signal to noise ratio gains in some nonlinear systems due to the addition of noise. Such an observation appears on the surface to contradict the data processing inequality. It is demonstrated that the data processing inequality is upheld for the case of a periodic input signals.

Introduction: The data processing inequality (DPI) states that given random variables $X$, $Y$ and $Z$ that form a Markov chain in the order $X \rightarrow Y \rightarrow Z$, then the mutual information between $X$ and $Y$ is greater than or equal to the mutual information between $X$ and $Z$ [1]. That is $I(X; Y) \geq I(X; Z)$. In other words, no signal processing on $Y$ can increase the information that $X$ contains about $Y$. However, in the field of stochastic resonance (SR) [2, 3] many papers report that it is possible to obtain a signal to noise ratio (SNR) gain in some nonlinear systems by the addition of noise [4, 5]. To a casual observer, this would seem to contradict the DPI.

Background: An SNR gain is not in itself a remarkable thing; SNR gains are routinely obtained by filtering. However in the SR literature, the reported SNR gains are said to be due to stochastic resonance, rather than a deliberately designed filter, which is why the SNR gains are taken to be quite remarkable. SR is the term given to the phenomenon where the optimal output of a nonlinear system occurs for nonzero noise in that system. It was at first thought to occur only in bistable dynamical systems, generally driven by a periodic input signal and broadband noise. The work on such systems showed that the ratio of the output power at the input frequency to the background noise spectral density at the frequency could be maximised by a nonzero value of noise intensity.

It has been proven using linear response theory that for the case of stationary Gaussian noise and a signal that is small compared to the noise, that for nonlinear systems the SNR gain must be less than or equal to unity, and that hence no SNR gain can be induced by utilising the SR effect [6]. Once this fact was established, researchers still hoping to be able to find systems in which SNR gains due to noise could occur, turned their attention to situations not covered by the proof, that is, the case of a signal that is not small compared to the noise, or broadband signals or non-Gaussian noise.

For broadband or aperiodic input signals, SNR is not appropriate, and methods such as cross-correlation [2] and mutual information [7] are the most commonly used to show that SR can occur for nonperiodic signals. However to compare the detectability at the output to the input for broadband input signals, a measure analogous to the input–output SNR ratio for periodic input signals is required. One such possible measure is channel capacity. This measure has been used by Chapeau-Blondeau [4], who stated that comparing the channel capacity at the input and output for a periodic input signal was analogous to a comparison of the input and output SNRs for periodic input signals. His work indicated that it was possible for the channel capacity at the output to exceed that at the input. This appears prima facie to be a clear contradiction of the DPI, and we resolve this in the following.

Model: Consider a system where a signal, $s$, is subject to independent additive random noise, $n$, to form another random signal, $y = s + n$. The signal $y$ is then subjected to a nonlinear function, $g$, to give a final random signal, $z = g(y)$. Since we have $z = g(y)$, $z$ is conditionally independent of $s$, our model forms a Markov chain and therefore the DPI applies. We consider the case of $s$ consisting of a random binary pulse train. Hence, our model is similar to those reported to show SNR gains for aperiodic input signals [4].

Example: Let the input $s$ take on values $\pm s_0$. The output, $z$, is one of three values ($s_0$, $-s_0$ and 0) and is determined by two thresholds, $\pm \theta$ such that $g(y)$ is given by

$$z = g(y) = \begin{cases} s_0 & \text{if } y = s_0 + n > \theta \\ -s_0 & \text{if } y = s_0 + n < -\theta \\ 0 & \text{otherwise} \end{cases}$$

where an output value of zero indicates the complete erasure of an input value, rather than its corruption.

Fig. 1 Channel capacity against RMS noise amplitude for three cases

If we have $\pm \theta = 0$, and a probability of error given the input, $p_e = p(z = s_0 | x = s_0) = p(z = s_0 | x = -s_0)$ then the channel is a binary symmetric channel, for which it is known that channel capacity occurs when $p(z = s_0) = 0.5$ and is given by $C_0 = 1 + p_e \log_2 (p_e) + (1 - p_e) \log_2 (1 - p_e)$ [1]. If the noise in the channel has an even